

Skill-Assessment Exercise 2.4

PROBLEM: Find the differential equation corresponding to the transfer function,

$$G(s) = \frac{2s + 1}{s^2 + 6s + 2}$$

ANSWER: $\frac{d^2c}{dt^2} + 6\frac{dc}{dt} + 2c = 2\frac{dr}{dt} + r$

The complete solution is at www.wiley.com/college/nise.

Skill-Assessment Exercise 2.5

PROBLEM: Find the ramp response for a system whose transfer function is

$$G(s) = \frac{s}{(s + 4)(s + 8)}$$

ANSWER: $c(t) = \frac{1}{32} - \frac{1}{16}e^{-4t} + \frac{1}{32}e^{-8t}$

The complete solution is at www.wiley.com/college/nise.

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In general, a physical system that can be represented by a linear, time-invariant differential equation can be modeled as a transfer function. The rest of this chapter will be devoted to the task of modeling individual subsystems. We will learn how to represent electrical networks, translational mechanical systems, rotational mechanical systems, and electromechanical systems as transfer functions. As the need arises, the reader can consult the Bibliography at the end of the chapter for discussions of other types of systems, such as pneumatic, hydraulic, and heat-transfer systems (*Cannon, 1967*).

2.4 Electrical Network Transfer Functions




In this section, we formally apply the transfer function to the mathematical modeling of electric circuits including passive networks and operational amplifier circuits. Subsequent sections cover mechanical and electromechanical systems.

Equivalent circuits for the electric networks that we work with first consist of three passive linear components: resistors, capacitors, and inductors.² Table 2.3 summarizes the components and the relationships between voltage and current and between voltage and charge under zero initial conditions.

We now combine electrical components into circuits, decide on the input and output, and find the transfer function. Our guiding principles are Kirchhoff's laws. We sum voltages around loops or sum currents at nodes, depending on which technique involves the least effort in algebraic manipulation, and then equate the result to zero. From these relationships we can write the differential equations for the circuit. Then we can take the Laplace transforms of the differential equations and finally solve for the transfer function.

² *Passive* means that there is no internal source of energy.

TABLE 2.3 Voltage-current, voltage-charge, and impedance relationships for capacitors, resistors, and inductors

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C} \int_0^t i(\tau) d\tau$	$i(t) = C \frac{dv(t)}{dt}$	$v(t) = \frac{1}{C} q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R} v(t)$	$v(t) = R \frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L \frac{di(t)}{dt}$	$i(t) = \frac{1}{L} \int_0^t v(\tau) d\tau$	$v(t) = L \frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ – V (volts), $i(t)$ – A (amps), $q(t)$ – Q (coulombs), C – F (farads), R – Ω (ohms), G – Ω (mhos), L – H (henries).

Simple Circuits via Mesh Analysis

Transfer functions can be obtained using Kirchoff's voltage law and summing voltages around loops or meshes.³ We call this method *loop* or *mesh analysis* and demonstrate it in the following example.

Example 2.6

Transfer Function—Single Loop via the Differential Equation

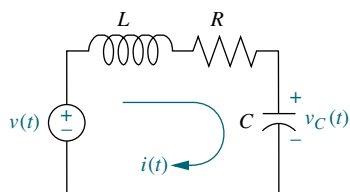


FIGURE 2.3 RLC network

PROBLEM: Find the transfer function relating the capacitor voltage, $V_C(s)$, to the input voltage, $V(s)$ in Figure 2.3.

SOLUTION: In any problem, the designer must first decide what the input and output should be. In this network, several variables could have been chosen to be the output—for example, the inductor voltage, the capacitor voltage, the resistor voltage, or the current. The problem statement, however, is clear in this case: We are to treat the capacitor voltage as the output and the applied voltage as the input.

Summing the voltages around the loop, assuming zero initial conditions, yields the integro-differential equation for this network as

$$L \frac{di(t)}{dt} + Ri(t) + \frac{1}{C} \int_0^t i(\tau) d\tau = v(t) \quad (2.61)$$

Changing variables from current to charge using $i(t) = dq(t)/dt$ yields

$$L \frac{d^2q(t)}{dt^2} + R \frac{dq(t)}{dt} + \frac{1}{C} q(t) = v(t) \quad (2.62)$$

From the voltage-charge relationship for a capacitor in Table 2.3,

$$q(t) = Cv_C(t) \quad (2.63)$$

Substituting Eq. (2.63) into Eq. (2.62) yields

$$LC \frac{d^2v_C(t)}{dt^2} + RC \frac{dv_C(t)}{dt} + v_C(t) = v(t) \quad (2.64)$$

³ A particular loop that resembles the spaces in a screen or fence is called a *mesh*.

Taking the Laplace transform assuming zero initial conditions, rearranging terms, and simplifying yields

$$(LCs^2 + RCs + 1)V_C(s) = V(s) \quad (2.65)$$

Solving for the transfer function, $V_C(s)/V(s)$, we obtain

$$\frac{V_C(s)}{V(s)} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \quad (2.66)$$

as shown in Figure 2.4.

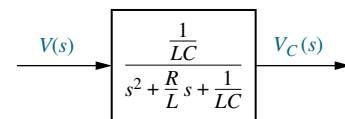


FIGURE 2.4 Block diagram of series RLC electrical network

Let us now develop a technique for simplifying the solution for future problems. First, take the Laplace transform of the equations in the voltage-current column of Table 2.3 assuming zero initial conditions.

For the capacitor,

$$V(s) = \frac{1}{Cs}I(s) \quad (2.67)$$

For the resistor,

$$V(s) = RI(s) \quad (2.68)$$

For the inductor,

$$V(s) = LsI(s) \quad (2.69)$$

Now define the following transfer function:

$$\frac{V(s)}{I(s)} = Z(s) \quad (2.70)$$

Notice that this function is similar to the definition of resistance, that is, the ratio of voltage to current. But, unlike resistance, this function is applicable to capacitors and inductors and carries information on the dynamic behavior of the component, since it represents an equivalent differential equation. We call this particular transfer function *impedance*. The impedance for each of the electrical elements is shown in Table 2.3.

Let us now demonstrate how the concept of impedance simplifies the solution for the transfer function. The Laplace transform of Eq. (2.61), assuming zero initial conditions, is

$$\left(Ls + R + \frac{1}{Cs}\right)I(s) = V(s) \quad (2.71)$$

Notice that Eq. (2.71), which is in the form

$$[\text{Sum of impedances}]I(s) = [\text{Sum of applied voltages}] \quad (2.72)$$

suggests the series circuit shown in Figure 2.5. Also notice that the circuit of Figure 2.5 could have been obtained immediately from the circuit of Figure 2.3 simply by replacing each element with its impedance. We call this altered circuit the *transformed circuit*. Finally, notice that the transformed circuit leads immediately to Eq. (2.71) if we add impedances in series as we add resistors in series. Thus, rather than writing the differential equation first and then taking the

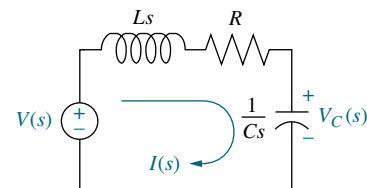


FIGURE 2.5 Laplace-transformed network

Laplace transform, we can draw the transformed circuit and obtain the Laplace transform of the differential equation simply by applying Kirchhoff's voltage law to the transformed circuit. We summarize the steps as follows:

1. Redraw the original network showing all time variables, such as $v(t)$, $i(t)$, and $v_C(t)$, as Laplace transforms $V(s)$, $I(s)$, and $V_C(s)$, respectively.
2. Replace the component values with their impedance values. This replacement is similar to the case of dc circuits, where we represent resistors with their resistance values.

We now redo Example 2.6 using the transform methods just described and bypass the writing of the differential equation.

Example 2.7

Transfer Function—Single Loop via Transform Methods

PROBLEM: Repeat Example 2.6 using mesh analysis and transform methods without writing a differential equation.

SOLUTION: Using Figure 2.5 and writing a mesh equation using the impedances as we would use resistor values in a purely resistive circuit, we obtain

$$\left(Ls + R + \frac{1}{Cs}\right)I(s) = V(s) \quad (2.73)$$

Solving for $I(s)/V(s)$,

$$\frac{I(s)}{V(s)} = \frac{1}{Ls + R + \frac{1}{Cs}} \quad (2.74)$$

But the voltage across the capacitor, $V_C(s)$, is the product of the current and the impedance of the capacitor. Thus,

$$V_C(s) = I(s) \frac{1}{Cs} \quad (2.75)$$

Solving Eq. (2.75) for $I(s)$, substituting $I(s)$ into Eq. (2.74), and simplifying yields the same result as Eq. (2.66).

Simple Circuits via Nodal Analysis

Transfer functions also can be obtained using Kirchhoff's current law and summing currents flowing from nodes. We call this method *nodal analysis*. We now demonstrate this principle by redoing Example 2.6 using Kirchhoff's current law and the transform methods just described to bypass writing the differential equation.

Example 2.8

Transfer Function—Single Node via Transform Methods

PROBLEM: Repeat Example 2.6 using nodal analysis and without writing a differential equation.

SOLUTION: The transfer function can be obtained by summing currents flowing out of the node whose voltage is $V_C(s)$ in Figure 2.5. We assume that currents leaving the node are positive and currents entering the node are negative. The currents consist of the current through the capacitor and the current flowing through the series resistor and inductor. From Eq. (2.70), each $I(s) = V(s)/Z(s)$. Hence,

$$\frac{V_C(s)}{1/Cs} + \frac{V_C(s) - V(s)}{R + Ls} = 0 \quad (2.76)$$

where $V_C(s)/(1/Cs)$ is the current flowing out of the node through the capacitor, and $[V_C(s) - V(s)]/(R + Ls)$ is the current flowing out of the node through the series resistor and inductor. Solving Eq. (2.76) for the transfer function, $V_C(s)/V(s)$, we arrive at the same result as Eq. (2.66).

Simple Circuits via Voltage Division

Example 2.6 can be solved directly by using voltage division on the transformed network. We now demonstrate this technique.

Example 2.9

Transfer Function—Single Loop via Voltage Division

PROBLEM: Repeat Example 2.6 using voltage division and the transformed circuit.

SOLUTION: The voltage across the capacitor is some proportion of the input voltage, namely the impedance of the capacitor divided by the sum of the impedances. Thus,

$$V_C(s) = \frac{1/Cs}{\left(Ls + R + \frac{1}{Cs}\right)} V(s) \quad (2.77)$$

Solving for the transfer function, $V_C(s)/V(s)$, yields the same result as Eq. (2.66).

Review Examples 2.6 through 2.9. Which method do you think is easiest for this circuit?

The previous example involves a simple, single-loop electrical network. Many electrical networks consist of multiple loops and nodes, and for these circuits we must write and solve simultaneous differential equations in order to find the transfer function, or solve for the output.

Complex Circuits via Mesh Analysis

To solve complex electrical networks—those with multiple loops and nodes—using mesh analysis, we can perform the following steps:

1. Replace passive element values with their impedances.
2. Replace all sources and time variables with their Laplace transform.
3. Assume a transform current and a current direction in each mesh.

4. Write Kirchhoff's voltage law around each mesh.
5. Solve the simultaneous equations for the output.
6. Form the transfer function.

Let us look at an example.

Example 2.10

Transfer Function—Multiple Loops

PROBLEM: Given the network of Figure 2.6(a), find the transfer function, $I_2(s)/V(s)$.

SOLUTION: The first step in the solution is to convert the network into Laplace transforms for impedances and circuit variables, assuming zero initial conditions. The result is shown in Figure 2.6(b). The circuit with which we are dealing requires two simultaneous equations to solve for the transfer function. These equations can be found by summing voltages around each mesh through which the assumed currents, $I_1(s)$ and $I_2(s)$, flow. Around Mesh 1, where $I_1(s)$ flows,

$$R_1 I_1(s) + Ls I_1(s) - Ls I_2(s) = V(s) \quad (2.78)$$

Around Mesh 2, where $I_2(s)$ flows,

$$Ls I_2(s) + R_2 I_2(s) + \frac{1}{Cs} I_2(s) - Ls I_1(s) = 0 \quad (2.79)$$

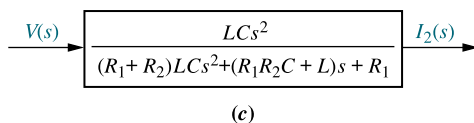
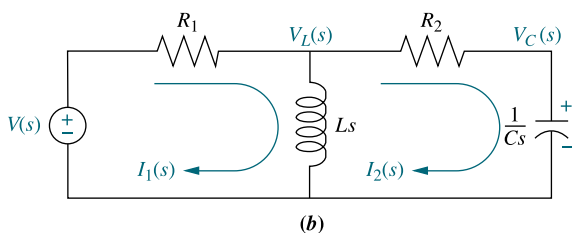
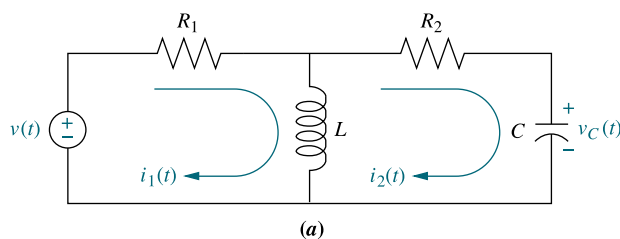


FIGURE 2.6 a. Two-loop electrical network; b. transformed two-loop electrical network; c. block diagram

Combining terms, Eqs. (2.78) and (2.79) become simultaneous equations in $I_1(s)$ and $I_2(s)$:

$$(R_1 + Ls)I_1(s) - LsI_2(s) = V(s) \quad (2.80a)$$

$$-LsI_1(s) + \left(Ls + R_2 + \frac{1}{Cs}\right)I_2(s) = 0 \quad (2.80b)$$

We can use Cramer's rule (or any other method for solving simultaneous equations) to solve Eq. (2.80) for $I_2(s)$.⁴ Hence,

$$I_2(s) = \frac{\begin{vmatrix} (R_1 + Ls) & V(s) \\ -Ls & 0 \end{vmatrix}}{\Delta} = \frac{LsV(s)}{\Delta} \quad (2.81)$$

where

$$\Delta = \begin{vmatrix} (R_1 + Ls) & -Ls \\ -Ls & \left(Ls + R_2 + \frac{1}{Cs}\right) \end{vmatrix}$$

Forming the transfer function, $G(s)$, yields

$$G(s) = \frac{I_2(s)}{V(s)} = \frac{Ls}{\Delta} = \frac{LCs^2}{(R_1 + R_2)LCs^2 + (R_1R_2C + L)s + R_1} \quad (2.82)$$

as shown in Figure 2.6(c).

We have succeeded in modeling a physical network as a transfer function: The network of Figure 2.6(a) is now modeled as the transfer function of Figure 2.6(c). Before leaving the example, we notice a pattern first illustrated by Eq. (2.72). The form that Eq. (2.80) take is

$$\begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{around Mesh 1} \end{bmatrix} I_1(s) - \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{common to the} \\ \text{two meshes} \end{bmatrix} I_2(s) = \begin{bmatrix} \text{Sum of applied} \\ \text{voltages around} \\ \text{Mesh 1} \end{bmatrix} \quad (2.83a)$$

$$- \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{common to the} \\ \text{two meshes} \end{bmatrix} I_1(s) + \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{around Mesh 2} \end{bmatrix} I_2(s) = \begin{bmatrix} \text{Sum of applied} \\ \text{voltages around} \\ \text{Mesh 2} \end{bmatrix} \quad (2.83b)$$

Recognizing the form will help us write such equations rapidly; for example, mechanical equations of motion (covered in Sections 2.5 and 2.6) have the same form.

Students who are performing the MATLAB exercises and want to explore the added capability of MATLAB's Symbolic Math Toolbox should now run `ch2sp4` in Appendix F at www.wiley.com/college/nise, where Example 2.10 is solved. You will learn how to use the Symbolic Math Toolbox to solve simultaneous equations using Cramer's rule. Specifically, the Symbolic Math Toolbox will be used to solve for the transfer function in Eq. (2.82) using Eq. (2.80).

Symbolic Math
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⁴ See Appendix G (Section G.4) at www.wiley.com/college/nise for Cramer's rule.

Complex Circuits via Nodal Analysis

Often, the easiest way to find the transfer function is to use nodal analysis rather than mesh analysis. The number of simultaneous differential equations that must be written is equal to the number of nodes whose voltage is unknown. In the previous example we wrote simultaneous mesh equations using Kirchhoff's voltage law. For multiple nodes we use Kirchhoff's current law and sum currents flowing from each node. Again, as a convention, currents flowing from the node are assumed to be positive, and currents flowing into the node are assumed to be negative.

Before progressing to an example, let us first define *admittance*, $Y(s)$, as the reciprocal of impedance, or

$$Y(s) = \frac{1}{Z(s)} = \frac{I(s)}{V(s)} \quad (2.84)$$

When writing nodal equations, it can be more convenient to represent circuit elements by their admittance. Admittances for the basic electrical components are shown in Table 2.3. Let us look at an example.

Example 2.11

Transfer Function—Multiple Nodes

PROBLEM: Find the transfer function, $V_C(s)/V(s)$, for the circuit in Figure 2.6(b). Use nodal analysis.

SOLUTION: For this problem, we sum currents at the nodes rather than sum voltages around the meshes. From Figure 2.6(b) the sum of currents flowing from the nodes marked $V_L(s)$ and $V_C(s)$ are, respectively,

$$\frac{V_L(s) - V(s)}{R_1} + \frac{V_L(s)}{Ls} + \frac{V_L(s) - V_C(s)}{R_2} = 0 \quad (2.85a)$$

$$CsV_C(s) + \frac{V_C(s) - V_L(s)}{R_2} = 0 \quad (2.85b)$$

Rearranging and expressing the resistances as conductances,⁵ $G_1 = 1/R_1$ and $G_2 = 1/R_2$, we obtain,

$$\left(G_1 + G_2 + \frac{1}{Ls}\right)V_L(s) - G_2V_C(s) = V(s)G_1 \quad (2.86a)$$

$$-G_2V_L(s) + (G_2 + Cs)V_C(s) = 0 \quad (2.86b)$$

Solving for the transfer function, $V_C(s)/V(s)$, yields

$$\frac{V_C(s)}{V(s)} = \frac{\frac{G_1G_2}{C}s}{(G_1 + G_2)s^2 + \frac{G_1G_2L + C}{LC}s + \frac{G_2}{LC}} \quad (2.87)$$

as shown in Figure 2.7.

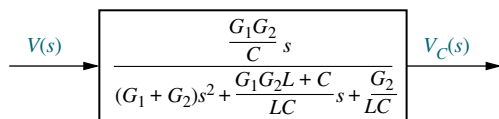


FIGURE 2.7 Block diagram of the network of Figure 2.6

⁵ In general, admittance is complex. The real part is called conductance and the imaginary part is called susceptance. But when we take the reciprocal of resistance to obtain the admittance, a purely real quantity results. The reciprocal of resistance is called conductance.

Another way to write node equations is to replace voltage sources by current sources. A voltage source presents a constant voltage to any load; conversely, a current source delivers a constant current to any load. Practically, a current source can be constructed from a voltage source by placing a large resistance in series with the voltage source. Thus, variations in the load do not appreciably change the current, because the current is determined approximately by the large series resistor and the voltage source. Theoretically, we rely on *Norton's theorem*, which states that a voltage source, $V(s)$, in series with an impedance, $Z_s(s)$, can be replaced by a current source, $I(s) = V(s)/Z_s(s)$, in parallel with $Z_s(s)$.

In order to handle multiple-node electrical networks, we can perform the following steps:

1. Replace passive element values with their admittances.
2. Replace all sources and time variables with their Laplace transform.
3. Replace transformed voltage sources with transformed current sources.
4. Write Kirchhoff's current law at each node.
5. Solve the simultaneous equations for the output.
6. Form the transfer function.

Let us look at an example.

Example 2.12

Transfer Function—Multiple Nodes with Current Sources

PROBLEM: For the network of Figure 2.6, find the transfer function, $V_C(s)/V(s)$, using nodal analysis and a transformed circuit with current sources.

SOLUTION: Convert all impedances to admittances and all voltage sources in series with an impedance to current sources in parallel with an admittance using Norton's theorem.

Redrawing Figure 2.6(b) to reflect the changes, we obtain Figure 2.8, where $G_1 = 1/R_1$, $G_2 = 1/R_2$, and the node voltages—the voltages across the inductor and the capacitor—have been identified as $V_L(s)$ and $V_C(s)$, respectively. Using the general relationship, $I(s) = Y(s)V(s)$, and summing currents at the node $V_L(s)$,

$$G_1 V_L(s) + \frac{1}{Ls} V_L(s) + G_2 [V_L(s) - V_C(s)] = V(s) G_1 \quad (2.88)$$

Summing the currents at the node $V_C(s)$ yields

$$Cs V_C(s) + G_2 [V_C(s) - V_L(s)] = 0 \quad (2.89)$$

Combining terms, Eqs. (2.88) and (2.89) become simultaneous equations in $V_C(s)$ and $V_L(s)$, which are identical to Eq. (2.86) and lead to the same solution as Eq. (2.87).

An advantage of drawing this circuit lies in the form of Eq. (2.86) and its direct relationship to Figure 2.8, namely

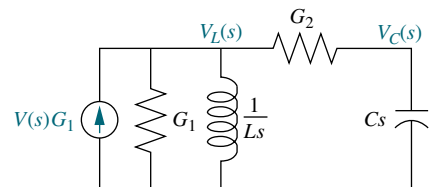


FIGURE 2.8 Transformed network ready for nodal analysis

$$\left[\begin{array}{c} \text{Sum of admittances} \\ \text{connected to Node 1} \end{array} \right] V_L(s) - \left[\begin{array}{c} \text{Sum of admittances} \\ \text{common to the two} \\ \text{nodes} \end{array} \right] V_C(s) = \left[\begin{array}{c} \text{Sum of applied} \\ \text{currents at Node 1} \end{array} \right] \quad (2.90a)$$

$$- \left[\begin{array}{c} \text{Sum of admittances} \\ \text{common to the two} \\ \text{nodes} \end{array} \right] V_L(s) + \left[\begin{array}{c} \text{Sum of admittances} \\ \text{connected to Node 2} \end{array} \right] V_C(s) = \left[\begin{array}{c} \text{Sum of applied} \\ \text{currents at Node 2} \end{array} \right] \quad (2.90b)$$

A Problem-Solving Technique

In all of the previous examples, we have seen a repeating pattern in the equations that we can use to our advantage. If we recognize this pattern, we need not write the equations component by component; we can sum impedances around a mesh in the case of mesh equations or sum admittances at a node in the case of node equations. Let us now look at a three-loop electrical network and write the mesh equations by inspection to demonstrate the process.

Example 2.13

Mesh Equations via Inspection

PROBLEM: Write, but do not solve, the mesh equations for the network shown in Figure 2.9.

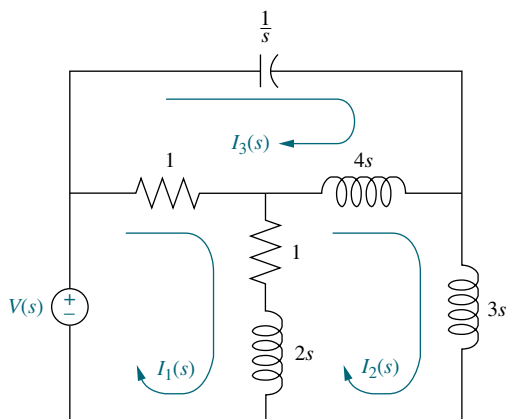


FIGURE 2.9 Three-loop electrical network

SOLUTION: Each of the previous problems has illustrated that the mesh equations and nodal equations have a predictable form. We use that knowledge to solve this three-loop problem. The equation for Mesh 1 will have the following form:

$$\begin{aligned} \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{around Mesh 1} \end{array} \right] I_1(s) - \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{common to} \\ \text{Mesh 1 and} \\ \text{Mesh 2} \end{array} \right] I_2(s) \\ - \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{common to} \\ \text{Mesh 1 and} \\ \text{Mesh 3} \end{array} \right] I_3(s) = \left[\begin{array}{l} \text{Sum of applied} \\ \text{voltages around} \\ \text{Mesh 1} \end{array} \right] \end{aligned} \quad (2.91)$$

Similarly, Meshes 2 and 3, respectively, are

$$\begin{aligned} - \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{common to} \\ \text{Mesh 1 and} \\ \text{Mesh 2} \end{array} \right] I_1(s) + \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{around Mesh 2} \end{array} \right] I_2(s) - \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{common to} \\ \text{Mesh 2 and} \\ \text{Mesh 3} \end{array} \right] I_3(s) = \left[\begin{array}{l} \text{Sum of applied} \\ \text{voltages around} \\ \text{Mesh 2} \end{array} \right] \end{aligned} \quad (2.92)$$

and

$$\begin{aligned} - \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{common to} \\ \text{Mesh 1 and} \\ \text{Mesh 3} \end{array} \right] I_1(s) - \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{common to} \\ \text{Mesh 2 and} \\ \text{Mesh 3} \end{array} \right] I_2(s) \\ + \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{around Mesh 3} \end{array} \right] I_3(s) = \left[\begin{array}{l} \text{Sum of applied} \\ \text{voltages around} \\ \text{Mesh 3} \end{array} \right] \end{aligned} \quad (2.93)$$

Substituting the values from Figure 2.9 into Eqs. (2.91) through (2.93) yields

$$+(2s+2)I_1(s) - (2s+1)I_2(s) - I_3(s) = V(s) \quad (2.94a)$$

$$-(2s+1)I_1(s) + (9s+1)I_2(s) - 4sI_3(s) = 0 \quad (2.94b)$$

$$-I_1(s) - 4sI_2(s) + (4s+1+\frac{1}{s})I_3(s) = 0 \quad (2.94c)$$

which can be solved simultaneously for any desired transfer function, for example, $I_3(s)/V(s)$.

TryIt 2.8

Use the following MATLAB and Symbolic Math Toolbox statements to help you solve for the electrical currents in Eq. (2.94).

```
syms s I1 I2 I3 V
A=[(2*s+2) -(2*s+1)...
  -1
  -(2*s+1) (9*s+1)...
  -4*s
  -1 -4*s...
  (4*s+1+1/s)];
B=[I1; I2; I3];
C=[V; 0; 0];
B=inv(A)*C;
pretty(B)
```

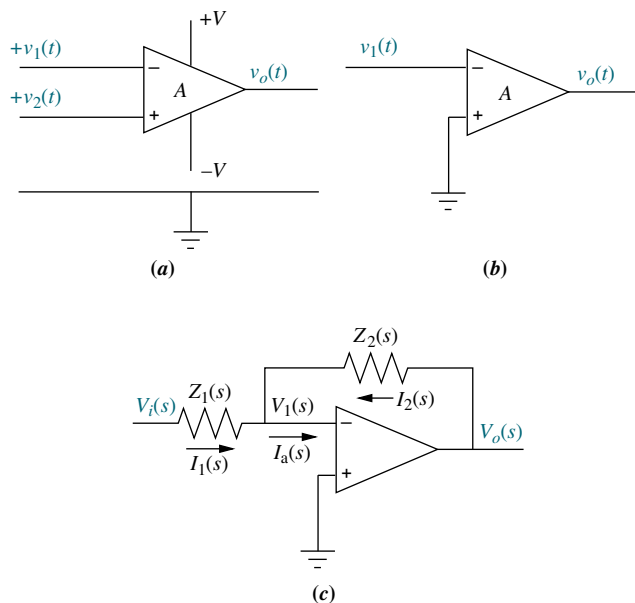


FIGURE 2.10 **a.** Operational amplifier; **b.** schematic for an inverting operational amplifier; **c.** inverting operational amplifier configured for transfer function realization. Typically, the amplifier gain, A , is omitted.

Passive electrical circuits were the topic of discussion up to this point. We now discuss a class of active circuits that can be used to implement transfer functions. These are circuits built around an operational amplifier.

Operational Amplifiers

An *operational amplifier*, pictured in Figure 2.10(a), is an electronic amplifier used as a basic building block to implement transfer functions. It has the following characteristics:

1. Differential input, $V_2(t) - v_1(t)$
2. High input impedance, $Z_i = \infty$ (ideal)
3. Low output impedance, $Z_o = 0$ (ideal)
4. High constant gain amplification, $A = \infty$ (ideal)

The output, $v_o(t)$, is given by

$$v_o(t) = A(v_2(t) - v_1(t)) \quad (2.95)$$

Inverting Operational Amplifier

If $v_2(t)$ is grounded, the amplifier is called an *inverting operational amplifier*, as shown in Figure 2.10(b). For the inverting operational amplifier, we have

$$v_o(t) = -Av_1(t) \quad (2.96)$$

If two impedances are connected to the inverting operational amplifier as shown in Figure 2.10(c), we can derive an interesting result if the amplifier has the characteristics mentioned in the beginning of this subsection. If the input impedance to the amplifier is high, then by Kirchhoff's current law, $I_a(s) = 0$ and $I_1(s) = -I_2(s)$.

Also, since the gain A is large, $v_1(t) \approx 0$. Thus, $I_1(s) = V_i(s)/Z_1(s)$, and $-I_2(s) = -V_o(s)/Z_2(s)$. Equating the two currents, $V_o(s)/Z_2(s) = -V_i(s)/Z_1(s)$, or the transfer function of the inverting operational amplifier configured as shown in Figure 2.10(c) is

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} \quad (2.97)$$

Example 2.14

Transfer Function—Inverting Operational Amplifier Circuit

PROBLEM: Find the transfer function, $V_o(s)/V_i(s)$, for the circuit given in Figure 2.11.

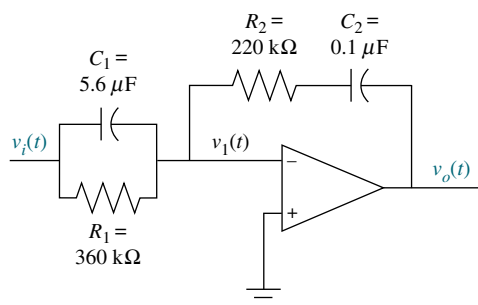


FIGURE 2.11 Inverting operational amplifier circuit for Example 2.14

SOLUTION: The transfer function of the operational amplifier circuit is given by Eq. (2.97). Since the admittances of parallel components add, $Z_1(s)$ is the reciprocal of the sum of the admittances, or

$$Z_1(s) = \frac{1}{C_1s + \frac{1}{R_1}} = \frac{1}{5.6 \times 10^{-6}s + \frac{1}{360 \times 10^3}} = \frac{360 \times 10^3}{2.016s + 1} \quad (2.98)$$

For $Z_2(s)$ the impedances add, or

$$Z_2(s) = R_2 + \frac{1}{C_2s} = 220 \times 10^3 + \frac{10^7}{s} \quad (2.99)$$

Substituting Eqs. (2.98) and (2.99) into Eq. (2.97) and simplifying, we get

$$\frac{V_o(s)}{V_i(s)} = -1.232 \frac{s^2 + 45.95s + 22.55}{s} \quad (2.100)$$

The resulting circuit is called a PID controller and can be used to improve the performance of a control system. We explore this possibility further in Chapter 9.

Noninverting Operational Amplifier

Another circuit that can be analyzed for its transfer function is the noninverting operational amplifier circuit shown in Figure 2.12. We now derive the transfer

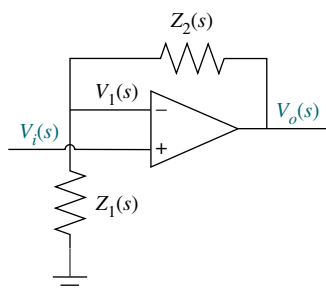


FIGURE 2.12 General noninverting operational amplifier circuit

function. We see that

$$V_o(s) = A(V_i(s) - V_1(s)) \quad (2.101)$$

But, using voltage division,

$$V_1(s) = \frac{Z_1(s)}{Z_1(s) + Z_2(s)} V_o(s) \quad (2.102)$$

Substituting Eq. (2.102) into Eq. (2.101), rearranging, and simplifying, we obtain

$$\frac{V_o(s)}{V_i(s)} = \frac{A}{1 + AZ_1(s)/(Z_1(s) + Z_2(s))} \quad (2.103)$$

For large A , we disregard unity in the denominator and Eq. (2.103) becomes

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{Z_1(s) + Z_2(s)}{Z_1(s)}} \quad (2.104)$$

Let us now look at an example.

Example 2.15

Transfer Function—Noninverting Operational Amplifier Circuit

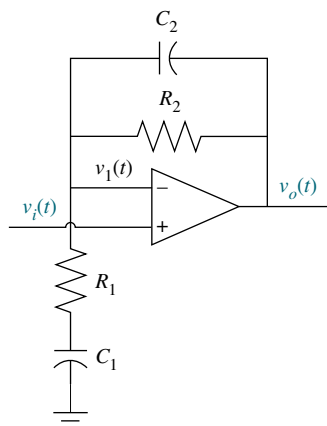


FIGURE 2.13 Noninverting operational amplifier circuit for Example 2.15

PROBLEM: Find the transfer function, $V_o(s)/V_i(s)$, for the circuit given in Figure 2.13.

SOLUTION: We find each of the impedance functions, $Z_1(s)$ and $Z_2(s)$, and then substitute them into Eq. (2.104). Thus,

$$Z_1(s) = R_1 + \frac{1}{C_1 s} \quad (2.105)$$

and

$$Z_2(s) = \frac{R_2(1/C_2 s)}{R_2 + (1/C_2 s)} \quad (2.106)$$

Substituting Eqs. (2.105) and (2.106) into Eq. (2.104) yields

$$\frac{V_o(s)}{V_i(s)} = \frac{C_2 C_1 R_2 R_1 s^2 + (C_2 R_2 + C_1 R_2 + C_1 R_1) s + 1}{C_2 C_1 R_2 R_1 s^2 + (C_2 R_2 + C_1 R_1) s + 1} \quad (2.107)$$

Skill-Assessment Exercise 2.6

PROBLEM: Find the transfer function, $G(s) = V_L(s)/V(s)$, for the circuit given in Figure 2.14. Solve the problem two ways—mesh analysis and nodal analysis. Show that the two methods yield the same result.

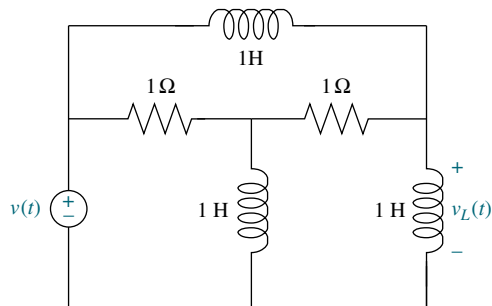


FIGURE 2.14 Electric circuit for Skill-Assessment Exercise 2.6

ANSWER: $V_L(s)/V(s) = (s^2 + 2s + 1)/(s^2 + 5s + 2)$

The complete solution is at www.wiley.com/college/nise.

Skill-Assessment Exercise 2.7

PROBLEM: If $Z_1(s)$ is the impedance of a $10 \mu\text{F}$ capacitor and $Z_2(s)$ is the impedance of a $100 \text{ k}\Omega$ resistor, find the transfer function, $G(s) = V_o(s)/V_i(s)$, if these components are used with (a) an inverting operational amplifier and (b) a noninverting amplifier as shown in Figures 2.10(c) and 2.12, respectively.

ANSWER: $G(s) = -s$ for an inverting operational amplifier; $G(s) = s + 1$ for a noninverting operational amplifier.

The complete solution is at www.wiley.com/college/nise.

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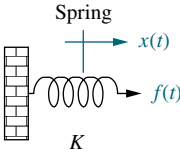
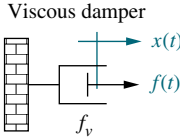
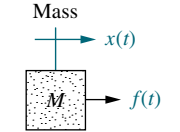
In this section, we found transfer functions for multiple-loop and multiple-node electrical networks, as well as operational amplifier circuits. We developed mesh and nodal equations, noted their form, and wrote them by inspection. In the next section we begin our work with mechanical systems. We will see that many of the concepts applied to electrical networks can also be applied to mechanical systems via analogies—from basic concepts to writing the describing equations by inspection. This revelation will give you the confidence to move beyond this textbook and study systems not covered here, such as hydraulic or pneumatic systems.

2.5 Translational Mechanical System Transfer Functions

We have shown that electrical networks can be modeled by a transfer function, $G(s)$, that algebraically relates the Laplace transform of the output to the Laplace transform of the input. Now we will do the same for mechanical systems. In this section we concentrate on translational mechanical systems. In the next section we extend the concepts to rotational mechanical systems. Notice that the end product, shown in Figure 2.2, will be mathematically indistinguishable from an electrical network. Hence, an electrical network can be interfaced to a mechanical system by cascading their transfer functions, provided that one system is not loaded by the other.⁶

⁶The concept of loading is explained further in Chapter 5.

TABLE 2.4 Force-velocity, force-displacement, and impedance translational relationships for springs, viscous dampers, and mass

Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
 <p>Spring</p>	$f(t) = K \int_0^t v(\tau) d\tau$	$f(t) = Kx(t)$	K
 <p>Viscous damper</p>	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
 <p>Mass</p>	$f(t) = M \frac{dv(t)}{dt}$	$f(t) = M \frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t) = \text{N}$ (newtons), $x(t) = \text{m}$ (meters), $v(t) = \text{m/s}$ (meters/second), $K = \text{N/m}$ (newtons/meter), $f_v = \text{N}\cdot\text{s/m}$ (newton-seconds/meter), $M = \text{kg}$ (kilograms = newton-seconds²/meter).

Mechanical systems parallel electrical networks to such an extent that there are analogies between electrical and mechanical components and variables. Mechanical systems, like electrical networks, have three passive, linear components. Two of them, the spring and the mass, are energy-storage elements; one of them, the viscous damper, dissipates energy. The two energy-storage elements are analogous to the two electrical energy-storage elements, the inductor and capacitor. The energy dissipator is analogous to electrical resistance. Let us take a look at these mechanical elements, which are shown in Table 2.4. In the table, K , f_v , and M are called *spring constant*, *coefficient of viscous friction*, and *mass*, respectively.

We now create analogies between electrical and mechanical systems by comparing Tables 2.3 and 2.4. Comparing the force-velocity column of Table 2.4 to the voltage-current column of Table 2.3, we see that mechanical force is analogous to electrical voltage and mechanical velocity is analogous to electrical current. Comparing the force-displacement column of Table 2.4 with the voltage-charge column of Table 2.3 leads to the analogy between the mechanical displacement and electrical charge. We also see that the spring is analogous to the capacitor, the viscous damper is analogous to the resistor, and the mass is analogous to the inductor. Thus, summing forces written in terms of velocity is analogous to summing voltages written in terms of current, and the resulting mechanical differential equations are analogous to mesh equations. If the forces are written in terms of displacement, the resulting mechanical equations resemble, but are not analogous to, the mesh equations. We, however, will use this model for mechanical systems so that we can write equations directly in terms of displacement.

Another analogy can be drawn by comparing the force-velocity column of Table 2.4 to the current-voltage column of Table 2.3 in reverse order. Here the analogy is between force and current and between velocity and voltage. Also, the

spring is analogous to the inductor, the viscous damper is analogous to the resistor, and the mass is analogous to the capacitor. Thus, summing forces written in terms of velocity is analogous to summing currents written in terms of voltage and the resulting mechanical differential equations are analogous to nodal equations. We will discuss these analogies in more detail in Section 2.9.

We are now ready to find transfer functions for translational mechanical systems. Our first example, shown in Figure 2.15(a), is similar to the simple RLC network of Example 2.6 (see Figure 2.3). The mechanical system requires just one differential equation, called the *equation of motion*, to describe it. We will begin by assuming a positive direction of motion, for example, to the right. This assumed positive direction of motion is similar to assuming a current direction in an electrical loop. Using our assumed direction of positive motion, we first draw a free-body diagram, placing on the body all forces that act on the body either in the direction of motion or opposite to it. Next we use Newton's law to form a differential equation of motion by summing the forces and setting the sum equal to zero. Finally, assuming zero initial conditions, we take the Laplace transform of the differential equation, separate the variables, and arrive at the transfer function. An example follows.

Example 2.16

Transfer Function—One Equation of Motion

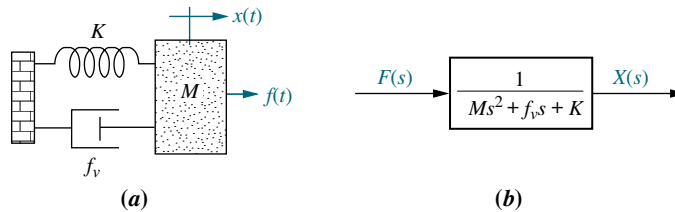


FIGURE 2.15 a. Mass, spring, and damper system; b. block diagram

PROBLEM: Find the transfer function, $X(s)/F(s)$, for the system of Figure 2.15(a).

SOLUTION: Begin the solution by drawing the free-body diagram shown in Figure 2.16(a). Place on the mass all forces felt by the mass. We assume the mass is traveling toward the right. Thus, only the applied force points to the right; all other forces impede the motion and act to oppose it. Hence, the spring, viscous damper, and the force due to acceleration point to the left.

We now write the differential equation of motion using Newton's law to sum to zero all of the forces shown on the mass in Figure 2.16(a):

$$M \frac{d^2 x(t)}{dt^2} + f_v \frac{dx(t)}{dt} + Kx(t) = f(t) \quad (2.108)$$

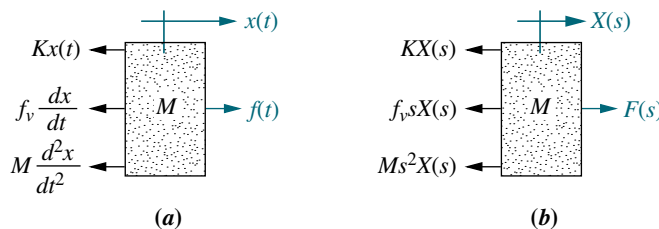


FIGURE 2.16 a. Free-body diagram of mass, spring, and damper system; b. transformed free-body diagram

Taking the Laplace transform, assuming zero initial conditions,

$$Ms^2X(s) + f_v sX(s) + KX(s) = F(s) \quad (2.109)$$

or

$$(Ms^2 + f_v s + K)X(s) = F(s) \quad (2.110)$$

Solving for the transfer function yields

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K} \quad (2.111)$$

which is represented in Figure 2.15(b).

Now can we parallel our work with electrical networks by circumventing the writing of differential equations and by defining impedances for mechanical components? If so, we can apply to mechanical systems the problem-solving techniques learned in the previous section. Taking the Laplace transform of the force-displacement column in Table 2.4, we obtain for the spring,

$$F(s) = KX(s) \quad (2.112)$$

for the viscous damper,

$$F(s) = f_v sX(s) \quad (2.113)$$

and for the mass,

$$F(s) = Ms^2X(s) \quad (2.114)$$

If we define impedance for mechanical components as

$$Z_M(s) = \frac{F(s)}{X(s)} \quad (2.115)$$

and apply the definition to Eqs. (2.112) through (2.114), we arrive at the impedances of each component as summarized in Table 2.4 (*Raven, 1995*).⁷

Replacing each force in Figure 2.16(a) by its Laplace transform, which is in the format

$$F(s) = Z_M(s)X(s) \quad (2.116)$$

we obtain Figure 2.16(b), from which we could have obtained Eq. (2.109) immediately without writing the differential equation. From now on we use this approach.

⁷Notice that the impedance column of Table 2.4 is not a direct analogy to the impedance column of Table 2.3, since the denominator of Eq. (2.115) is displacement. A direct analogy could be derived by defining mechanical impedance in terms of velocity as $F(s)/V(s)$. We chose Eq. (2.115) as a convenient definition for writing the equations of motion in terms of displacement, rather than velocity. The alternative, however, is available.

Finally, notice that Eq. (2.110) is of the form

$$[\text{Sum of impedances}]X(s) = [\text{Sum of applied forces}] \quad (2.117)$$

which is similar, but not analogous, to a mesh equation (see footnote 7).

Many mechanical systems are similar to multiple-loop and multiple-node electrical networks, where more than one simultaneous differential equation is required to describe the system. In mechanical systems, the number of equations of motion required is equal to the number of *linearly independent* motions. Linear independence implies that a point of motion in a system can still move if all other points of motion are held still. Another name for the number of linearly independent motions is the number of *degrees of freedom*. This discussion is not meant to imply that these motions are not coupled to one another; in general, they are. For example, in a two-loop electrical network, each loop current depends on the other loop current, but if we open-circuit just one of the loops, the other current can still exist if there is a voltage source in that loop. Similarly, in a mechanical system with two degrees of freedom, one point of motion can be held still while the other point of motion moves under the influence of an applied force.

In order to work such a problem, we draw the free-body diagram for each point of motion and then use superposition. For each free-body diagram we begin by holding all other points of motion still and finding the forces acting on the body due only to its own motion. Then we hold the body still and activate the other points of motion one at a time, placing on the original body the forces created by the adjacent motion.

Using Newton's law, we sum the forces on each body and set the sum to zero. The result is a system of simultaneous equations of motion. As Laplace transforms, these equations are then solved for the output variable of interest in terms of the input variable from which the transfer function is evaluated. Example 2.17 demonstrates this problem-solving technique.

Example 2.17

Transfer Function—Two Degrees of Freedom

PROBLEM: Find the transfer function, $X_2(s)/F(s)$, for the system of Figure 2.17(a).

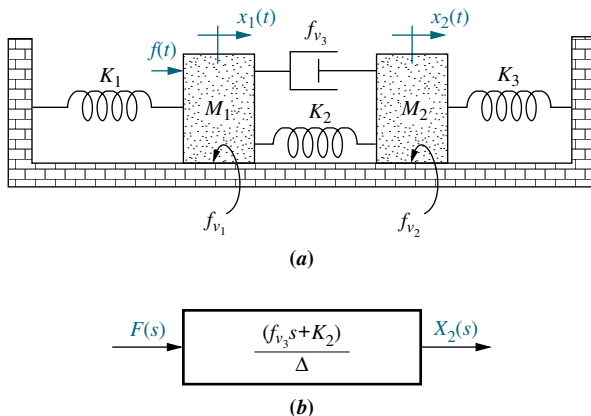


FIGURE 2.17 a. Two-degrees-of-freedom translational mechanical system;⁸ b. block diagram

Virtual Experiment 2.1 Automobile Suspension

Put theory into practice exploring the dynamics of another two degree of freedom system—an automobile suspension system driving over a bumpy road demonstrated with the Quanser Active Suspension System modeled in LabVIEW.



Virtual experiments are found on WileyPLUS.

⁸Friction shown here and throughout the book, unless otherwise indicated, is viscous friction. Thus, f_{v1} and f_{v2} are not Coulomb friction, but arise because of a viscous interface.

SOLUTION: The system has two degrees of freedom, since each mass can be moved in the horizontal direction while the other is held still. Thus, two simultaneous equations of motion will be required to describe the system. The two equations come from free-body diagrams of each mass. Superposition is used to draw the free-body diagrams. For example, the forces on M_1 are due to (1) its own motion and (2) the motion of M_2 transmitted to M_1 through the system. We will consider these two sources separately.

If we hold M_2 still and move M_1 to the right, we see the forces shown in Figure 2.18(a). If we hold M_1 still and move M_2 to the right, we see the forces shown in Figure 2.18(b). The total force on M_1 is the superposition, or sum, of the forces just discussed. This result is shown in Figure 2.18(c). For M_2 , we proceed in a similar fashion: First we move M_2 to the right while holding M_1 still; then we move M_1 to the right and hold M_2 still. For each case we evaluate the forces on M_2 . The results appear in Figure 2.19.

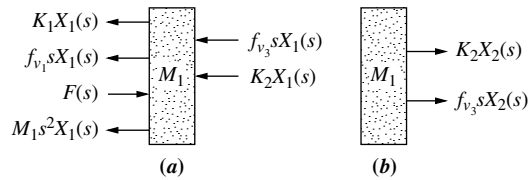


FIGURE 2.18 a. Forces on M_1 due only to motion of M_1 ; b. forces on M_1 due only to motion of M_2 ; c. all forces on M_1

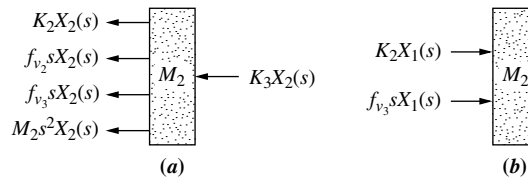
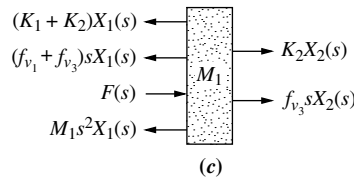
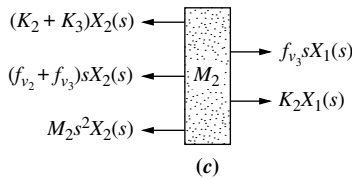


FIGURE 2.19 a. Forces on M_2 due only to motion of M_2 ; b. forces on M_2 due only to motion of M_1 ; c. all forces on M_2



The Laplace transform of the equations of motion can now be written from Figures 2.18(c) and 2.19(c) as

$$[M_1s^2(f_{v_1} + f_{v_3})s + (K_1 + K_2)]X_1(s) - (f_{v_3}s + K_2)X_2(s) = F(s) \quad (2.118a)$$

$$-(f_{v_3}s + K_2)X_1(s) + [M_2s^2 + (f_{v_2} + f_{v_3})s + (K_2 + K_3)]X_2(s) = 0 \quad (2.118b)$$

From this, the transfer function, $X_2(s)/F(s)$, is

$$\frac{X_2(s)}{F(s)} = G(s) = \frac{(f_{v_3}s + K_2)}{\Delta} \quad (2.119)$$

as shown in Figure 2.17(b) where

$$\Delta = \begin{vmatrix} [M_1s^2 + (f_{v_1} + f_{v_3})s + (K_1 + K_2)] & -(f_{v_3}s + K_2) \\ -(f_{v_3}s + K_2) & [M_2s^2 + (f_{v_2} + f_{v_3})s + (K_2 + K_3)] \end{vmatrix}$$

Notice again, in Eq. (2.118), that the form of the equations is similar to electrical mesh equations:

$$\begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{connected} \\ \text{to the motion} \\ \text{at } x_1 \end{bmatrix} X_1(s) - \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_1 \text{ and } x_2 \end{bmatrix} X_2(s) = \begin{bmatrix} \text{Sum of} \\ \text{applied forces} \\ \text{at } x_1 \end{bmatrix} \quad (2.120a)$$

$$- \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_1 \text{ and } x_2 \end{bmatrix} X_1(s) + \begin{bmatrix} \text{Sum of} \\ \text{impedances} \\ \text{connected} \\ \text{to the motion} \\ \text{at } x_2 \end{bmatrix} X_2(s) = \begin{bmatrix} \text{Sum of} \\ \text{applied forces} \\ \text{at } x_2 \end{bmatrix} \quad (2.120b)$$

The pattern shown in Eq. (2.120) should now be familiar to us. Let us use the concept to write the equations of motion of a three-degrees-of-freedom mechanical network by inspection, without drawing the free-body diagram.

Example 2.18

Equations of Motion by Inspection

PROBLEM: Write, but do not solve, the equations of motion for the mechanical network of Figure 2.20.

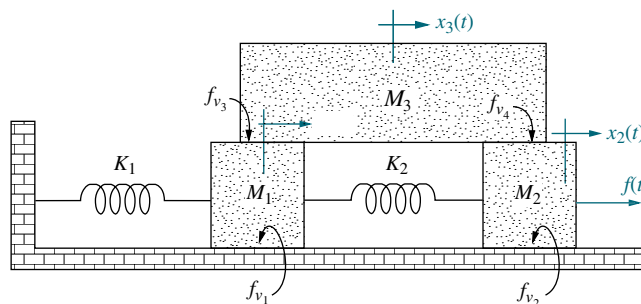


FIGURE 2.20 Three-degrees-of-freedom translational mechanical system

SOLUTION: The system has three degrees of freedom, since each of the three masses can be moved independently while the others are held still. The form of the equations will be similar to electrical mesh equations. For M_1 ,

$$\begin{aligned} \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{connected} \\ \text{to the motion} \\ \text{at } x_1 \end{array} \right] X_1(s) - \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_1 \text{ and } x_2 \end{array} \right] X_2(s) \\ - \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_1 \text{ and } x_3 \end{array} \right] X_3(s) = \left[\begin{array}{l} \text{Sum of} \\ \text{applied forces} \\ \text{at } x_1 \end{array} \right] \end{aligned} \quad (2.121)$$

Similarly, for M_2 and M_3 , respectively,

$$\begin{aligned} - \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_1 \text{ and } x_2 \end{array} \right] X_1(s) + \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{connected} \\ \text{to the motion} \\ \text{at } x_2 \end{array} \right] X_2(s) \\ - \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_2 \text{ and } x_3 \end{array} \right] X_3(s) = \left[\begin{array}{l} \text{Sum of} \\ \text{applied forces} \\ \text{at } x_2 \end{array} \right] \end{aligned} \quad (2.122)$$

$$\begin{aligned} - \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_1 \text{ and } x_3 \end{array} \right] X_1(s) - \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{between} \\ x_2 \text{ and } x_3 \end{array} \right] X_2(s) \\ + \left[\begin{array}{l} \text{Sum of} \\ \text{impedances} \\ \text{connected} \\ \text{to the motion} \\ \text{at } x_3 \end{array} \right] X_3(s) = \left[\begin{array}{l} \text{Sum of} \\ \text{applied forces} \\ \text{at } x_3 \end{array} \right] \end{aligned} \quad (2.123)$$

M_1 has two springs, two viscous dampers, and mass associated with its motion. There is one spring between M_1 and M_2 and one viscous damper between M_1 and M_3 . Thus, using Eq. (2.121),

$$[M_1 s^2 + (f_{v_1} + f_{v_3})s + (K_1 + K_2)]X_1(s) - K_2 X_2(s) - f_{v_3} s X_3(s) = 0 \quad (2.124)$$

Similarly, using Eq. (2.122) for M_2 ,

$$-K_2 X_1(s) + [M_2 s^2 + (f_{v_2} + f_{v_4})s + K_2]X_2(s) - f_{v_4} s X_3(s) = F(s) \quad (2.125)$$

and using Eq. (2.123) for M_3 ,

$$-f_{v_3}sX_1(s) - f_{v_4}sX_2(s) + [M_3s^2 + (f_{v_3} + f_{v_4})s]X_3(s) = 0 \quad (2.126)$$

Equations (2.124) through (2.126) are the equations of motion. We can solve them for any displacement, $X_1(s)$, $X_2(s)$, or $X_3(s)$, or transfer function.

Skill-Assessment Exercise 2.8

PROBLEM: Find the transfer function, $G(s) = X_2(s)/F(s)$, for the translational mechanical system shown in Figure 2.21.

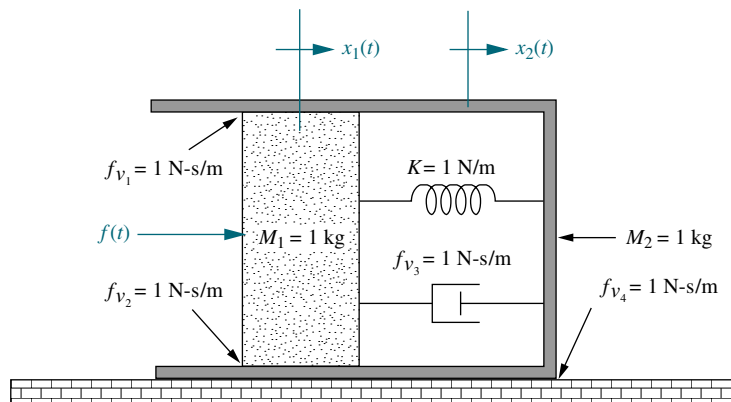


FIGURE 2.21 Translational mechanical system for Skill-Assessment Exercise 2.8

ANSWER:
$$G(s) = \frac{3s + 1}{s(s^3 + 7s^2 + 5s + 1)}$$

The complete solution is at www.wiley.com/college/nise.

2.6 Rotational Mechanical System Transfer Functions

Having covered electrical and translational mechanical systems, we now move on to consider rotational mechanical systems. Rotational mechanical systems are handled the same way as translational mechanical systems, except that torque replaces force and angular displacement replaces translational displacement. The mechanical components for rotational systems are the same as those for translational systems, except that the components undergo rotation instead of translation. Table 2.5 shows the components along with the relationships between torque and angular velocity, as well as angular displacement. Notice that the symbols for the