

⇒ Z-score Normalized distribution

It is also called a standard score which gives you an idea of how far from the mean of data. But more technically its measure of how many standard deviations below or above the population mean a raw score is.

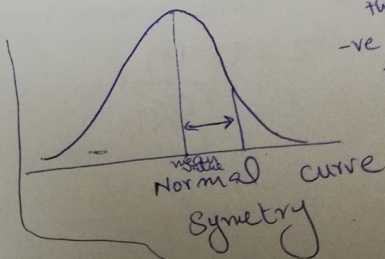
A z-score can be placed on a normal distribution curve. Z-score range from -3σ (which would fall to the far left of normal distribution curve) up to $+3\sigma$ (which would fall to the far right of the normal distribution curve). In order to use a z-score we need to know the mean μ and also the population standard deviation σ .

Formula

The basic z-score formula for a sample

$$\text{is } z = \frac{x - \mu}{\sigma}$$

(for one sample)



+ve lie above the mean.
-ve lie below the mean.

~~e.g.~~ Let's say you have a test score of 190. The test has a mean (μ) of 150 and standard deviation (σ) of 25.

Assuming normal dist.

$$\begin{aligned} z &= \frac{x - \mu}{\sigma} \\ &= \frac{190 - 150}{25} = 1.6 \end{aligned}$$

The z score tells you how many S.D from the mean your score is. In this your score is 1.6 S.D above the mean.

Z-score formula

Standard error of mean.

$$Z = \frac{\overset{\text{sample mean}}{x} - \underset{\text{pop. mean}}{\mu}}{\sigma/\sqrt{n}}$$

Z-score will tell you how many standard errors there are between the sample mean and population mean.

Q In general the mean height of women is 65 with a S.D of 3.5. What is the probability of finding a random sample of 50 women with a mean height of 70 assuming the heights are normally distributed.

$$Z = \frac{(x - \mu)}{\sigma/\sqrt{n}}$$

$$= \frac{70 - 65}{3.5/\sqrt{50}} = \frac{5}{0.495} = 10.1$$

$$\frac{5}{100} = 5\%$$

As we know that 99% of values fall within 3 standard deviation from the mean in normal Probability distribution. therefore there is less than 1% probability that any sample of women will have a mean height of 70"

Assignment How is it used in Real life

⇒ Standardized distribution :

A normal distribution whose value have undergone transformation so as to have a mean of 0 and standard deviation of 1. Also called standard normal distribution.

⇒ Normal Distribution -

It is defined by the (P.d.f)

Probability density function.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{[x-\mu]^2}{2\sigma^2}} \quad \text{for } -\infty < x < \infty$$

and $\sigma > 0$

where μ is the mean, σ (sigma) is the standard deviation and $\pi = 3.1416$ and $e = 2.7183$ are constant.

It has 2 parameter μ and σ , its mean and standard deviation.

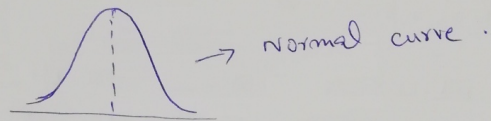
* Normal distribution having mean μ and variance σ^2 is usually denoted by $N(\mu, \sigma^2)$

Thus we also said that a random variable X is normally distributed with mean (μ) and variance (σ^2) i.e.

$$X \sim N(\mu, \sigma^2)$$

* The graph of normal dist, which is symmetrical bell shaped curve, is called Normal curve. The location & shape of the normal curve is determined by μ & σ .

⇒



Q

Assignment

Importance of Normal Distribution
what is its importance in research.

⇒ Standardized Normal Distribution

Normal Probability dist depends on the values of Parameter μ and σ^2 and the various possible values for these 2 parameter. will result in an ~~limited~~ no. of diff Normal dist. The r.v. $Z = \frac{X - \mu}{\sigma}$, and having zero mean & unit variance ~~etc~~. Every normally dist r.v X with mean $= \mu$ & var $= \sigma^2$ is \therefore transformed into a new normal r.v Z .

So the P.d.f of Z denoted $\phi(z)$

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad \text{for } -\infty < z < \infty$$

So the normal probability dist of Z which has zero mean and unit variance is called Standardized normal dist or unit normal dist and is denoted by $N(0,1)$.

Properties

- 1) Mean $\Rightarrow E(X) = \mu$.
- 2) variance $\Rightarrow \text{var}(X) = \sigma^2$
- 3) mean = mode = median i.e unimodal

⇒ Standardizing a distribution.

It is a process of transforming a variable to one with a mean of 0 and standard deviation of 1
 $\sim (\mu, \sigma^2) \rightarrow \sim (0, 1)$

⇒ Normal distribution can also be standardized

$$\sim N(\mu, \sigma^2) \rightarrow \sim N(0, 1)$$

$$z = \frac{x - \mu}{\sigma}$$

Q Why would you want to transform a set of raw scores into a set of z-scores?

To make it possible to compare scores from two different distributions and to make a distribution with a mean of 0 and S.D of 1.

⇒ Applications of Normal Distribution.