14.2 PROBABILITY OR RANDOM SAMPLES

A sample is called a *random sample* if the probability of selection for each unit in the population is known prior to sample selection. The important kinds of random samples which differ in the manner in which the sampling units are selected, are discussed in the subsections that follow:

14.2.1 Simple Random Sample. A sample is defined to be a *simple random sample* (*SRS*) if it is selected in such a manner that (i) each unit in the population has an equal probability of being included in the sample and (ii) each possible sample of the same size has an equal probability of being the sample selected.

Suppose a finite population contains N units and a sample of n units is to be selected. If we sample with replacement, the number of all *possible* samples of size n that could be selected is N^n, as the first unit of the sample can be selected in N different ways, the second unit can also be selected in N ways and so on. When we sample without replacement, then the number of all possible samples when the *order* of the units is considered, is the number of *permutations* of n units from N, *i.e.* ${}^{N}P_{n} = N(N-1) \ldots (N-n+1)$. But in practical problems, we ignore the *order* in which the n units are drawn. Then the number of different samples of n units that can be selected when the *order* is disregarded, is the number of *combinations* of n units from a finite population of N units, *i.e.*, $\binom{N}{n} = \frac{N!}{n!\,(N-n)!}$. Thus there are $\binom{N}{n}$ samples that could be selected and these samples occur with equal probabilities.

As an illustration, suppose we wish to select random samples of size 2 from a population of say, 5 students, identified as A, B, C, D and E. If we sample *with replacement*, then there are $(5)^2 = 25$ possible samples, which are listed below:

AA	*BA*	*CA*	*DA*	*EA*
AB	*BB*	*CB*	*DB*	*EB*
AC	*BC*	*CC*	*DC*	*EC*
AD	*BD*	*CD*	*DD*	*ED*
AE	*BE*	*CE*	*DE*	*EE*

Example 14.1. Assume that a population consists of 5 students and the marks obtained by them in a certain statistics class are 20, 15, 12, 16 and 18. Draw all possible random samples of two students when sampling is performed (i) with replacement, (ii) without replacement. Calculate the mean marks for each sample.

Let the five students be identified as A, B, C, D and E. Then (i) number of possible random samples of 2 students which can be selec *with replacement* from this population is $(5)^2 = 25$. Let X_1 denote marks of the student selected first and X_2, the marks of the stud selected on the second draw. Then the possible random samples of $n = 2$ with values of $\bar{X}$ are given below:

Sample No.	Sample Students	Sample Marks (X_1, X_2)	Sample Mean Marks $(\bar{X})$
1	*A, A*	20, 20	20
2	*A, B*	20, 15	17.5
3	*A, C*	20, 12	16
4	*A, D*	20, 16	18
5	*A, E*	20, 18	19
6	*B, A*	15, 20	17.5
7	*B, B*	15, 15	15
8	*B, C*	15, 12	13.5
9	*B, D*	15, 16	15.5
10	*B, E*	15, 18	16.5
11	*C, A*	12, 20	16
12	*C, B*	12, 15	13.5
13	*C, C*	12, 12	12
14	*C, D*	12, 16	14
15	*C, E*	12, 18	15
16	*D, A*	16, 20	18
17	*D, B*	16, 15	15.5
18	*D, C*	16, 12	14
19	*D, D*	16, 16	16
20	*D, E*	16, 18	17
21	*E, A*	18, 20	19
22	*E, B*	18, 15	16.5
23	*E, C*	18, 12	15
24	*E, D*	18, 16	17
25	*E, E*	18, 18	18

(ii) the number of random samples of 2 students that can be drawn *without replacement* is $\binom{5}{2} = 10$. These samples with values of mean marks are given below:

Sample No.	Sample Students	Sample Marks (X_1, X_2)	Sample Mean Marks $(\bar{X})$
1	*A, B*	20, 15	17.5
2	*A, C*	20, 12	16
3	*A, D*	20, 16	18
4	*A, E*	20, 18	19
5	*B, C*	15, 12	13.5
6	*B, D*	15, 16	15.5
7	*B, E*	15, 18	16.5
8	*C, D*	12, 16	14
9	*C, E*	12, 18	15
10	*D, E*	16, 18	17