Electrodynamics-II

By Muhammad Amer Mustafa
UOS, Sub Campus Bhakkar

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The Biot-Savart Law

Stationary charges	$\Rightarrow$	**constant electric fields: electrostatics.**
Steady currents	$\Rightarrow$	**constant magnetic fields: magnetostatics.**

- By steady current I mean a continuous flow that has been going forever
- Notice that a moving point charge cannot be a steady current

The Biot-Savart Law

1.1 Steady Currents

Stationary charges	$\Rightarrow$	constant electric fields: electrostatics.
Steady currents	$\Rightarrow$	constant magnetic fields: magnetostatics.

- By steady current I mean a continuous flow that has been going forever
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1.1 Steady Currents

Continuity Equation, that we have derived it previously, is

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

When a steady current flows in a wire, its magnitude I must be the same all along the line; otherwise, charge would be piling up somewhere, and it wouldn't be a steady current.

In case of steady state currents

$$\partial \rho / \partial t = 0$$

Thus, Equation of Continuity becomes

$$\nabla \cdot \mathbf{J} = 0.$$

1.2 Magnetic field of a steady current

The magnetic field of steady line current is given by Biot-Savart Law

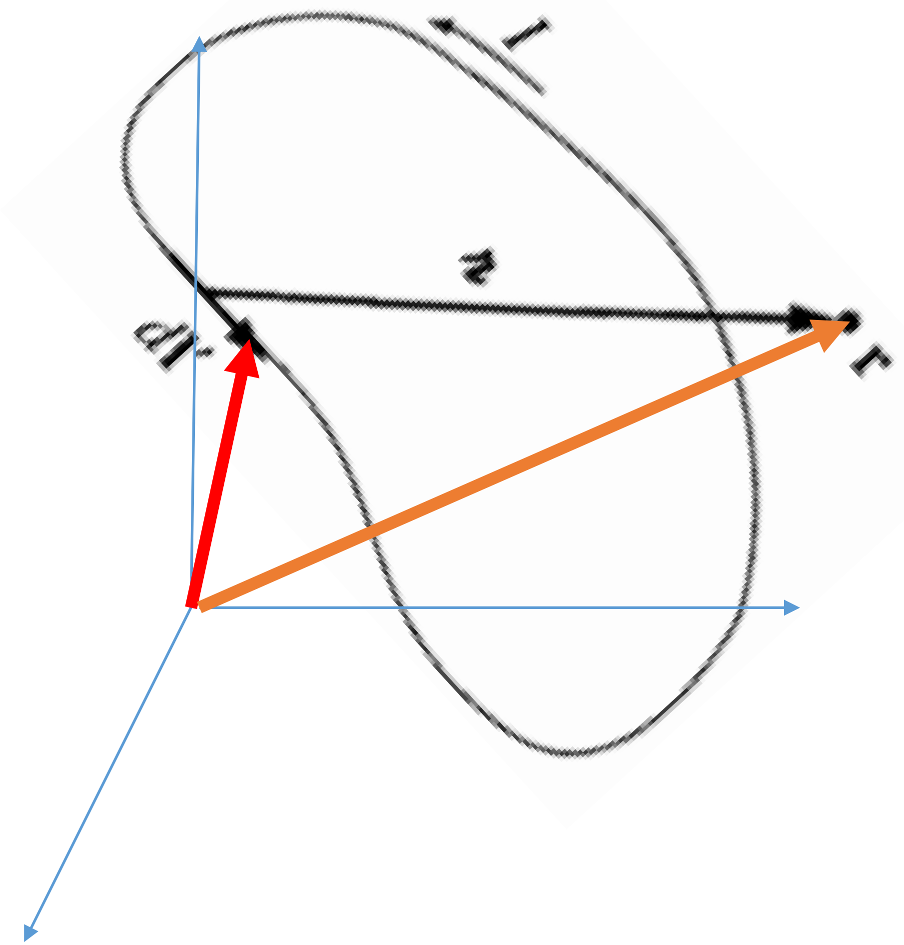
$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{I}\times\hat{\boldsymbol{\imath}}}{\imath^2}\,dl' = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l}'\times\hat{\boldsymbol{\imath}}}{\imath^2}.$$

Where

$$\mu_0 = 4\pi\times10^{-7}\ \mathrm{N/A^2}$$

Unit of Magnetic Field is

$$1\ \mathrm{T} = 1\ \mathrm{N/(A\cdot m)}$$



Resemblence Between Coloumb’s Law and Biot-Savart Law

Coloumb’s Law	**Biot-Savart Law**
Starting point of Electrostatics	Starting point of Magnetostatics
Used to calculate the Electric Field	Used to calculate the Magnetic Field
$1/r^2$ dependence	$1/r^2$ dependence

Example 5.5

Find the magnetic field a distance s from a long straight wire carrying a steady current I

SOLUTION

Formula Used

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{I}\times\hat{\mathbf{r}}}{r^2}\,dl' = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l}'\times\hat{\mathbf{r}}}{r^2}.$$

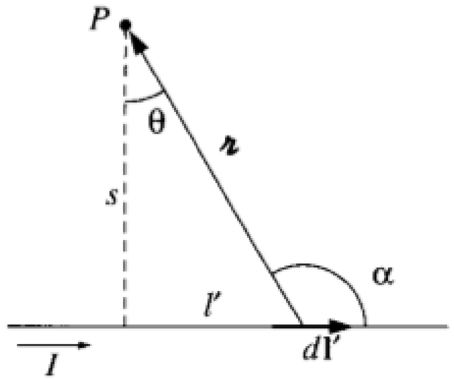


Fig.1

Using Fig.1

$\hat{\mathbf{r}}$

$$d\mathbf{l}'\times\hat{\mathbf{r}} = d\mathbf{l}'\sin\alpha = d\mathbf{l}'\sin(90-\theta) = d\mathbf{l}'\cos\theta$$

and $l'/s = \tan\theta \Rightarrow l' = s\tan\theta$

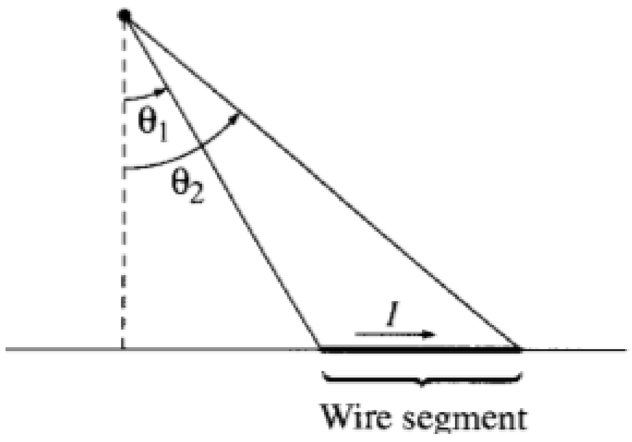
$\Rightarrow d\mathbf{l}' = s\sec^2\theta\, d\theta$

$\Rightarrow d\mathbf{l}' = s/\cos^2\theta$

Example 5.5

$s = \mathscr{r}\cos\theta \;\rightarrow\; \boxed{\dfrac{1}{\mathscr{r}^2} = \dfrac{\cos^2\theta}{s^2}.}$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{I}\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,dl' = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l}'\times\hat{\mathscr{r}}}{\mathscr{r}^2}.$$



Putting the values of all variables

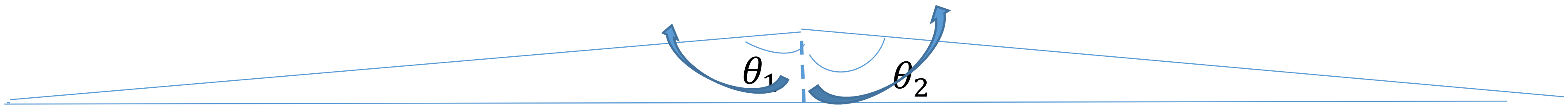
$$B = \frac{\mu_0 I}{4\pi}\int_{\theta_1}^{\theta_2}\left(\frac{\cos^2\theta}{s^2}\right)\left(\frac{s}{\cos^2\theta}\right)\cos\theta\,d\theta$$

$$= \frac{\mu_0 I}{4\pi s}\int_{\theta_1}^{\theta_2}\cos\theta\,d\theta = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1).$$

Which is value of magnetic field for segment of wire as indicated in the figure

Example 5.5

For infinitely long straight wire $\theta_1 = \frac{-\pi}{2}$ and $\theta_2 = \frac{\pi}{2}$



We have calculated it for any angle limits

$$B = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1).$$

$$B = \frac{\mu_0 I}{4\pi s}\left(\sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{-\pi}{2}\right)\right)$$

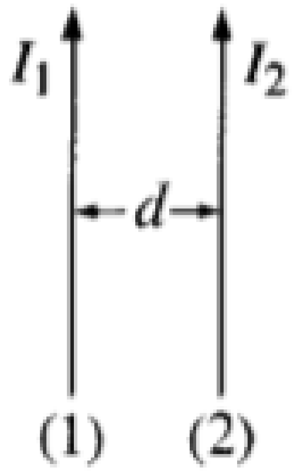
$$B = \frac{\mu_0 I}{2\pi s}.$$

Example 5.5

- **Find the force of attraction between two long parallel wires distance d apart carrying currrents I_1 and I_2**

The field at (2) due to (1) is

$$B = \frac{\mu_0 I_1}{2\pi d},$$



And it point into the page and using the Lorentz force law $\mathbf{F}_{\text{mag}} = I \int (d\mathbf{l} \times \mathbf{B}).$

$$\mathbf{F}_{\text{mag}} = I_2 \int (d\mathbf{l} \times \mathbf{B}).$$ and $\theta = 90^0$

We get force due to wire

$$F = I_2 \left(\frac{\mu_0 I_1}{2\pi d}\right) \int dl.$$

$\int dl.$ → Represents the length of wire 2 here

Force per unit length acting on wire 2 is

$$f = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d}.$$

Where $f = F / \int dl.$

Force on **(2) due to (1) is Attractive. You can determine it using right hand rule**