

5. The Hydrogen Atom

Table 5.2. Normalized total wave functions of an electron in the Coulomb potential $E_{\text{pot}} = -Ze^2/(4\pi\epsilon_0 r)$

n	l	m	Eigenfunction $\psi_{n,l,m}(r, \vartheta, \varphi)$
1	0	0	$\frac{1}{\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$
2	0	0	$\frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \left(2 - \frac{Zr}{a_0}\right) e^{-Zr/2a_0}$
2	1	0	$\frac{1}{4\sqrt{2\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \frac{Zr}{a_0} e^{-Zr/2a_0} \cos \vartheta$
2	1	± 1	$\frac{1}{8\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \frac{Zr}{a_0} e^{-Zr/2a_0} \sin \vartheta e^{\pm i\varphi}$
3	0	0	$\frac{1}{81\sqrt{3\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \left(27 - 18\frac{Zr}{a_0} + 2\frac{Z^2 r^2}{a_0^2}\right) e^{-Zr/3a_0}$
3	1	0	$\frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \left(6 - \frac{Zr}{a_0}\right) \frac{Zr}{a_0} e^{-Zr/3a_0} \cos \vartheta$
3	1	± 1	$\frac{1}{\sqrt{81\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \left(6 - \frac{Zr}{a_0}\right) \frac{Zr}{a_0} e^{-Zr/3a_0} \sin \vartheta e^{\pm i\varphi}$
3	2	0	$\frac{1}{81\sqrt{6\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \frac{Z^2 r^2}{a_0^2} e^{-Zr/3a_0} (3 \cos^2 \vartheta - 1)$
3	2	± 1	$\frac{1}{81\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \frac{Z^2 r^2}{a_0^2} e^{-Zr/3a_0} \sin \vartheta \cos \vartheta e^{\pm i\varphi}$
3	2	± 2	$\frac{1}{162\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \frac{Z^2 r^2}{a_0^2} e^{-Zr/3a_0} \sin^2 \vartheta e^{\pm 2i\varphi}$

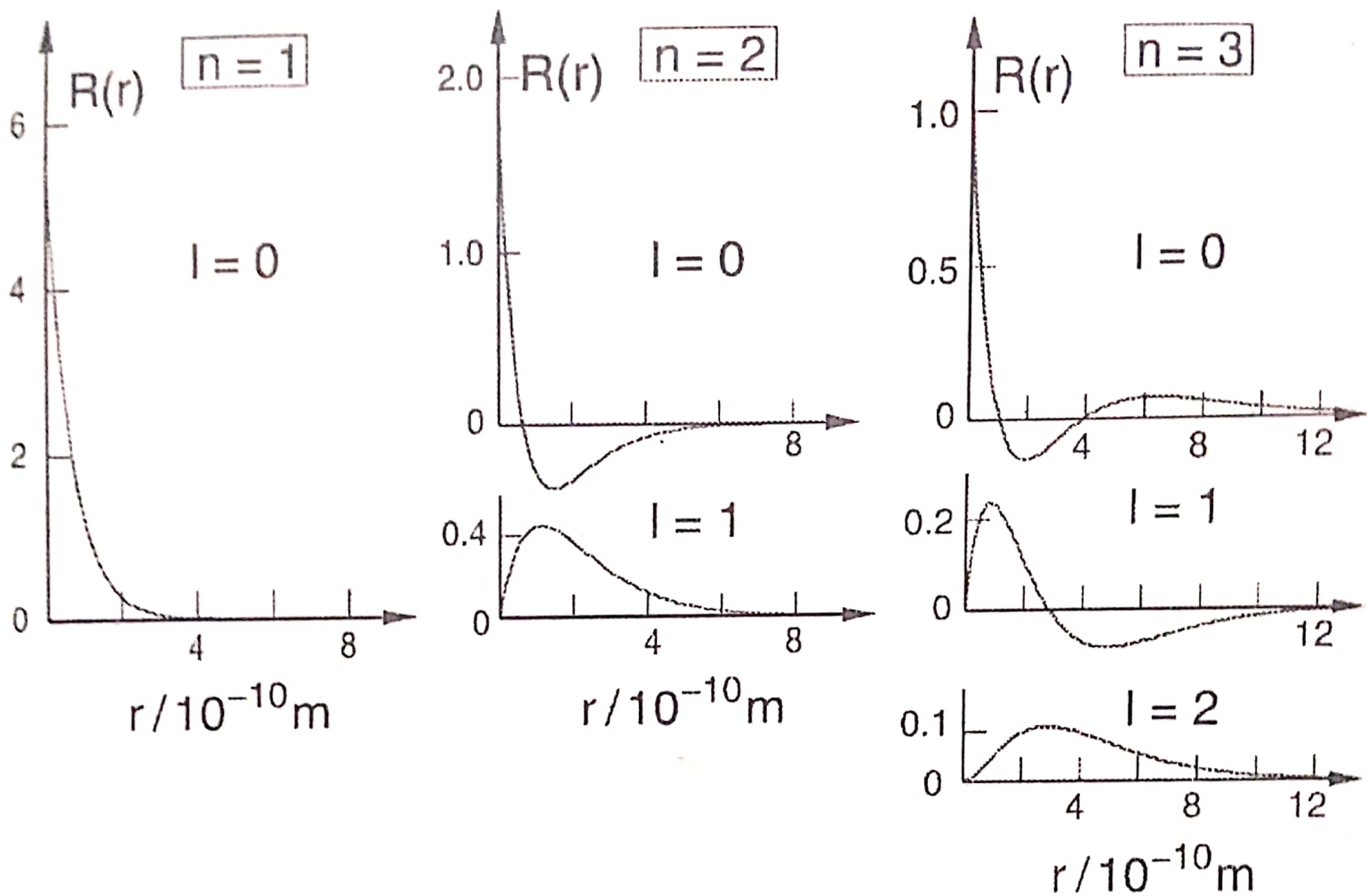


Fig. 5.3. The radial wave function $R_{n,l}(r)$ for the principle quantum numbers $n = 1, 2, 3$. The ordinate is scaled in units of $10^8 \text{ m}^{-3/2}$

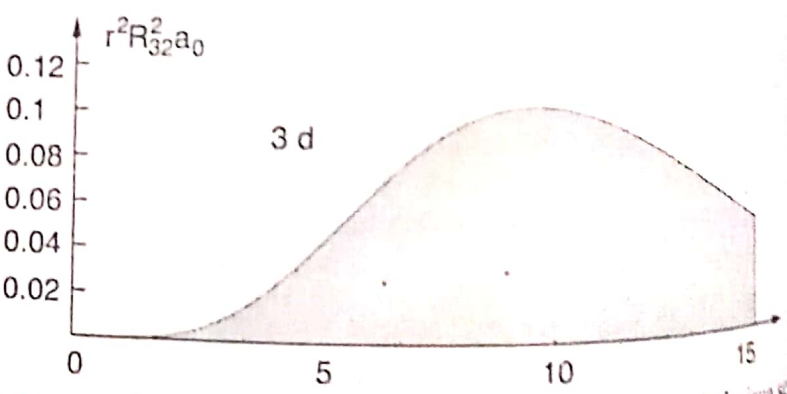
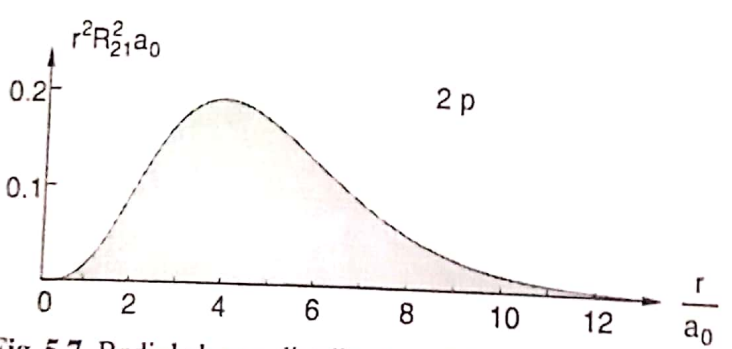
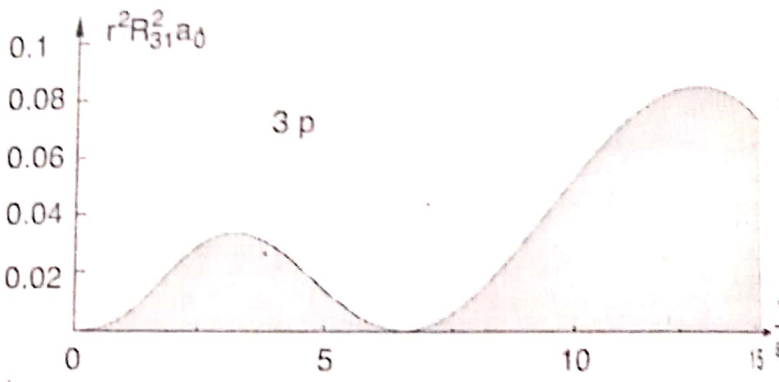
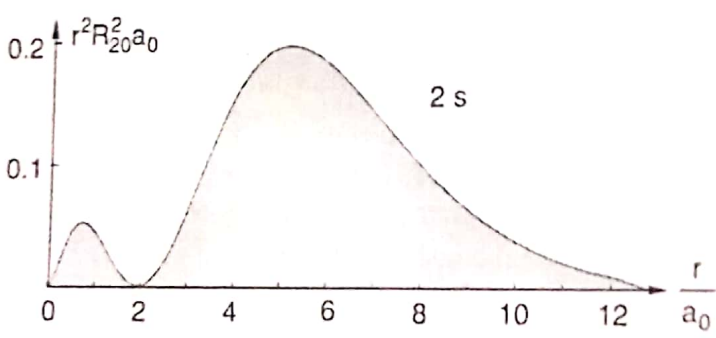
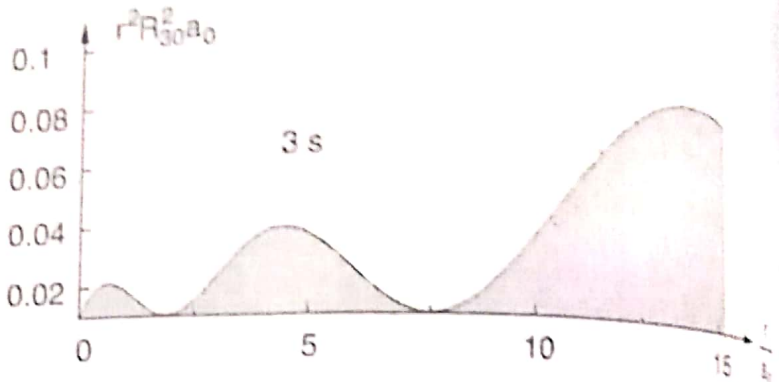
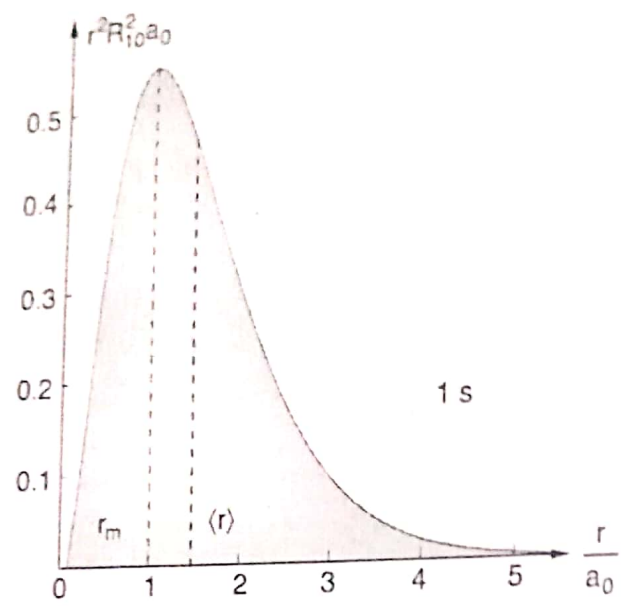


Fig. 5.7. Radial charge distribution of the electron in different states of the H atom. Note the different ordinate and abscissa scales.