## Evaluating Integrals

Evaluate the indefinite integrals in Exercises 1–12 by using the given substitutions to reduce the integrals to standard form.

1. $\int \sin 3x\, dx, \quad u = 3x$

2. $\int x \sin(2x^2)\, dx, \quad u = 2x^2$

3. $\int \sec 2t \tan 2t\, dt, \quad u = 2t$

4. $\int \left(1 - \cos\frac{t}{2}\right)^2 \sin\frac{t}{2}\, dt, \quad u = 1 - \cos\frac{t}{2}$





5. $\int 28(7x - 2)^{-5}\, dx, \quad u = 7x - 2$

6. $\int x^3(x^4 - 1)^2\, dx, \quad u = x^4 - 1$

7. $\int \frac{9r^2\, dr}{\sqrt{1 - r^3}}, \quad u = 1 - r^3$

8. $\int 12(y^4 + 4y^2 + 1)^2(y^3 + 2y)\, dy, \quad u = y^4 + 4y^2 + 1$

9. $\int \sqrt{x} \sin^2(x^{3/2} - 1)\, dx, \quad u = x^{3/2} - 1$

10. $\int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right) dx, \quad u = -\frac{1}{x}$

11. $\int \csc^2 2\theta \cot 2\theta\, d\theta$

 a. Using $u = \cot 2\theta$
 b. Using $u = \csc 2\theta$

12. $\int \frac{dx}{\sqrt{5x + 8}}$

 a. Using $u = 5x + 8$
 b. Using $u = \sqrt{5x + 8}$

Evaluate the integrals in Exercises 13–48.

13. $\int \sqrt{3 - 2s}\, ds$

14. $\int (2x + 1)^3\, dx$

15. $\int \frac{1}{\sqrt{5s + 4}}\, ds$

16. $\int \frac{3\, dx}{(2 - x)^2}$

17. $\int \theta \sqrt[4]{1 - \theta^2}\, d\theta$

18. $\int 8\theta \sqrt[3]{\theta^2 - 1}\, d\theta$

19. $\int 3y\sqrt{7 - 3y^2}\, dy$

20. $\int \frac{4y\, dy}{\sqrt{2y^2 + 1}}$

21. $\int \frac{1}{\sqrt{x}(1 + \sqrt{x})^2}\, dx$

22. $\int \frac{(1 + \sqrt{x})^3}{\sqrt{x}}\, dx$

23. $\int \cos(3z + 4)\, dz$

24. $\int \sin(8z - 5)\, dz$

25. $\int \sec^2(3x + 2)\, dx$

26. $\int \tan^2 x \sec^2 x\, dx$

27. $\int \sin^5\frac{x}{3}\cos\frac{x}{3}\, dx$

28. $\int \tan^7\frac{x}{2}\sec^2\frac{x}{2}\, dx$

29. $\int r^2\left(\frac{r^3}{18} - 1\right)^5 dr$

30. $\int r^4\left(7 - \frac{r^5}{10}\right)^3 dr$

31. $\int x^{1/2}\sin(x^{3/2} + 1)\, dx$

32. $\int x^{1/3}\sin(x^{4/3} - 8)\, dx$

33. $\int \sec\left(v + \frac{\pi}{2}\right)\tan\left(v + \frac{\pi}{2}\right) dv$

34. $\int \csc\left(\frac{v - \pi}{2}\right)\cot\left(\frac{v - \pi}{2}\right) dv$

35. $\int \frac{\sin(2t + 1)}{\cos^2(2t + 1)}\, dt$

36. $\int \frac{6\cos t}{(2 + \sin t)^3}\, dt$

37. $\int \sqrt{\cot y}\csc^2 y\, dy$

38. $\int \frac{\sec z \tan z}{\sqrt{\sec z}}\, dz$

39. $\int \frac{1}{t^2}\cos\left(\frac{1}{t} - 1\right) dt$

40. $\int \frac{1}{\sqrt{t}}\cos(\sqrt{t} + 3)\, dt$

41. $\int \frac{1}{\theta^2}\sin\frac{1}{\theta}\cos\frac{1}{\theta}\, d\theta$

42. $\int \frac{\cos\sqrt{\theta}}{\sqrt{\theta}\sin^2\sqrt{\theta}}\, d\theta$

43. $\int (s^3 + 2s^2 - 5s + 5)(3s^2 + 4s - 5)\, ds$

44. $\int (\theta^4 - 2\theta^2 + 8\theta - 2)(\theta^3 - \theta + 2)\, d\theta$

45. $\int t^3(1 + t^4)^3\, dt$

46. $\int \sqrt{\frac{x - 1}{x^5}}\, dx$

47. $\int x^3\sqrt{x^2 + 1}\, dx$

48. $\int 3x^5\sqrt{x^3 + 1}\, dx$

## Simplifying Integrals Step by Step

If you do not know what substitution to make, try reducing the integral step by step, using a trial substitution to simplify the integral a bit and then another to simplify it some more. You will see what we mean if you try the sequences of substitutions in Exercises 49 and 50.

49. $\int \frac{18\tan^2 x \sec^2 x}{(2 + \tan^3 x)^2}\, dx$

 a. $u = \tan x$, followed by $v = u^3$, then by $w = 2 + v$
 b. $u = \tan^3 x$, followed by $v = 2 + u$
 c. $u = 2 + \tan^3 x$

50. $\int \sqrt{1 + \sin^2(x - 1)}\sin(x - 1)\cos(x - 1)\, dx$

 a. $u = x - 1$, followed by $v = \sin u$, then by $w = 1 + v^2$
 b. $u = \sin(x - 1)$, followed by $v = 1 + u^2$
 c. $u = 1 + \sin^2(x - 1)$

Evaluate the integrals in Exercises 51 and 52.

51. $\int \frac{(2r - 1)\cos\sqrt{3(2r - 1)^2 + 6}}{\sqrt{3(2r - 1)^2 + 6}}\, dr$

52. $\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta\cos^3\sqrt{\theta}}}\, d\theta$

## Initial Value Problems

Solve the initial value problems in Exercises 53–58.

53. $\frac{ds}{dt} = 12t(3t^2 - 1)^3, \quad s(1) = 3$

54. $\frac{dy}{dx} = 4x(x^2 + 8)^{-1/3}, \quad y(0) = 0$

55. $\frac{ds}{dt} = 8\sin^2\left(t + \frac{\pi}{12}\right), \quad s(0) = 8$

56. $\frac{dr}{d\theta} = 3\cos^2\left(\frac{\pi}{4} - \theta\right), \quad r(0) = \frac{\pi}{8}$

# Ex # 5.5

(7) $\int \frac{9r^2\,dr}{\sqrt{1-r^3}}$ — (1) Put $u = 1 - r^3$ — (2)

(2) $u = 1 - r^3$

Taking derivative both sides w.r.t 'r'

$$\frac{du}{dr} = 0 - 3r^2$$

$$du = -3r^2 \cdot dr$$

$$\frac{du}{-3r^2} = dr \Rightarrow dr = \frac{du}{-3r^2} \quad \text{— (3)}$$

using (2) and (3) in (1)

$$\int \frac{9r^2}{\sqrt{1-r^3}}\,dr = \int \frac{9\cancel{r^2}}{\sqrt{u}} \cdot \frac{du}{-3\cancel{r^2}} = \int \frac{\overset{3}{\cancel{9}}\,du}{\sqrt{u} \cdot -\cancel{3}}$$

$$= \int -\frac{3\,du}{\sqrt{u}} = -3\int \frac{1}{\sqrt{u}}\,du$$

$$\int \frac{9r^2}{\sqrt{1-r^3}}\,dr = -3\int u^{-1/2}\,du$$

$$\because \int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$= -3\,\frac{u^{-1/2+1}}{-1/2+1} + c \qquad \frac{-1+2}{2} = \frac{1}{2}$$

$$= -3\,\frac{u^{1/2}}{1/2} + c = -3 \times \frac{2}{1}\,u^{1/2} + c$$

$$= -6u^{1/2} + c \quad \text{Put } u = 1 - r^3$$

$$\int \frac{9r^2}{\sqrt{1-r^3}}\,dr = -6(1-r^3)^{1/2} + c \quad \underline{\text{Ans}}$$

+ c

hx + c

+ c

(9) $\int \sqrt{x} \sin^2(x^{3/2}-1)\, dx$ — (1)

$u = x^{3/2} - 1$ — (2)

(2) Taking derivative both sides w.r.t 'x'

$$\frac{du}{dx} = \frac{d}{dx}(x^{3/2}-1) = \frac{d}{dx}(x)^{3/2} - \cancel{\frac{d}{dx}(1)}^{\,0}$$

$$\frac{du}{dx} = \frac{3}{2}x^{3/2-1}\cancel{\frac{d}{dx}(x)}^{\,1}$$

$$\frac{du}{dx} = \frac{3}{2}x^{1/2} \Rightarrow du = \frac{3}{2}x^{1/2}\,dx$$

$$\Rightarrow \frac{2}{3}\frac{du}{x^{1/2}} = dx$$

$$\Rightarrow dx = \frac{2}{3}\frac{du}{x^{1/2}} \quad \text{— (3)}$$

Using (2) & (3) in (1)

$$\int \sqrt{x} \sin^2(x^{3/2}-1)\, dx = \int \cancel{\sqrt{x}} \sin^2 u \cdot \frac{2}{3}\frac{du}{\cancel{x^{1/2}}}$$

$$= \frac{2}{3}\int \sin^2 u\, du$$

$$\because \boxed{\sin^2 x = \frac{1-\cos 2x}{2}}$$

$$= \frac{\cancel{2}}{3}\int \frac{1-\cos 2u}{\cancel{2}}\, du$$

$$= \frac{1}{3}\int (1-\cos 2u)\, du$$

$$= \frac{1}{3}\left[\int 1\, du - \int \cos 2u\, du\right]$$

$\because \int 1\,dx = x \qquad \because \int \cos x\,dx = \frac{\sin x}{1} + C$

$= \frac{1}{3}\left[\int 1\,du - \int \cos 2u\,du\right]$

$= \frac{1}{3}\left[u - \frac{\sin 2u}{2}\right] + C$

Put $u = x^{3/2} - 1$

$= \frac{1}{3}\left[(x^{3/2} - 1) - \frac{\sin 2(x^{3/2} - 1)}{2}\right] + C$

Ans

$\because \boxed{\cos^2 x = \frac{1 + \cos 2x}{2}}$

(11) $\int \operatorname{cosec}^2 2\theta \cot 2\theta \, d\theta$ — (1)

(a) $u = \cot 2\theta$ — (2)

Diff (2) both sides w.r.t $\theta$

$\frac{du}{d\theta} = \frac{d}{d\theta}(\cot 2\theta) \qquad \because \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x\ (1)$

$\frac{du}{d\theta} = -\operatorname{cosec}^2 2\theta \cdot \frac{d}{d\theta}(2\theta) = -\operatorname{cosec}^2 2\theta \cdot 2(1)$

$\frac{du}{d\theta} = -2\operatorname{cosec}^2 2\theta$

$du = -2\operatorname{cosec}^2 2\theta \, d\theta$

$\frac{du}{-2\operatorname{cosec}^2 2\theta} = d\theta \Rightarrow d\theta = \frac{-du}{2\operatorname{cosec}^2 2\theta}$ — (3)

using (2) & (3) in (1)

$\int \operatorname{cosec}^2(2\theta)\cot(2\theta)\,d\theta = \int \cancel{\operatorname{cosec}^2 2\theta} \cdot u \times \frac{-du}{2\cancel{\operatorname{cosec}^2 2\theta}}$

$= \int \frac{-u\,du}{2} = -\frac{1}{2}\int u\,du = -\frac{1}{2}\left[\frac{u^2}{2}\right] + C$

$$= -\frac{u^2}{4} + c$$

put $u = \cot 2\theta$

$$\int \operatorname{cosec}^2 2\theta \cot 2\theta \, d\theta = -\frac{(\cot 2\theta)^2}{4} + c \quad \text{Ans}$$

(b) put $u = \operatorname{cosec} 2\theta$ Do yourself

Q 13 - 48 Evaluate

(14) $\int (2x+1)^3 dx$ — (1)

put $u = (2x+1)$ — (2)

Diff (2) w.r.t $x$

$$\frac{du}{dx} = \frac{d}{dx}(2x+1) = 2(1)+0 = 2$$

$$du = 2dx$$

$$\frac{du}{2} = dx \Rightarrow dx = \frac{du}{2} \quad \text{— (3)}$$

using (2) & (3) in (1)

$$\int (2x+1)^3 dx = \int u^3 \frac{du}{2} = \frac{1}{2}\int u^3 du$$

$$= \frac{1}{2}\left[\frac{u^4}{4}\right] + c$$

$$= \frac{1}{8}u^4 + c$$

Put $u = (2x+1)$

$$\int (2x+1)^3 dx = \frac{1}{8}(2x+1)^4 + c \quad \text{Ans}$$

$\boxed{5, 6, 10, 12, 15, 13}$ Homework