15-22

Ex # ~~8.7~~ 5.3

$$\boxed{\because \int x^n\,dx = \frac{x^{n+1}}{n+1} + C}$$

upper limit (always large)

$$\int_a^b f(x)\,dx$$

lower limit (always small)

$$\because \int_a^b f(x)dx = F(b) - F(a)$$

$$\because \int 1dx = x + C$$

(16) $\int_{\frac{1}{2}}^{3/2} (-2x+4)\,dx$

$$= \int_{\frac{1}{2}}^{3/2} -2x\,dx + \int_{1/2}^{3/2} 4\,dx$$

$$= -2\int_{1/2}^{3/2} x^1\,dx + 4\int_{1/2}^{3/2} 1\,dx$$

$$= -2\left[\frac{x^2}{2}\Big|_{1/2}^{3/2}\right] + 4\left[x\Big|_{1/2}^{3/2}\right]$$

$$= -\frac{\cancel{2}}{\cancel{2}}\left[x^2\Big|_{1/2}^{3/2}\right] + 4\left[\left(\frac{3}{2}\right) - \left(\frac{1}{2}\right)\right]$$

$$= -1\left[\left(\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right] + 4\left[\frac{3}{2} - \frac{1}{2}\right]$$

$$= -1\left[\frac{9}{4} - \frac{1}{4}\right] + 4\left[\frac{3-1}{2}\right] = -1\left[\frac{9-1}{4}\right] + 4\left[\frac{\cancel{2}}{\cancel{2}}\right]$$

$$= -1\left(\frac{\cancel{8}^{\,2}}{\cancel{4}_{\,1}}\right) + 4$$

$$= -2 + 4 = 2 \text{ Ans}$$

(18) $\int_{-4}^{0} \sqrt{16-x^2}\, dx$

$$\because \int \sqrt{a^2-x^2}\, dx = \frac{1}{2} x\sqrt{a^2-x^2} + \frac{1}{2} a^2 \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int_{-4}^{0} \sqrt{16-x^2}\, dx = \int_{-4}^{0} \sqrt{(4)^2-(x)^2}\, dx$$

$$= \frac{1}{2} x\sqrt{(4)^2-(x)^2} + \frac{1}{2}(4)^2 \sin^{-1}\left(\frac{x}{4}\right) \Bigg|_{-4}^{0}$$

$$= \left[\frac{1}{2}(0)\cancel{\sqrt{16-(0)^2}}^{\,0} + \frac{16}{2}\sin^{-1}\cancel{\left(\frac{0}{4}\right)}^{\,0}\right] - \left[\frac{1}{\cancel{2}_1}(-\cancel{4}^{\,2})\sqrt{(4)^2-(-4)^2} + \frac{\cancel{16}^{\,8}}{\cancel{2}_1}\sin^{-1}\left(-\frac{4}{4}\right)\right]$$

$$= [0-0] - \left[-2\cancel{\sqrt{16-16}}^{\,0} + 8\sin^{-1}(-1)\right]$$

$$= -\left[0 + \cancel{8}^{\,4}\left(-\frac{\pi}{\cancel{2}_1}\right)\right]$$

$$= -[-4\pi] = 4\pi$$ **Ans**

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(20) $\int_{-1}^{1} (1-|x|)\, dx$

$$= \int_{-1}^{1} 1\, dx - \int_{-1}^{1} |x|\, dx$$

$$= x\Big|_{-1}^{1} - \int_{-1}^{1} |x|\,dx$$

$$|x| = \begin{cases} +x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$= [1-(-1)] - \left[\int_{-1}^{0} -x\,dx + \int_{0}^{1} x\,dx\right]$$

$$= [1+1] - \left[-\int_{-1}^{0} x\,dx + \int_{0}^{1} x\,dx\right]$$

$$= 2 - \left[-\left(\frac{x^2}{2}\right)\Big|_{-1}^{0} + \left(\frac{x^2}{2}\right)\Big|_{0}^{1}\right]$$

$$= 2 + \left[\frac{x^2}{2}\Big|_{-1}^{0} - \frac{x^2}{2}\Big|_{0}^{1}\right]$$

$$= 2 + \left[\left(\frac{(0)^2}{2} - \frac{(-1)^2}{2}\right) - \left(\frac{(1)^2}{2} - \frac{(0)^2}{2}\right)\right]$$

$$= 2 + \left[\frac{-1}{2} - \frac{1}{2}\right] = 2 + \left[\frac{-1-1}{2}\right]$$

$$= 2 + \left[\frac{-2}{2}\right] = 2 - 1$$

$$= 1 \quad \underline{\text{Ans}}$$

(22) $\int_{-1}^{1} (1 + \sqrt{1-x^2})\, dx$

$= \int_{-1}^{1} 1\, dx + \int_{-1}^{1} \sqrt{1-x^2}\, dx$

$\because \int \sqrt{a^2-x^2}\, dx = \frac{1}{2} x \sqrt{a^2-x^2} + \frac{1}{2} a^2 \sin^{-1} \frac{x}{a} + c.$

$= x \Big|_{-1}^{1} + \int_{-1}^{1} \sqrt{(1)^2 - (x)^2}\, dx$

$= [(1) - (-1)] + \left(\frac{1}{2} x \sqrt{(1)^2 - (x)^2} + \frac{1}{2} (1)^2 \sin^{-1} \frac{x}{1} \Big|_{-1}^{1} \right)$

$= [1+1] + \left[\left(\frac{1}{2} (1) \sqrt{1-(1)^2} + \frac{1}{2} \sin^{-1}(1) \right) - \left(\frac{1}{2} (-1) \sqrt{(1)^2 - (-1)^2} + \frac{1}{2} \sin^{-1}(-1) \right) \right]$

$= 2 + \left[\left(\frac{1}{2}(0) + \frac{1}{2}\left(\frac{\pi}{2} \right) \right) - \left(-\frac{1}{2}(0) + \frac{1}{2}\left(-\frac{\pi}{2} \right) \right) \right]$

$= 2 + \left(\frac{\pi}{4} + \frac{\pi}{4} \right) = 2 + \left(\frac{\pi + \pi}{4} \right)$

$= 2 + \left(\frac{2\pi}{4} \right) = 2 + \frac{\pi}{2}$

$= \frac{4+\pi}{2}$ Ans

$\boxed{15, 17, 19, 21}$