11 ANALYSIS OF TIME SERIES

11.1 Introduction There are four important bases for classification of data, namely, qualitative, quantitative, geographical and chronological. In the classification on chronological basis, the data are arranged by successive time periods, e.g. years, quarters, months, etc. An arrangement of data by successive time periods is called a *time series.*

Table 11.1

Year	Production of Sugar (thousand tons)
1980	851
1981	1301
1982	1127
1983	1147
1984	1306
1985	1116
1986	1286
1987	1771
1988	1858

Source: Federal Bureau of Statistics

Examples of time series are the annual yield of a crop in a country for a number of years, the total monthly sales receipts in a departmental store, the daily closing prices of a share on the stock exchange, the hourly temperatures recorded by the weather bureau in a city and so on. Table 11.1, which shows the production of sugar in Pakistan for the years 1980 to 1988, is an example of time series.

11.2 Graphs of Time Series The graph of a time series is called a *historigram*. To construct a historigram, we mark the time units along the X-axis and the time series variable along the Y-axis. Points plotted corresponding to different time periods and the values of the time series variable are joined by straight lines. Often the minimum value of the time series variable is distant from zero and the variations are restricted to a certain range only. For example, in the data of Table 11.1, values range between 851 and 1858. In such a case, if we take the scale of Y as 0, 500, 1000, 1500, 2000, the lower portion of the graph would be blank and the graph will be confined to a portion towards the top looking like a line. The purpose of a graph is to show variations at a glance. A graph, therefore, must show the variations clearly. To make the variations clear, we stretch the Y-axis by using what is called a *false base line*. In taking the false base line, we break or zig-zag the Y-axis immediately above the origin and take the next stage equal or nearly equal to the minimum value. Figure 11.1 shows the graph (historigram) of the data in Table 11.1 in which a false base line has been used.

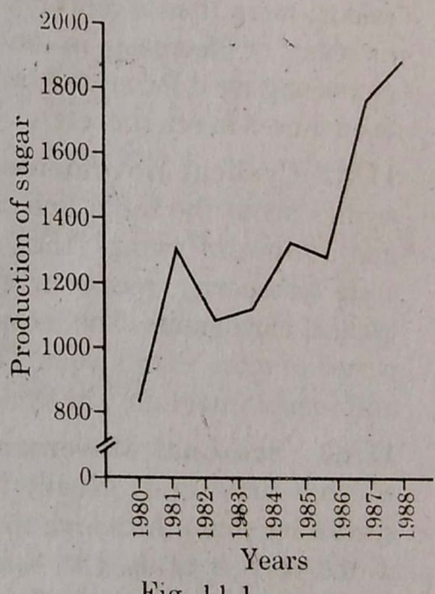

Fig. 11.1

11.3 Signal and Noise Time series may be considered as made up of two types of sequences. The sequence which follows some regular patterns of variation and can be completely determined or specified is called the *systematic sequence* or *signal*. The sequence which follows random or irregular patterns of variation is called the *random sequence* or *noise*.

A time series may, therefore, be expressed as a function of these two sequences – the signal and the noise. Let the values of the time series variable Y be $Y_1, Y_2, \ldots, Y_n$ observed at equal intervals of time $t_1, t_2, \ldots, t_n$. Then the time series may be represented by the model

$$Y = f(t) + U$$

where $f(t)$ denotes the systematic sequence (signal) and U denotes the random sequence (noise). In the early days of development of analysis of time series, only such models were considered in which the effect of time was applied only to the systematic component $f(t)$ and not to the random component and time series was expressed by the model $Y = f(t)$.

11.4 Characteristic Movements of Time Series We have seen that a time series is an arrangement of data by successive time periods. It is, therefore, affected by all the changes which may occur in a period of time. Experience with many time series has revealed certain characteristic movements. These movements are classified into four main types also called the *components of a time series*: (i) Long-Term or Secular Movements (ii) Cyclical Movements (iii) Seasonal Movements and (iv) Irregular or Random Movements.

11.4.1 Secular Movements The word 'secular' is used to mean 'long-term' or 'relating to long period of time'. Thus the secular trend refers to the movement of a time series in one direction over a fairly long period of time. The movement is smooth, steady and regular in nature. It is spread over several years or decades (but seldom more than a century). Such a movement characterizes the general pattern of increase or decrease in an economic or social phenomenon, e.g. a continually increasing need for more food due to population increase, a decline in death rate due to advances in science, etc.

11.4.2 Cyclical Movements These movements refer to the long term oscillations or swings about the trend line or curve. Since the movements take the form of upward and downward swings, they are also called 'cycles'. The four phases of a business cycle (prosperity, recession, depression and revival) provide important examples of cyclical movements. The movements are considered cyclical only if they recur after a period of more than a year. The cycle may or may not follow exactly similar patterns after equal intervals. The cycles may also vary in their intensity.

11.4.3 Seasonal Movements These movements refer to short-term variations which a time series usually follows during the corresponding months or seasons of successive years. Although the word 'seasonal' means 'as connected with the seasons' of the year, it is used to refer to any variation of repeating nature provided it is repeated within a period of one year. Since these movements are caused by recurring events, they may recur within a year, a month, a week or even a day. The increased demand for woolen clothes during winter, increase in sales at a departmental store before Eid, number of passengers travelling by rail or by buses on week-ends or morning and evening hours are examples of seasonal movements.

11.4.4 Irregular or Random Movements These movements refer to fluctuations of irregular nature caused by chance events such as wars, strikes, accidents, famines, earthquakes, floods, etc. Rise in prices during war, delay in production due to a labour strike or electricity failures, etc. are examples of irregular movements. Since the movements are caused by chance events, they are also called 'accidental' or 'erratic' movements.

The first three components, namely, secular trend, cyclical movements and seasonal movements, which follow regular patterns of variation fall under 'signal' while the last one, namely, irregular movements fall under 'noise'.

11.5 Analysis of Time Series The analysis of time series consists of describing, measuring and isolating the various components present in the time series. A graphic illustration of these components is given in Figs.11.2. Figure 11.2(a) shows a graph of long-term or secular trend. Figure 11.2(b) shows a long-term trend line along with a cyclical movement. Figure 11.2(c) shows a seasonal movement superimposed on graph (b). If we superimpose on graph (c) some random movements, the result will look more like a time series occurring in practice.

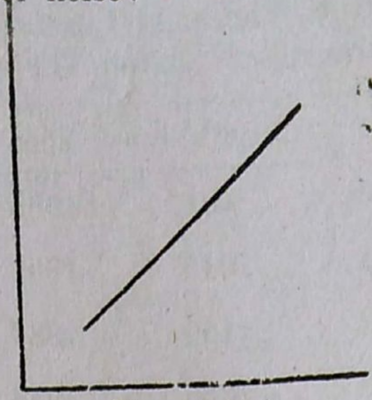

Fig. 11.2(a)

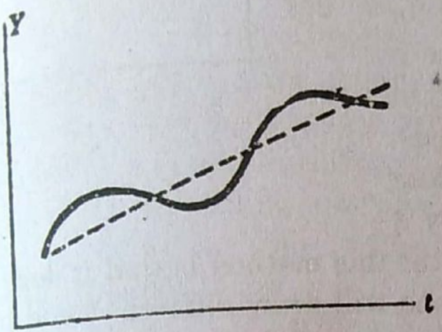

Fig. 11.2(b)

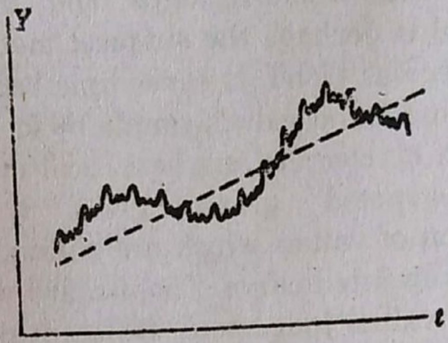

Fig. 11.2(c)

It is assumed that the components of a time series follow a product (multiplicative) law. This means that any value (Y) of a time series is the product of the effects of four components, viz. trend (T), cyclical (C), seasonal (S) and irregular (I) movements. In symbols,

$$Y = T \times C \times S \times I \tag{11.1}$$

It may be mentioned that some statisticians consider that the components of a time series follow an additive law: $Y = T + C + S + I$

In the analysis of a time series, we investigate the factors T, C, S and I. This analysis is often called the *decomposition* of a time series into basic component movements. Such an analysis enables us to understand and interpret the movements constituting the time series and to make forecasts about the trends of future business and economic activity.

11.6 Measurement of Secular Trend Following methods are used for measuring or estimating the secular trend: (i) The Freehand Curve Method (ii) The Method of Semi-averages (iii) The Method of Moving Averages and (iv) The Method of Least Squares.

11.6.1 The Freehand Curve Method In this method, the data are plotted on a graph measuring the time units (years, months, etc.) along the X-axis and the values of the time series variable along the Y-axis. A trend line or smooth curve is drawn through the graph in such a way that it shows the general tendency of the values. The trend values for different years (or months) are read from the trend line or curve.

Example 11.1 Use the freehand curve method to find trend values for the data in Table 11.1.

Solution Figure 11.3 gives the graph of the data of Table 11.1 along with a trend line drawn by freehand. The trend values as read from the trend line are given below:

Year	Trend Value	Year	Trend Value
1980	940	1985	1465
1981	1045	1986	1570
1982	1150	1987	1675
1983	1255	1988	1780
1984	1360		

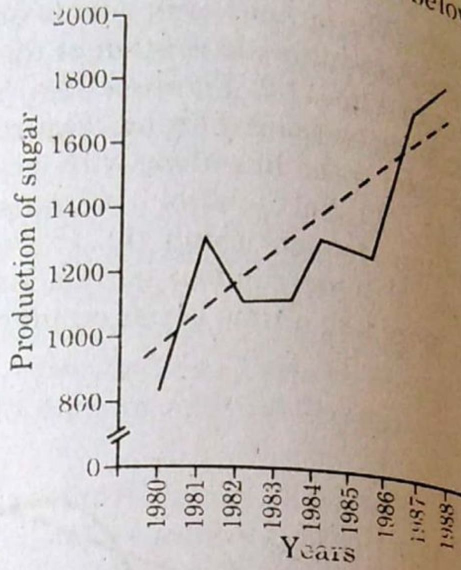

Fig. 11.3

Advantages and Disadvantages The freehand curve method is perhaps the simplest method for measuring secular trend. It saves time because it is easier to find trend values simply by looking at a graph than by complex mathematical methods. A well-constructed graph gives a close approximation of values which are obtained by a mathematically fitted curve. The disadvantage of this method is that it depends too much on individual judgement. Different persons will draw different trend lines for the same data. This method is, therefore, subjective and not suitable for making accurate estimates.

11.6.2 The Method of Semi-averages The freehand curve method depends too much on personal judgement and gives subjective results. Another simple method for measuring secular trend is the *method of semi-averages*. In this method, the data are divided into two equal parts. (If the number of values is odd, either the middle value is left out or the series is divided unevenly). The averages for each part are computed and placed against the centre of each part. The averages are plotted and joined by a line. The line is extended to cover the whole data. Trend values corresponding to different time periods can be read from this trend line. The trend values can also be computed directly as shown below in Examples 11.2 and 11.3.

Example 11.2 Use the semi-average method to find trend values for the following data showing per capita (gross) income of Pakistan for the years 1959–60 to 1968–69.

Year	1959–60	1960–61	1961–62	1962–63	1963–64	1964–65	1965–66	1966–67	1967–68	1968–69
Per Capita Income (Rs.)	318	326	337	340	359	365	372	381	402	412

Source: Federal Bureau of Statistics

Solution Computation of semi-averages is outlined in the following table. As we can see from the table, the series has been divided into two parts; the totals and averages for each part have been placed in the centre of each part.

Year	Per Capita Income (Rs.)	Semi Total	Semi Average	X	Trend Value
1959–60	318			−2	316
1960–61	326			−1	326
1961–62	337	1680	$\frac{1680}{5} = 336$	0	336
1962–63	340			1	346
1963–64	359			2	356
1964–65	365			3	366
1965–66	372			4	376
1966–67	381	1930	$\frac{1930}{5} = 386$	5	386
1967–68	402			6	396
1968–69	410			7	406

Figure 11.4 gives the graph of the original data The semi-averages

can, [illegible] therefore, give poor results when the [illegible] we usually use arithmetic mean to find the semi-averages. The arithmetic mean [illegible] greatly affected by extremely large or extremely small values. In that case, we can use median to find the semi-averages.

11.6.3 The Method of Moving Averages We have seen that the freehand curve method is subjective because it is based purely on individual judgment. The method of semi-averages is appropriate only when the trend is linear. Another simple method which can also be used to eliminate seasonal, cyclical and irregular movements is the *method of moving averages*. In this method we find the simple averages successively by taking specific number of values at a time. For example, if we want to find a 3-year moving average, we shall find the average of the first three values. Next we drop the first value and include the fourth value and obtain the average of these values. The process will be continued till all the values in the series are exhausted. The averages so obtained are placed in the middle of the group for which the average is calculated. When we find the moving average taking an even number of values, the middle of the group will lie between two years. In order to make the averages coincide with a particular year, we centre the averages by calculating further a 2-year moving average of the even order moving averages. The averages so obtained are called *moving averages(centred)*.

The moving averages tend to reduce the variation in the values of time series averages. We can, therefore, eliminate seasonal, cyclical and irregular variations using moving average of appropriate order. For example, if we want to eliminate cyclical variation, we may use a moving average equal to the length of the cycle.

Example 11.4 Find (i) 3-year and (ii) 5-year moving average for the following time series. Also draw a graph of the original data and the trend values.

Year	Value	Year	Value
1948	20	1954	32
1949	23	1955	26
1950	26	1956	38
1951	29	1957	35
1952	23	1958	32
1953	29	1959	35

Solution Computation of moving averages is outlined in the following table. Figure 11.6 gives the graph of the original data and the moving averages.

Year	Value	3-year moving total	3-year moving average	5-year moving total	5-year moving average
1948	20				
1949	23	69	23		
1950	26	78	26	121	24.2
1951	29	78	26	130	26.0
1952	23	81	27	139	27.8
1953	29	84	28	139	27.8
1954	32	87	29	148	29.6
1955	26	96	32	160	32.0
1956	38	99	33	163	32.6
1957	35	105	35	166	33.2
1958	32	102	34		
1959	35				

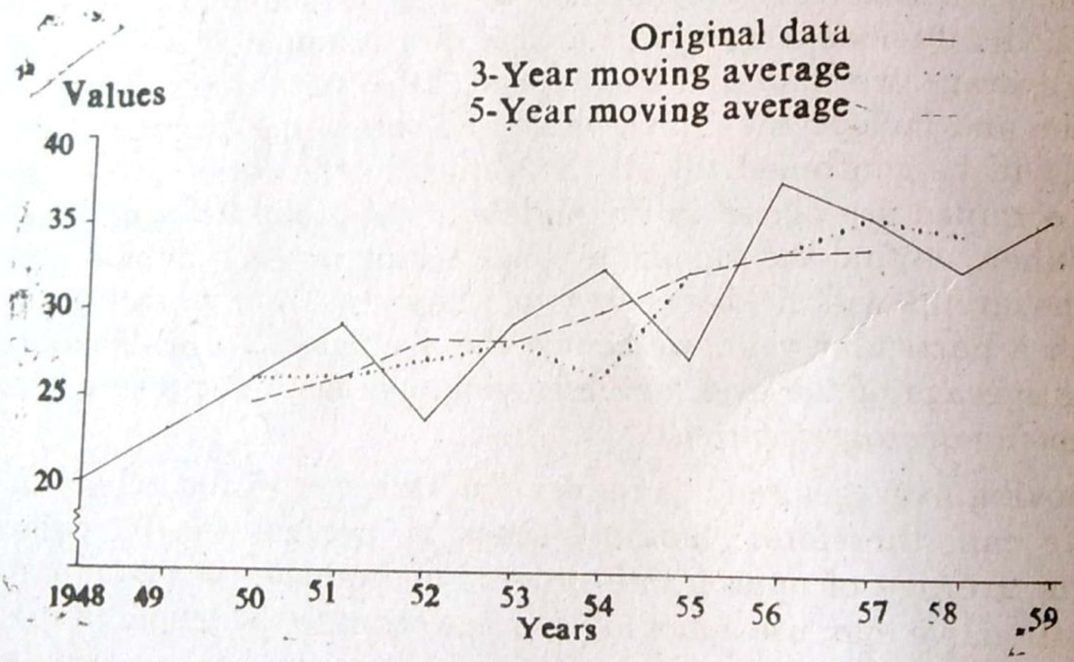

Fig. 11.6