Electrodynamics II

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Example 5.10

A toroidal coil consists of a circular ring, or "donut," around which a long wire is wrapped (Fig. 5.38). The winding is uniform and tight enough so that each turn can be considered a closed loop. The cross-sectional shape of the coil is immaterial. I made it rectangular in Fig. 5.38 for the sake of simplicity, but it could just as well be circular or even some weird asymmetrical form, as in Fig. 5.39, just as long as the shape remains the same all the way around the ring. In that case it follows that the *magnetic field of the toroid is circumferential at all points, both inside and outside the coil.*

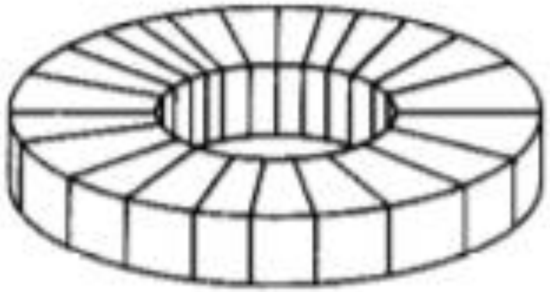


Figure 5.38

Example 5.10

According to Biot-Savart law, the field at **r** due to current element at **r'** is

$$d\vec{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\frac{\vec{I}\times\hat{\mathscr{r}}}{\mathscr{r}^2}dl' = \frac{\mu_0}{4\pi}\frac{\vec{I}\times\vec{\mathscr{r}}}{\mathscr{r}^3}dl'$$

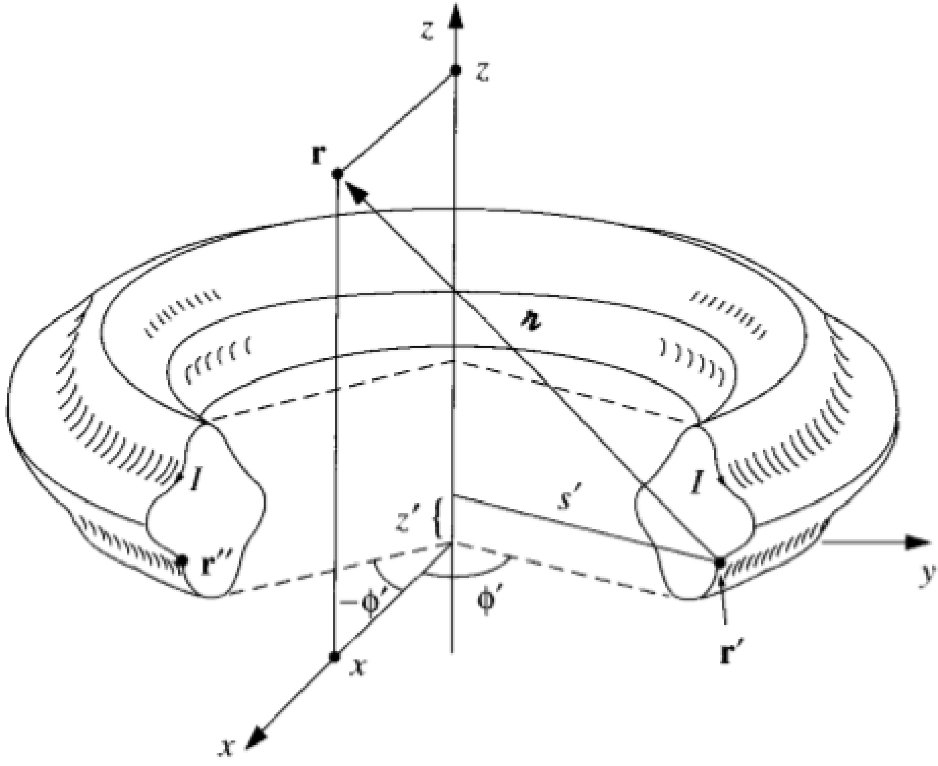


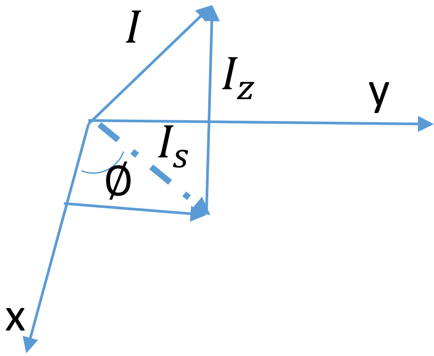
Figure 5.39

We may as well put **r** in xz plane (fig.5.39)

So its Cartesian coordinates are $\vec{r} = (x, 0, z)$

While the source coordinates are

$$\overrightarrow{r'} = (s'cos\emptyset', s'sin\emptyset', z') \qquad \vec{r} = (x, 0, z)$$



Example 5.10

$$\vec{r} = (x - s'cos\emptyset', -s'sin\emptyset', z - z')$$

Since the current has no $\emptyset$ component. $\vec{I} = I_s\hat{s} + I_z\hat{z}$
or in cartisian coordinates

$$\vec{I} = (I_s cos\emptyset', I_s sin\emptyset', I_z)$$

Example 5.10

$$\vec{I} \times \vec{r} = \begin{bmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ I_s \cos\phi' & I_s \sin\phi' & I_z \\ (x - s'\cos\phi') & (-s'\sin\phi') & (z - z') \end{bmatrix}$$

$$= [\sin\phi'(I_s(z - z') + s' I_z)]\,\hat{\mathbf{x}}$$

$$+ [I_z(x - s'\cos\phi') - I_s\cos\phi'(z - z')]\,\hat{\mathbf{y}} + [-I_s x \sin\phi']\,\hat{\mathbf{z}}.$$

Example 5.10

But there is a symmetrically situated current element at $\vec{r}''$,with the same s', the same $\mathscr{r}$, the same dl', the same I_s, the same I_z, but the negative $\emptyset'$ (Fig. 5.39) Because $sin\emptyset'$changes sign, the $\widehat{x}$ and $\hat{z}$ contributions from $\vec{r}'$ and $\vec{r}''$ cancel, leaving Only a $\hat{y}$ term.
Thus the field at $\vec{r}$ is in the $\hat{y}$ direction, and in general the field points in the $\widehat{\emptyset}$ direction.

Now field inside the toroid is circumferential. So applying the Ampere's Law

$$\oint \vec{B}.\overrightarrow{dl}=\mu_0 I_{enc}$$

$$B2\pi s = \mu_0 I_{enc}$$

Example 5.10

and hence

$$\mathbf{B}(\mathbf{r}) = \begin{cases} \dfrac{\mu_0 N I}{2\pi s}\hat{\boldsymbol{\phi}}, & \text{for points inside the coil,} \\ 0, & \text{for points outside the coil,} \end{cases} \tag{5.58}$$

where N is the total number of turns.

Problem 5.14 A thick slab extending from $z = -a$ to $z = +a$ carries a uniform volume current $\mathbf{J} = J\,\hat{\mathbf{x}}$ (Fig. 5.41). Find the magnetic field, as a function of z, both inside and outside the slab.

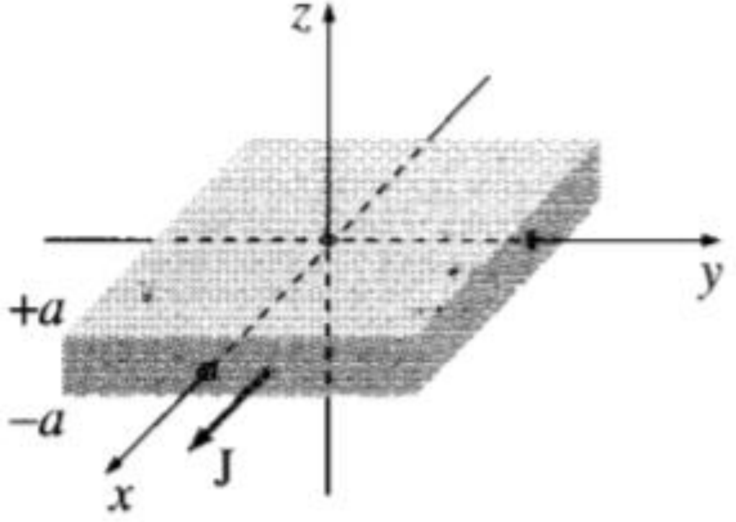


Figure 5.41

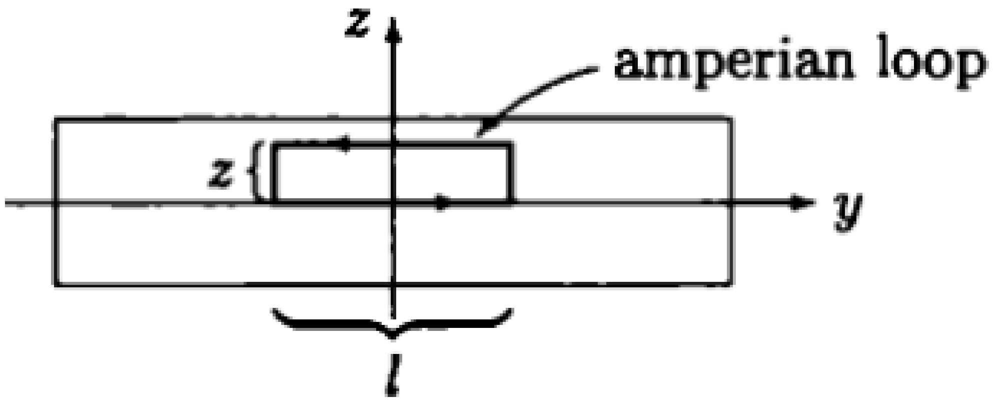
Problem 5.14

Inside the Slabe

By the right hand rule, the field points in the $-\hat{y}$ direction for z>0, and in the $+\hat{y}$ direction for Z<0

At z=0, B=0. Use the amperian loop shown:

$$\oint \vec{B}.\vec{dl} = Bl = \mu_0 I_{enc} = \mu_0 lzJ$$



$\vec{B} = -\mu_0 Jz\hat{y}$ For $(-a < z < a)$

Problem 5.14

Outside the Slab

For z $> a$

$$I_{enc} = \mu_0 l a J$$

$$\oint \vec{B}.\overrightarrow{dl} = Bl = \mu_0 I_{enc} = \mu_0 l a J$$

so $\mathbf{B} = \begin{cases} -\mu_0 J a\,\hat{\mathbf{y}}, & \text{for } z > +a; \\ +\mu_0 J a\,\hat{\mathbf{y}}, & \text{for } z > -a. \end{cases}$