Subject: Electrodynamics II

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Lecture Content

- Example 5.6, Problem 5.8 (Magnetostatics)

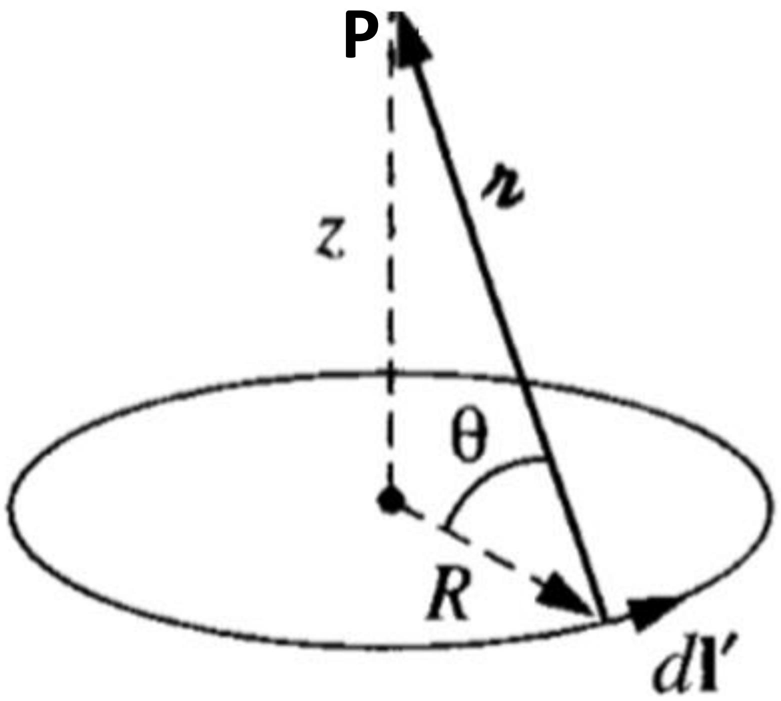
Student Assignment:

- Problem 5.9, 5.10

(Magnetostatics)

Example 5.6

Find the magnetic field a distance z above the center of a circular loop of radius R, which carries a steady current I (Fig. 5.21).



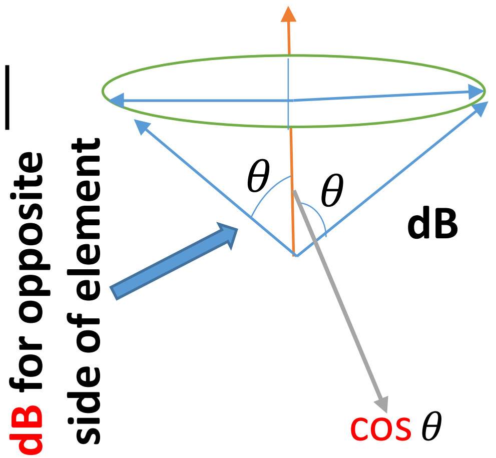
Formula Used:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l}' \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}.$$

Mentally split the whole circular wire into small elements, Each element $d\mathbf{l}'$ produces the magnetic field $d\mathbf{B}$

$$|d\mathbf{l}' \times \hat{\boldsymbol{\mathscr{r}}}| = |d\mathbf{l}'| \;\Longrightarrow\; |d\boldsymbol{B}| = \frac{\mu_0 I}{4\pi R}|d\acute{l}|$$

$$|\mathbf{B}(\mathbf{r})| = \frac{\mu_0}{4\pi} I \int \frac{|d\mathbf{l}'|}{\mathscr{r}^2}.$$



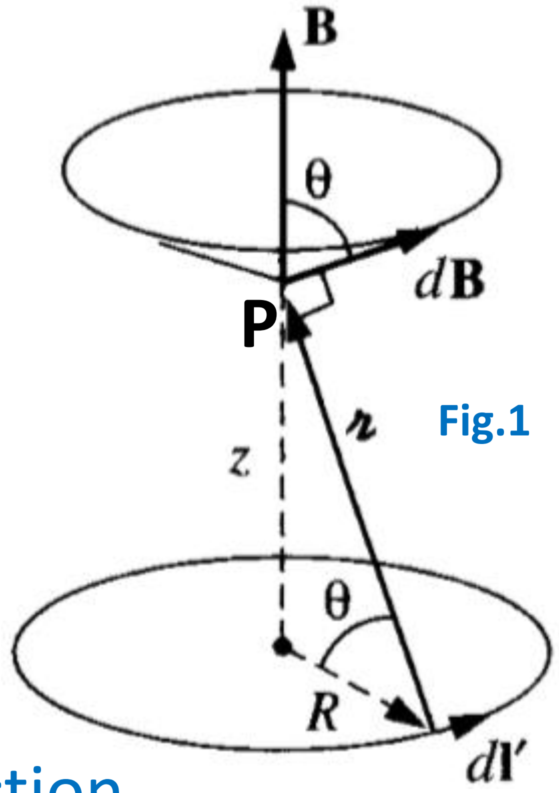


Fig.1

And direction of magnetic field $d\mathbf{B}$ is from point P to space making an angle θ with the Z axis.

Vertical components of all elements are towards the Z direction and horizontal components of all elements are cancelled to each Another. So all components along axis are added i.e.

$$B(z) = |\mathbf{B}(\mathbf{r})| \cos\theta = \frac{\mu_0}{4\pi} I \int \frac{dl'}{\mathscr{r}^2} \cos\theta$$

$$B(z) = \frac{\mu_0}{4\pi} I \int \frac{dl'}{r^2} \cos\theta$$

r^2 and $\cos\theta$ are constant can be taken out of integral.

$$B(z) = \frac{\mu_0}{4\pi} I \frac{\cos\theta}{r^2} \int dl'$$

$\int dl'$ is the length of circular wire $= 2\pi R$

From the figure 1 we get value of

$$cos\theta = \frac{R^2}{(R^2 + z^2)^{1/2}}$$

So we get

$$B(z) = \frac{\mu_0 I}{4\pi} \left(\frac{\cos\theta}{r^2} \right) 2\pi R = \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}}.$$

Problem 5.8

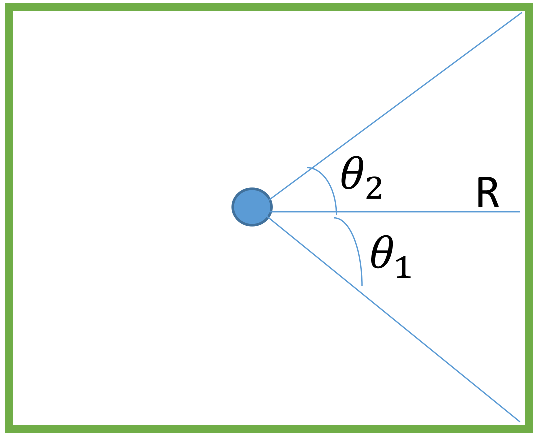
(a) Find the magnetic field at the center of a square loop, which carries a steady current I. Let R be the distance from center to side (Fig. 5.22).

(b) Find the field at the center of a regular n-sided polygon, carrying a steady current I. Again, let R be the distance from the center to any side.

(c) Check that your formula reduces to the field at the center of a circular loop, in the limit $n \to \infty$.

Formulas Used

$$B = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1).$$



$$z = R, \theta_2 = -\theta_1 = 45°$$

$$\theta_1 = -\theta_2 = \frac{\pi}{4}$$

$$sin\theta_1 = \frac{1}{\sqrt{2}} \quad , \quad sin\theta_1 = \frac{-1}{\sqrt{2}}$$

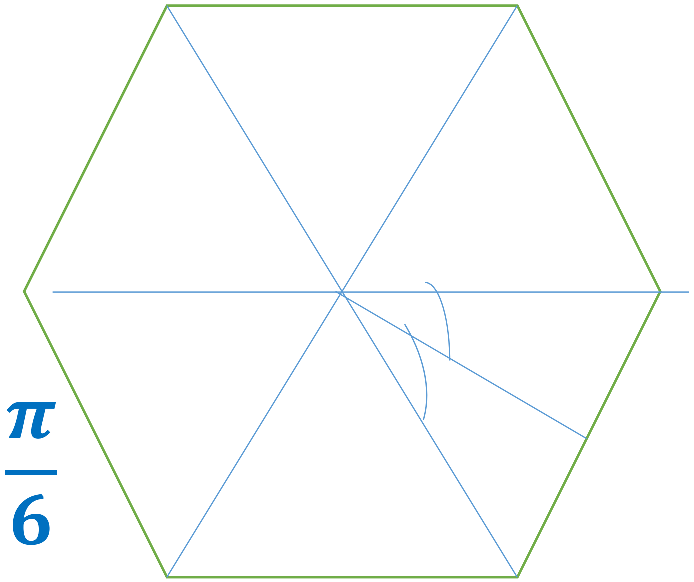
$$= \frac{\mu_0 I}{4\pi R}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

For one side

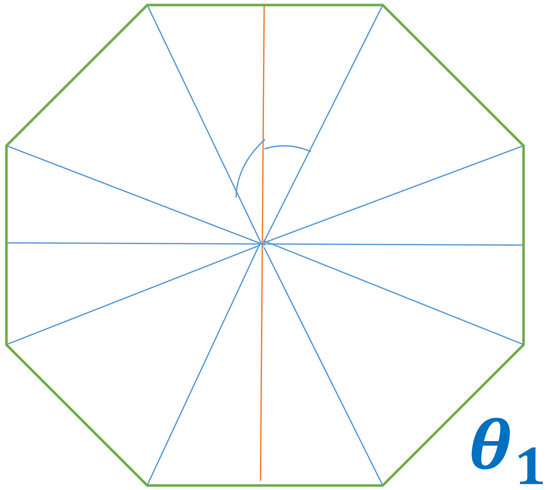
For four side, Result is multiplied with 4

$$B = \boxed{\frac{\sqrt{2}\mu_0 I}{\pi R}}.$$

(b) Find the field at the center of a regular n-sided polygon, carrying a steady current I. Again, let R be the distance from the center to any side.



$$\theta_1 = -\theta_2 = \frac{\pi}{6}$$



$$\theta_1 = -\theta_2 = \frac{\pi}{8}$$

For Polygon

$$z = R, \ \theta_2 = -\theta_1 = \frac{\pi}{n},$$

$$B = \frac{\mu_0 I}{4\pi s} 2 \ \sin(\pi/n).$$

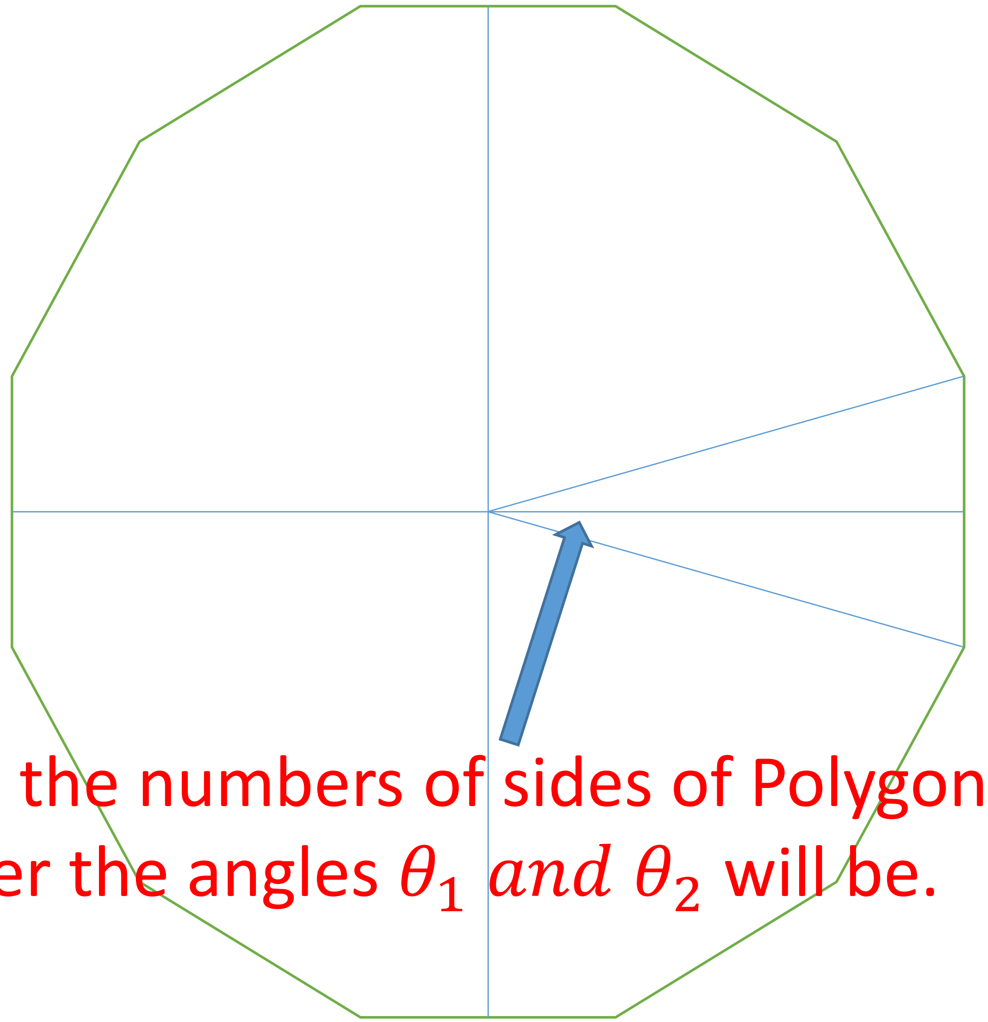
For n sides, Total magnetic field will be n times

$$B = \boxed{\frac{n\mu_0 I}{2\pi R} \sin(\pi/n).}$$

(c) Check that your formula reduces to the field at the center of a circular loop, in the limit $n \to \infty$.

(c) For small θ, $\sin\theta \approx \theta$. So as $n \to \infty$,

$$B \to \frac{n\mu_0 I}{2\pi R}\left(\frac{\pi}{n}\right) = \boxed{\frac{\mu_0 I}{2R}}$$



The more the numbers of sides of Polygon, the smaller the angles θ_1 and θ_2 will be.