Electrodynamics II

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Induced Electric Field

- Electrostatic rule

$$\vec{\nabla} \times \vec{E} = 0 \text{ (Electrostatics)}$$

- Generalization of this rule to Faraday's Law is

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

- Divergence of $\vec{E}$ is still given by Gauss's Law $\vec{\nabla} . \vec{E} = \frac{\rho}{\epsilon_0}$

- If $\vec{E}$ is purely Faraday Field, then

$$\vec{\nabla} . \vec{E} = 0$$

Think!
What is Faraday Field?

Comparison

Amper's Law

- $\vec{\nabla}.\vec{B} = 0$
- $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$
- $\vec{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\boldsymbol{r}') \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'$

Faraday's Law

- $\vec{\nabla}.\vec{E} = 0$
- $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
- By using the same token, we can write $\vec{E}$

$$\vec{E}(\boldsymbol{r}) = \frac{1}{4\pi} \int \frac{\left(-\frac{\partial \vec{B}}{\partial t}\right) \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'$$

Comparison

Amper's Law

- *Ampere's law in integral form*

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 I_{enc}$$

Faraday's Law

- $\vec{E}(\boldsymbol{r}) = \frac{-1}{4\pi}\int \frac{\left(\frac{\partial \vec{B}}{\partial t}\right)\times\hat{\mathscr{r}}}{\mathscr{r}^2}d\tau'$
- $\vec{E}(\boldsymbol{r}) = \frac{-1}{4\pi}\frac{\partial}{\partial t}\int \frac{\vec{B}\times\hat{\mathscr{r}}}{\mathscr{r}^2}d\tau'$
- Faraday's Law in integral form

$$\int \vec{\nabla}\times\vec{E}\,.\overrightarrow{da} = -\int \frac{\partial \vec{B}}{\partial t}.\overrightarrow{da}$$

$$\oint \vec{E}.\overrightarrow{dl} = -\frac{d\emptyset}{dt}$$

The rate of change of magnetic flux plays the same role as the $\mu_0 I_{enc}$ in Ampere's Law

Example 7.7. A uniform magnetic field **B**(t), pointing straight up, fills the shaded circular region of Fig. 7.25. If **B** is changing with time, what is the induced electric field?

$$\oint \vec{E}.\overrightarrow{dl} = -\frac{d\emptyset}{dt}$$

The induced electric field be in such a direction that it will opposes the magnetic field.

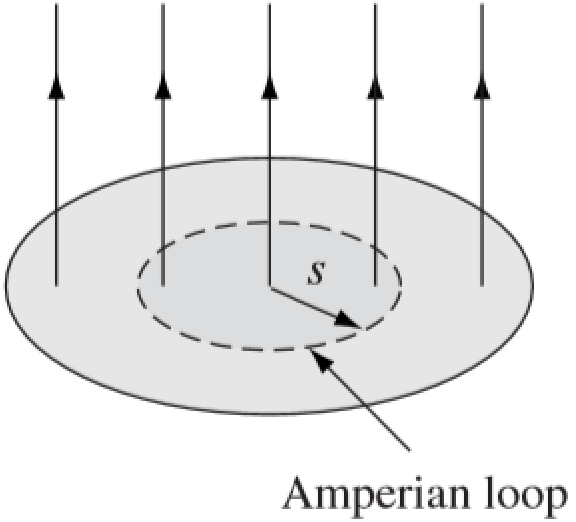

FIGURE 7.25

At any time t flux passig through Amperian Loop is

$$\emptyset = B\,A(area\ of\ Amperian\ loop)$$

$$\emptyset = B(t)\pi s^2$$

$$\frac{d\emptyset}{dt} = \frac{d\,(B(t)\pi s^2)}{dt} = \pi s^2 \frac{dB(t)}{dt}$$

$\vec{E}$ and $\overrightarrow{dl}$are in same direction so

$$\oint \vec{E}.\overrightarrow{dl} = E\oint dl \ = E(2\pi s)$$

Using the Faraday's Law $\oint \vec{E}.\overrightarrow{dl} = -\frac{d\emptyset}{dt}$

$$E(2\pi s) = -\pi s^2 \frac{dB(t)}{dt}$$

$$E = -\frac{s}{2}\frac{dB(t)}{dt}$$

$$\vec{E} = -\frac{s}{2}\frac{dB(t)}{dt}\hat{\emptyset}$$