Subject: Electrodynamics II

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Lecture Content

- Problem 5.4 (Magnetostatics)

Student Assignment:

Problem 5.7 (Magnetostatics)

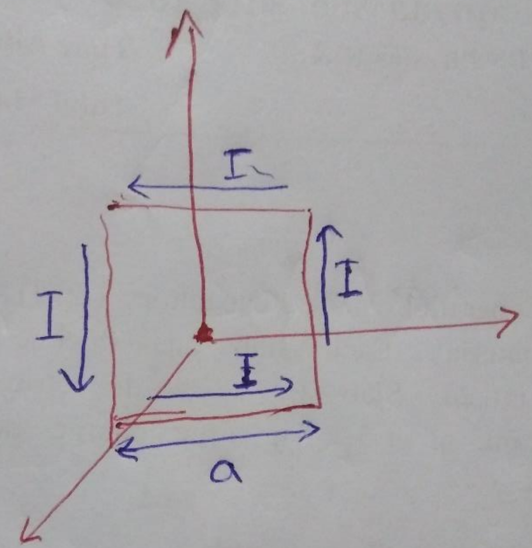
Problem 5.4 Suppose that the magnetic field in some region has the form

$$\mathbf{B} = kz\,\hat{\mathbf{x}}$$

(where k is a constant). Find the force on a square loop (side a), lying in the yz plane and centered at the origin, if it carries a current I, flowing counterclockwise, when you look down the x axis.

Pr. 5.4 solution

Formula $\vec{F} = I \int d\vec{l} \times \vec{B}$



Fig

As $d\vec{l}$ is perpendicular to magnetic field $\vec{B}$

$$|d\vec{l} \times \vec{B}| = dl\,B \sin 90$$
$$= dl\,B$$

so $|\vec{F}| = I\int dl\, B$

$= IB\int dl$

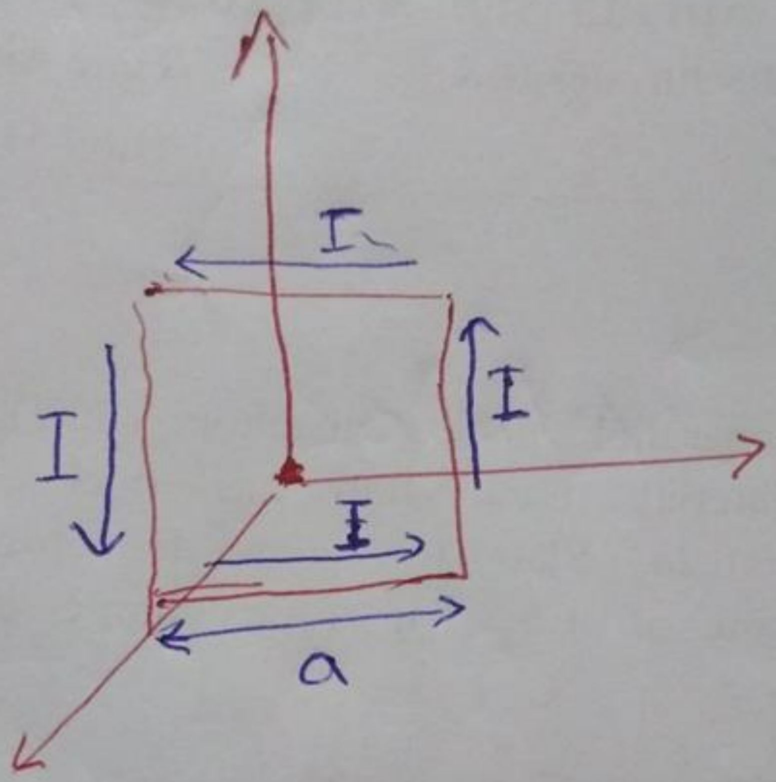
$\int dl =$ length of one side of loop $= a$

So force acting on one side of loop is

$|\vec{F}| = IBa$

In vector form

$\vec{F} = I(\vec{a} \times \vec{B})$



Fig

For upper side

$$\vec{a} = a\hat{y}$$

$$\vec{B} = kz\hat{x}$$

$$= k\left(\frac{a}{2}\right)\hat{x} \quad \because z = \frac{a}{2}$$

$$\vec{F} = I\left(\vec{a} \times \vec{B}\right)$$

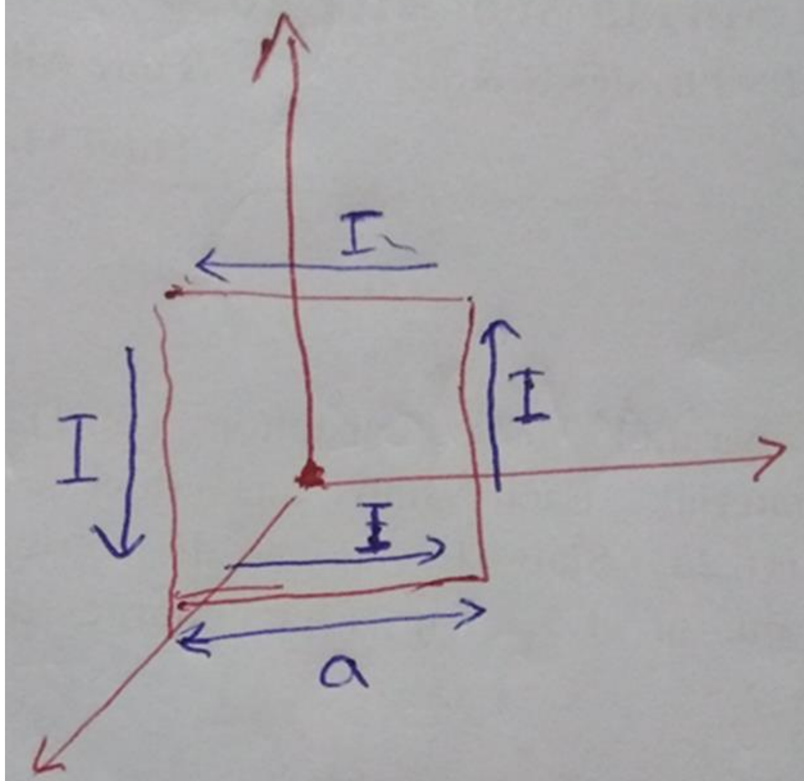
$$= I\left(a\hat{y} \times \frac{ka}{2}\hat{x}\right)$$

$$= -Ik\frac{a^2}{2}\left(\hat{y} \times \hat{x}\right)$$

$$= -\frac{Ika^2}{2}\left(-\hat{z}\right)$$

$$\boxed{\vec{F} = \frac{Ika^2}{2}\hat{z}} \quad (i)\ \hat{x} \times \hat{y} = 1\hat{z}$$

(Force is upward).



Fig

$$\vec{F} = I(\vec{a} \times \vec{B})$$

$$\vec{a} = a\hat{y}$$

$$\vec{B} = kz\hat{x}$$

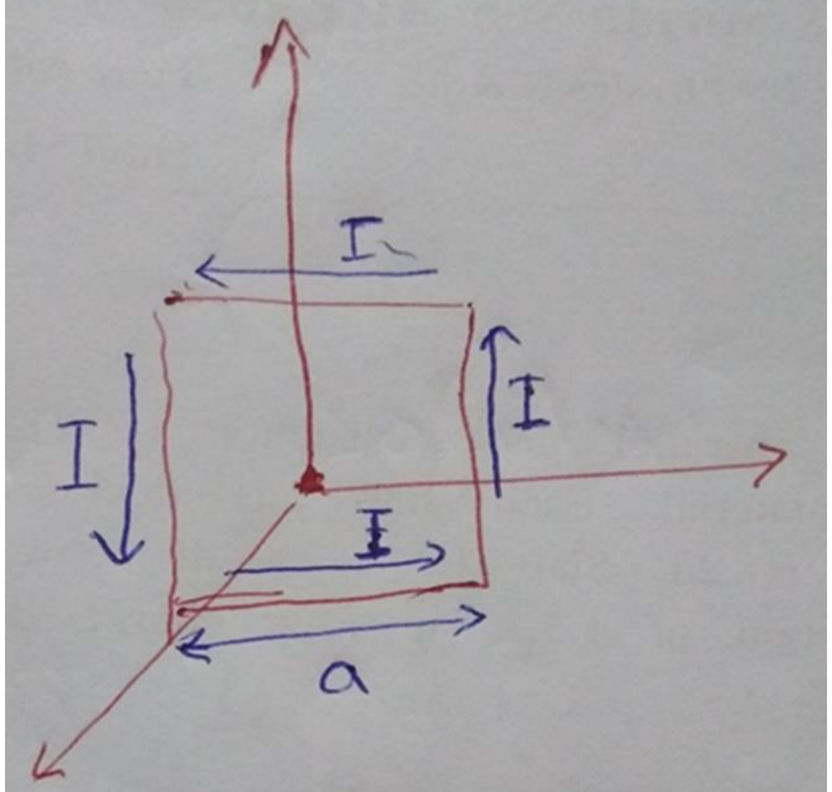
$$= -k\frac{a}{2}\hat{x} \quad \because z = -\frac{a}{2}$$

$$\vec{F} = I\left(a\hat{y} \times \left(-k\frac{a}{2}\hat{x}\right)\right)$$

$$= -\frac{Ia^2k}{2}(\hat{y} \times \hat{x})$$

$$= -\frac{Ia^2k}{2}(-\hat{z}) \rightarrow \text{(ii)}$$

$$\boxed{\vec{F} = \frac{Ia^2k}{2}\hat{z}}$$ (upward force)



Fig

For left side

$$\vec{F} = I \int (d\vec{l} \times \vec{B})$$

$$= I \int dl (-\hat{z}) \times kz\,\hat{x}$$

$dl = dz$

$$= Ik \int_{-a/2}^{a/2} z\,dz\,(-\hat{z} \times \hat{x})$$

$$= -Ik \int_{-a/2}^{+a/2} z\,dz\,\hat{y}$$

$$\boxed{\vec{F} = -Ik \int_{-a/2}^{+a/2} z\,dz\,\hat{y}}$$

For Right Side

Similarly

$$F = Ik \int_{-a/2}^{a/2} z\,dz\,\hat{y}$$

In this case $dl = dz\,\hat{z}$

The force on left side (towards the left) cancel the force on right side (toward right)

So Net force is (from (i) and (ii))

$$\vec{F} = \frac{Ika^2\hat{z}}{2} + \frac{Ia^2k}{2}\hat{z}$$

$$\boxed{\vec{F} = Ika^2\hat{z}}$$

Thank You