

which implies a response similar to output 2 of Figure 7.2(b). Notice that the integration in the forward path yields a finite error for a ramp input, rather than the infinite error found in Example 7.2.

For the input, $5t^2u(t)$, whose Laplace transform is $10/s^3$, the steady-state error will be 10 times as large as that given by Eq. (7.20), or

$$e(\infty) = e_{\text{parabola}}(\infty) = \frac{10}{\lim_{s \rightarrow 0} s^2 G(s)} = \frac{10}{0} = \infty \quad (7.29)$$

Notice that the integration in the forward path does not yield any improvement in steady-state error over that found in Example 7.2 for a parabolic input.

Skill-Assessment Exercise 7.1

PROBLEM: A unity feedback system has the following forward transfer function:

$$G(s) = \frac{10(s+20)(s+30)}{s(s+25)(s+35)}$$

- Find the steady-state error for the following inputs: $15u(t)$, $15tu(t)$, and $15t^2u(t)$.
- Repeat for

$$G(s) = \frac{10(s+20)(s+30)}{s^2(s+25)(s+35)(s+50)}$$

ANSWERS:

- The closed-loop system is stable. For $15u(t)$, $e_{\text{step}}(\infty) = 0$; for $15tu(t)$, $e_{\text{ramp}}(\infty) = 2.1875$; for $15(t^2)u(t)$, $e_{\text{parabola}}(\infty) = \infty$.
- The closed-loop system is unstable. Calculations cannot be made.

The complete solution is at www.wiley.com/college/nise.

WileyPLUS
WPCS
 Control Solutions

7.3 Static Error Constants and System Type

We continue our focus on unity negative feedback systems and define parameters that we can use as steady-state error performance specifications, just as we defined damping ratio, natural frequency, settling time, percent overshoot, and so on as performance specifications for the transient response. These steady-state error performance specifications are called *static error constants*. Let us see how they are defined, how to calculate them, and, in the next section, how to use them for design.

Static Error Constants

In the previous section we derived the following relationships for steady-state error. For a step input, $u(t)$,

$$e(\infty) = e_{\text{step}}(\infty) = \frac{1}{1 + \lim_{s \rightarrow 0} G(s)} \quad (7.30)$$

For a ramp input, $tu(t)$,

$$e(\infty) = e_{\text{ramp}}(\infty) = \frac{1}{\lim_{s \rightarrow 0} sG(s)} \quad (7.31)$$

For a parabolic input, $\frac{1}{2}t^2u(t)$.

$$e(\infty) = e_{\text{parabola}}(\infty) = \frac{1}{\lim_{s \rightarrow 0} s^2G(s)} \quad (7.32)$$

The three terms in the denominator that are taken to the limit determine the steady-state error. We call these limits *static error constants*. Individually, their names are *position constant*, K_p , where

$$K_p = \lim_{s \rightarrow 0} G(s) \quad (7.33)$$

velocity constant, K_v , where

$$K_v = \lim_{s \rightarrow 0} sG(s) \quad (7.34)$$

acceleration constant, K_a , where

$$K_a = \lim_{s \rightarrow 0} s^2G(s) \quad (7.35)$$

As we have seen, these quantities, depending upon the form of $G(s)$, can assume values of zero, finite constant, or infinity. Since the static error constant appears in the denominator of the steady-state error. Eqs. (7.30) through (7.32), the value of the steady-state error decreases as the static error constant increases.

In Section 7.2, we evaluated the steady-state error by using the final value theorem. An alternate method makes use of the static error constants. A few examples follow.

Example 7.4

Steady-State Error via Static Error Constants

PROBLEM: For each system of Figure 7.7, evaluate the static error constants and find the expected error for the standard step, ramp, and parabolic inputs.

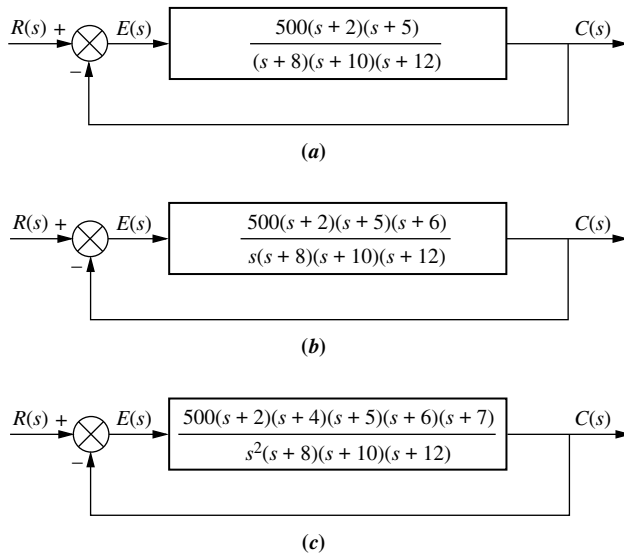


FIGURE 7.7 Feedback control systems for Example 7.4

SOLUTION: First verify that all closed-loop systems shown are indeed stable. For this example we leave out the details. Next, for Figure 7.7(a),

$$K_p = \lim_{s \rightarrow 0} G(s) = \frac{500 \times 2 \times 5}{8 \times 10 \times 12} = 5.208 \quad (7.36)$$

$$K_v = \lim_{s \rightarrow 0} sG(s) = 0 \quad (7.37)$$

$$K_a = \lim_{s \rightarrow 0} s^2 G(s) = 0 \quad (7.38)$$

Thus, for a step input,

$$e(\infty) = \frac{1}{1 + K_p} = 0.161 \quad (7.39)$$

For a ramp input,

$$e(\infty) = \frac{1}{K_v} = \infty \quad (7.40)$$

For a parabolic input,

$$e(\infty) = \frac{1}{K_a} = \infty \quad (7.41)$$

Now, for Figure 7.7(b),

$$K_p = \lim_{s \rightarrow 0} G(s) = \infty \quad (7.42)$$

$$K_v = \lim_{s \rightarrow 0} sG(s) = \frac{500 \times 2 \times 5 \times 6}{8 \times 10 \times 12} = 31.25 \quad (7.43)$$

and

$$K_a = \lim_{s \rightarrow 0} s^2 G(s) = 0 \quad (7.44)$$

Thus, for a step input,

$$e(\infty) = \frac{1}{1 + K_p} = 0 \quad (7.45)$$

For a ramp input,

$$e(\infty) = \frac{1}{K_v} = \frac{1}{31.25} = 0.032 \quad (7.46)$$

For a parabolic input,

$$e(\infty) = \frac{1}{K_a} = \infty \quad (7.47)$$

Finally, for Figure 7.7(c),

$$K_p = \lim_{s \rightarrow 0} G(s) = \infty \quad (7.48)$$

$$K_v = \lim_{s \rightarrow 0} sG(s) = \infty \quad (7.49)$$

and

$$K_a = \lim_{s \rightarrow 0} s^2 G(s) = \frac{500 \times 2 \times 4 \times 5 \times 6 \times 7}{8 \times 10 \times 12} = 875 \quad (7.50)$$

Thus, for a step input,

$$e(\infty) = \frac{1}{1 + K_p} = 0 \quad (7.51)$$

For a ramp input,

$$e(\infty) = \frac{1}{K_v} = 0 \quad (7.52)$$

For a parabolic input,

$$e(\infty) = \frac{1}{K_a} = \frac{1}{875} = 1.14 \times 10^{-3} \quad (7.53)$$

MATLAB

ML

Students who are using MATLAB should now run ch7p1 in Appendix B. You will learn how to test the system for stability, evaluate static error constants, and calculate steady-state error using MATLAB. This exercise applies MATLAB to solve Example 7.4 with System (b).

System Type

Let us continue to focus on a unity negative feedback system. The values of the static error constants, again, depend upon the form of $G(s)$, especially the number of pure integrations in the forward path. Since steady-state errors are dependent upon the

number of integrations in the forward path, we give a name to this system attribute. Given the system in Figure 7.8, we define *system type* to be the value of n in the denominator or, equivalently, the number of pure integrations in the forward path. Therefore, a system with $n = 0$ is a Type 0 system. If $n = 1$ or $n = 2$, the corresponding system is a Type 1 or Type 2 system, respectively.

Table 7.2 ties together the concepts of steady-state error, static error constants, and system type. The table shows the static error constants and the steady-state errors as functions of input waveform and system type.

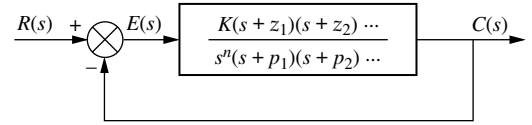


FIGURE 7.8 Feedback control system for defining system type

TABLE 7.2 Relationships between input, system type, static error constants, and steady-state errors

Input	Steady-state error formula	Type 0		Type 1		Type 2	
		Static error constant	Error	Static error constant	Error	Static error constant	Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p = \text{Constant}$	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v = \text{Constant}$	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a = \text{Constant}$	$\frac{1}{K_a}$

Skill-Assessment Exercise 7.2

PROBLEM: A unity feedback system has the following forward transfer function:

$$G(s) = \frac{1000(s+8)}{(s+7)(s+9)}$$

- Evaluate system type, K_p , K_v , and K_a .
- Use your answers to **a.** to find the steady-state errors for the standard step, ramp, and parabolic inputs.

ANSWERS:

- The closed-loop system is stable. System type = Type 0. $K_p = 127$, $K_v = 0$, and $K_a = 0$.
- $e_{\text{step}}(\infty) = 7.8 \times 10^{-3}$, $e_{\text{ramp}}(\infty) = \infty$, and $e_{\text{parabola}}(\infty) = \infty$

The complete solution is at www.wiley.com/college/nise.

TryIt 7.1

Use MATLAB, the Control System Toolbox, and the following statements to find K_p , $e_{\text{step}}(\infty)$, and the closed-loop poles to check for stability for the system of Skill-Assessment Exercise 7.2.

```
numg=1000*[1 8];
deng=poly([-7 -9]);
G=tf(numg,deng);
Kp=dcgain(G);
estep=1/(1+Kp);
T=feedback(G,1);
poles=pole(T)
```

In this section, we defined steady-state errors, static error constants, and system type. Now the specifications for a control system's steady-state errors will be formulated, followed by some examples.

7.4 Steady-State Error Specifications

Static error constants can be used to specify the steady-state error characteristics of control systems, such as that shown in Figure 7.9. Just as damping ratio, ζ , settling time, T_s , peak time, T_p , and percent overshoot, %OS, are used as specifications for a



FIGURE 7.9 A robot used in the manufacturing of semiconductor random-access memories (RAMs) similar to those in personal computers. Steady-state error is an important design consideration for assembly-line robots.

control system's transient response, so the position constant, K_p , velocity constant, K_v , and acceleration constant, K_a , can be used as specifications for a control system's steady-state errors. We will soon see that a wealth of information is contained within the specification of a static error constant.

For example, if a control system has the specification $K_v = 1000$, we can draw several conclusions:

1. The system is stable.
2. The system is of Type 1, since only Type 1 systems have K_v 's that are finite constants. Recall that $K_v = 0$ for Type 0 systems, whereas $K_v = \infty$ for Type 2 systems.
3. A ramp input is the test signal. Since K_v is specified as a finite constant, and the steady-state error for a ramp input is inversely proportional to K_v , we know the test input is a ramp.
4. The steady-state error between the input ramp and the output ramp is $1/K_v$ per unit of input slope.

Let us look at two examples that demonstrate analysis and design using static error constants.

Example 7.5

Interpreting the Steady-State Error Specification

PROBLEM: What information is contained in the specification $K_p = 1000$?

SOLUTION: The system is stable. The system is Type 0, since only a Type 0 system has a finite K_p . Type 1 and Type 2 systems have $K_p = \infty$. The input test signal is a step, since K_p is specified. Finally, the error per unit step is

$$e(\infty) = \frac{1}{1 + K_p} = \frac{1}{1 + 1000} = \frac{1}{1001} \quad (7.54)$$

Example 7.6

Gain Design to Meet a Steady-State Error Specification

PROBLEM: Given the control system in Figure 7.10, find the value of K so that there is 10% error in the steady state.

SOLUTION: Since the system is Type 1, the error stated in the problem must apply to a ramp input; only a ramp yields a finite error in a Type 1 system. Thus,

$$e(\infty) = \frac{1}{K_v} = 0.1 \quad (7.55)$$

Therefore,

$$K_v = 10 = \lim_{s \rightarrow 0} sG(s) = \frac{K \times 5}{6 \times 7 \times 8} \quad (7.56)$$

which yields

$$K = 672 \quad (7.57)$$

Applying the Routh-Hurwitz criterion, we see that the system is stable at this gain.

Although this gain meets the criteria for steady-state error and stability, it may not yield a desirable transient response. In Chapter 9 we will design feedback control systems to meet all three specifications.

Students who are using MATLAB should now run ch7 p2 in Appendix B. You will learn how to find the gain to meet a steady-state error specification using MATLAB. This exercise solves Example 7.6 using MATLAB.

MATLAB
ML

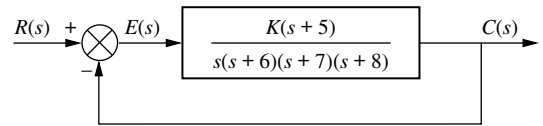


FIGURE 7.10 Feedback control system for Example 7.6

Skill-Assessment Exercise 7.3

PROBLEM: A unity feedback system has the following forward transfer function:

$$G(s) = \frac{K(s+12)}{(s+14)(s+18)}$$

Find the value of K to yield a 10% error in the steady state.

ANSWER: $K = 189$

The complete solution is at www.wiley.com/college/nise.

WileyPLUS
WPCS
Control Solutions

TryIt 7.2

Use MATLAB, the Control System Toolbox, and the following statements to solve Skill-Assessment Exercise 7.3 and check the resulting system for stability.

```
numg=[1 12];
deng=poly([-14 -18]);
G=tf(numg,deng);
Kpdk=dcgain(G);
estep=0.1;
K=(1/estep-1)/Kpdk;
T=feedback(G,1);
poles=pole(T)
```