14.4 NUCLEAR FISSION

Nuclear fission occurs when a very heavy nucleus such as ^{235}U splits, or fissions, into two particles of comparable mass. In such a reaction, **the total mass of the product particles is less than the original mass.** Fission is initiated by the capture of a thermal neutron by a heavy nucleus and involves the release of about 200 MeV per fission. This energy release occurs because the smaller fission-product nuclei are more tightly bound by about 1 MeV per nucleon than the original heavy nucleus.

The process of nuclear fission was first observed in 1938 by Otto Hahn (1879–1968, German chemist), Lise Meitner (1878–1968, Austrian physicist), and Fritz Strassmann (1902–1980, German chemist), following some basic studies by Fermi concerning the interaction of thermal neutrons with uranium. After bombarding uranium ($Z = 92$) with neutrons, Hahn and Strassmann performed a chemical analysis and discovered among the products two medium-mass elements, barium and lanthanum. Shortly thereafter, Lise Meitner and her nephew Otto Frisch (1904–1979, German-British physicist) explained what had happened and coined the term *fission*. The uranium nucleus could split into nearly equal fragments after absorbing a neutron. Measurements showed that about 200 MeV of energy was released in each fission event.

The fission of ^{235}U by thermal neutrons can be represented by

Fission of ^{235}U

$$^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \longrightarrow {}^{236}_{92}\text{U}^* \longrightarrow \text{X} + \text{Y} + \text{neutrons} \tag{14.8}$$

where ^{236}U* is an intermediate excited state that lasts for only about 10^{-12} s before splitting into X and Y. The resulting nuclei X and Y are called **fission fragments.** There are many combinations of X and Y that satisfy the requirements of conservation of mass–energy, charge, and nucleon number. Figure 14.4 shows the actual mass distribution of fragments in the fission of ^{235}U. The

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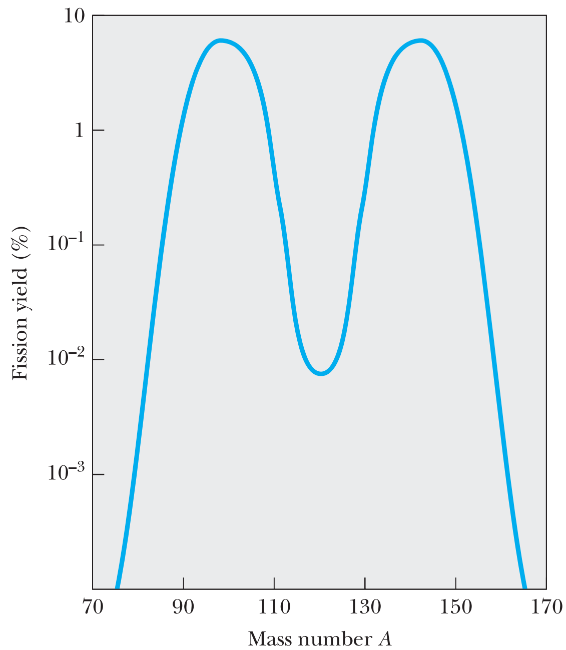

Figure 14.4 The distribution of fission products versus mass number for the fission of ^{235}U bombarded with slow neutrons. Note that the ordinate has a logarithmic scale.

process results in the production of several neutrons, typically two or three. On the average, about 2.5 neutrons are released per event. A typical reaction of this type is

$$^{1}_{0}n + ^{235}_{92}U \longrightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n \qquad (14.9)$$

Of the 200 MeV or so released in this reaction, most goes into the kinetic energy of the heavy fragments barium and krypton.

The breakup of the uranium nucleus can be compared to what happens to a drop of water when excess energy is added to it. All the atoms in the drop have energy, but not enough to break up the drop. However, if enough energy is added to set the drop into vibration, it undergoes elongation and compression until the amplitude of vibration becomes large enough to cause the drop to break. In the uranium nucleus, a similar process occurs. Figure 14.5 shows various stages of the nucleus as the result of neutron capture. The sequence of events for ^{235}U can be described as follows:

1. The ^{235}U nucleus captures a thermal (slow-moving) neutron.
2. This capture results in the formation of $^{236}U^*$, and the excess energy of this nucleus causes it to undergo violent oscillations.
3. The $^{236}U^*$ nucleus becomes highly distorted, and the force of repulsion between protons in the two halves of the dumbbell shape tends to increase the distortion.
4. The nucleus splits into two fragments, emitting several neutrons in the process.

As can be seen in Figure 14.4, the most probable fission events correspond to fission fragments with mass numbers $A \approx 140$ and $A \approx 95$. These

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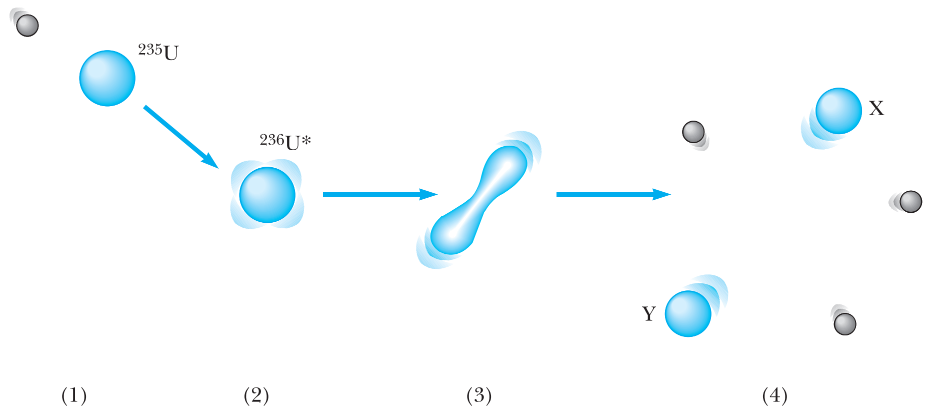

Figure 14.5 The stages in a nuclear fission event as described by the liquid-drop model of the nucleus.

fragments, which share the protons and neutrons of the mother nucleus, both fall on the neutron-rich side of the stability line in Figure 13.4 (Chapter 13). Since fragments that have a large excess of neutrons are unstable, the neutron-rich fragments almost instantaneously release two or three neutrons. The remaining fragments are still rich in neutrons and proceed to decay to more stable nuclei through a succession of beta decays. In the process of such decays, gamma rays are also emitted by nuclei in excited states.

Let us estimate the disintegration energy Q released in a typical fission process. From Figure 13.10 we see that the binding energy per nucleon for heavy nuclei ($A \approx 240$) is about 7.6 MeV, whereas in the intermediate mass range, the binding energy per nucleon is about 8.5 MeV. Taking the mass number of the mother nucleus to be $A = 240$, we see that the energy released per nucleon is estimated to be

$$Q = (240 \text{ nucleons})\left(8.5\,\frac{\text{MeV}}{\text{nucleon}} - 7.6\,\frac{\text{MeV}}{\text{nucleon}}\right) = 200 \text{ MeV}$$

About 85% of this energy appears in the form of kinetic energy in the heavy fragments. This energy is very large compared to the energy released in chemical processes. For example, the energy released in the combustion of one molecule of octane used in gasoline engines is about one-millionth the energy released in a single uranium fission event!

EXAMPLE 14.3 The Fission of Uranium

In addition to the barium–lanthanum reaction observed by Meitner and Frisch and the barium–krypton reaction of Equation 14.9, two other ways in which ^{235}U can fission when bombarded with a neutron are (1) by forming ^{140}Xe and ^{94}Sr and (2) by forming ^{132}Sn and ^{101}Mo. In each case, neutrons are also released. Find the number of neutrons released in each event.

Solution By balancing mass numbers and atomic numbers, we find that these reactions can be written

$$^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \longrightarrow {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + 2({}^{1}_{0}\text{n})$$
$$^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \longrightarrow {}^{132}_{50}\text{Sn} + {}^{101}_{42}\text{Mo} + 3({}^{1}_{0}\text{n})$$

Thus two neutrons are released in the first reaction and three in the second.

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EXAMPLE 14.4 The Energy Released in the Fission of ^{235}U

Calculate the total energy released if 1.00 kg of ^{235}U undergoes fission, taking the disintegration energy per event to be $Q = 208$ MeV (a more accurate value than the estimate given before).

Solution We need to know the number of nuclei in 1 kg of uranium. Since $A = 235$, the number of nuclei is

$$N = \frac{6.02 \times 10^{23}\ \text{nuclei/mol}}{235\ \text{g/mol}}(1.00 \times 10^3\ \text{g})$$
$$= 2.56 \times 10^{24}\ \text{nuclei}$$

Hence the disintegration energy is

$$E = NQ = (2.56 \times 10^{24}\ \text{nuclei})\left(208\ \frac{\text{MeV}}{\text{nucleus}}\right)$$
$$= 5.32 \times 10^{26}\ \text{MeV}$$

Since 1 MeV is equivalent to 4.14×10^{-20} kWh, we find that $E = 2.37 \times 10^7$ kWh. If this amount of energy were released suddenly, it would be equivalent to detonating 20,000 tons of TNT!

EXAMPLE 14.5 A Rough Mechanism for the Fission Process

Consider the fission reaction $^{235}_{92}\text{U} + \text{n} \rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 2^{1}_{0}\text{n}$. Show that the model of the fission process in which an excited $^{235}_{92}$U nucleus elongates enough to overcome attractive nuclear forces and separates into two charged fragments can be used to estimate the energy released in this fission process. In this model, we assume that the incident neutron provides a few MeV of excitation energy to separate the Ba and Kr nuclei within the uranium nucleus so that the two fragments $^{141}_{56}$Ba and $^{92}_{36}$Kr are driven apart by Coulomb repulsion. We also neglect the small amount of kinetic energy (several MeV) carried off by the neutrons produced in the reaction.

Solution First we calculate the separation, r, of the Ba and Kr nuclei at which the nuclear force between them falls to zero. This is $r \cong r_{\text{Ba}} + r_{\text{Kr}}$, where r_{Ba} and r_{Kr} are the nuclear radii of Ba and Kr given by Equation 13.1. Thus

$$r_{\text{Ba}} = (1.2 \times 10^{-15}\ \text{m})(141)^{1/3} = 6.2 \times 10^{-15}\ \text{m}$$
$$r_{\text{Kr}} = (1.2 \times 10^{-15}\ \text{m})(92)^{1/3} = 5.4 \times 10^{-15}\ \text{m}$$
$$r \cong 12 \times 10^{-15}\ \text{m}$$

Next calculate the Coulomb potential energy for two charges of $Z_1 = 56$ and $Z_2 = 36$ separated by a distance of 12 fm. The potential energy of the two nuclei on the brink of separation is

$$U = \frac{k(Z_1 e)(Z_2 e)}{r} = \frac{(1.440\ \text{eV}\cdot\text{nm})(56)(36)}{12 \times 10^{-15}\ \text{m}}$$
$$= 240\ \text{MeV}$$

As the two fragments separate, this potential energy is converted to an amount of kinetic energy consistent with the total measured energy release of about 200 MeV. This shows that the simple fission mechanism suggested here is plausible.

14.5 NUCLEAR REACTORS

In the last section we saw that when ^{235}U fissions, an average of 2.5 neutrons are emitted per event. These neutrons can in turn trigger other nuclei to fission, with the possibility of a chain reaction (Fig. 14.6). Calculations show that if the chain reaction is not controlled (that is, if it does not proceed slowly), it can result in a violent explosion, with the release of an enormous amount of energy. For example, if the energy in 1 kg of ^{235}U were released, it would be equivalent to detonating about 20,000 tons of TNT! This, of course, is the principle behind the first nuclear bomb, an uncontrolled fission reaction.

A nuclear reactor is a system designed to maintain what is called a **self-sustained chain reaction.** This important process was first achieved in 1942 by Fermi at the University of Chicago, with natural uranium as the fuel (Fig. 14.7). Most reactors in operation today also use uranium as fuel. Natural uranium contains only about 0.7% of the ^{235}U isotope, with the remaining 99.3% being ^{238}U. This fact is important for the operation of a reactor, because ^{238}U almost never fissions. Instead, it tends to absorb neutrons,

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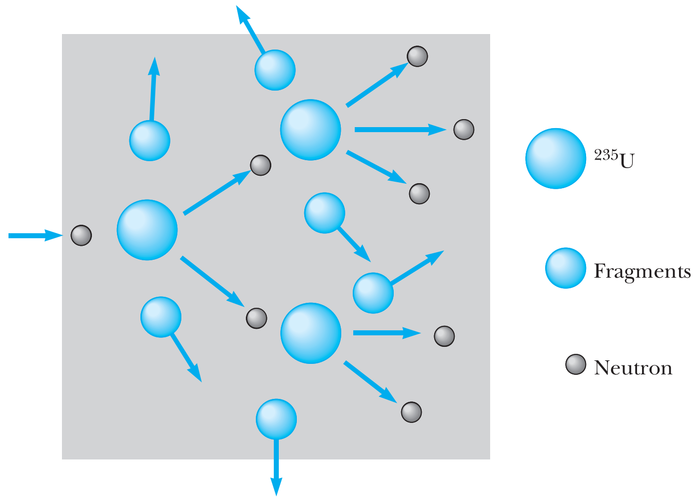

Figure 14.6 A nuclear chain reaction initiated by the capture of a neutron. (Many pairs of different isotopes are produced.)

producing neptunium and plutonium. For this reason, reactor fuels must be artificially enriched to contain at least a few percent ^{235}U.

Chain reaction

To achieve a self-sustained chain reaction, on average one of the neutrons emitted in ^{235}U fission must be captured by another ^{235}U nucleus and cause it to undergo fission. A useful parameter for describing the level of reactor

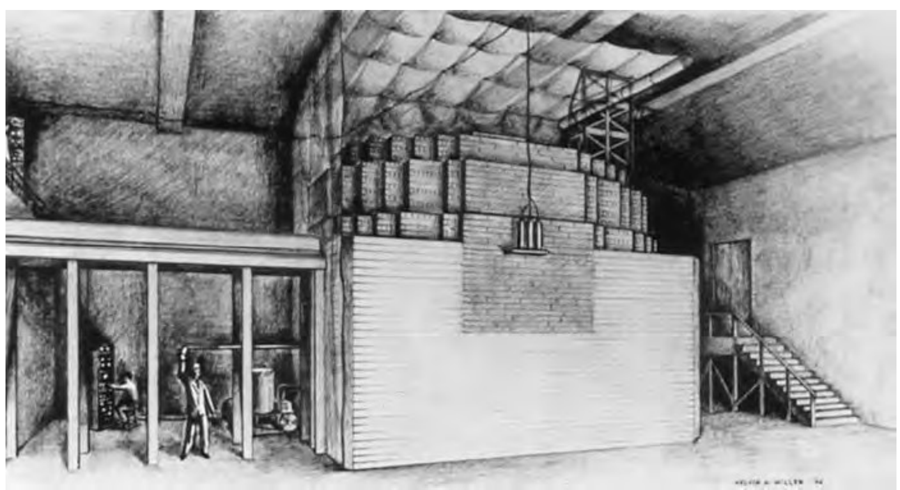

Figure 14.7 A sketch of the world's first reactor, which was composed of layers of graphite interspersed with uranium. (Because of wartime secrecy, there are no photographs of the completed reactor.) The first self-sustained chain reaction was achieved on December 2, 1942. Word of the success was telephoned immediately to Washington, D.C., in the form of this coded message: "The Italian navigator has landed in the New World and found the natives very friendly." The historic event took place in an improvised laboratory in the racquet court under the west stands of the University of Chicago's Stagg Field, and the Italian navigator guiding the work was Fermi.

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operation is the **reproduction constant** K, defined as the average number of neutrons from each fission event that actually cause another fission event. As we have seen, K can have a maximum value of 2.5 in the fission of uranium. In practice, however, K is less than this because of several factors, to be discussed.

Reproduction constant

A self-sustained chain reaction is achieved when $K = 1$. Under this condition the reactor is said to be **critical.** When $K < 1$, the reactor is subcritical and the reaction dies out. When K is substantially greater than unity, the reactor is said to be supercritical and a runaway reaction occurs. In a nuclear reactor run by a utility company to furnish power, it is necessary to maintain a value of K slightly greater than unity.

Figure 14.8 shows the basic ingredients of a nuclear reactor core. The fuel elements consist of enriched uranium. The function of the remaining parts of the reactor and some aspects of its design will now be described.

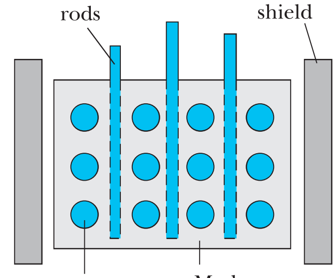

Figure 14.8 A cross section of a reactor core showing the control rods, fuel elements, and moderating material surrounded by a radiation shield.

Neutron Leakage

In any reactor, a fraction of the neutrons produced in fission leak out of the core before inducing other fission events. If the fraction leaking out is too great, the reactor will not operate. The percentage lost is large if the reactor is very small because leakage is a function of the ratio of surface area to volume. Therefore, a critical feature of the design of a reactor is to choose the correct surface-area-to-volume ratio so that a sustained reaction can be achieved.

Regulating Neutron Energies

Recall that the neutrons released in fission events are very energetic, having kinetic energies of about 2 MeV. It is necessary to slow these neutrons to thermal energies to allow them to be captured and produce fission of other ^{235}U nuclei, because the probability of neutron-induced fission increases with decreasing energy, as shown in Figure 14.9. The energetic neutrons are slowed down by a moderator substance surrounding the fuel, as shown in Figure 14.8.

Neutron Capture

In the process of being slowed down, neutrons may be captured by nuclei that do not fission. The most common event of this type is neutron capture by ^{238}U, which constitutes over 90% of the uranium in the fuel elements. The probability of neutron capture by ^{238}U is very high when the neutrons have high kinetic energies and very low when they have low kinetic energies. Thus the slowing down of the neutrons by the moderator serves the secondary purpose of making them available for reaction with ^{235}U and decreasing their chances of being captured by ^{238}U.

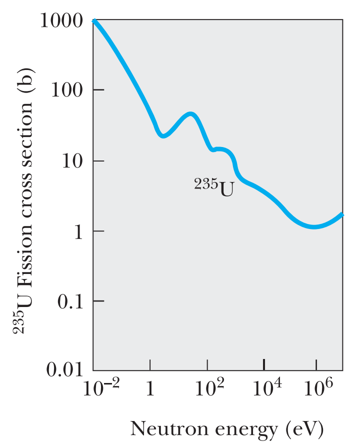

Figure 14.9 The cross section for neutron-induced fission of ^{235}U. The average cross section for room-temperature neutrons is about 500 b.

Control of Power Level

It is possible for a reactor to reach the critical stage ($K = 1$) after all the neutron losses just described are minimized. However, some method of control is needed to maintain a K value near unity. If K were to rise above this value, the heat produced in the runaway reaction would melt the reactor. To control the power level, control rods are inserted into the reactor core (see Fig. 14.8). These rods are made of materials such as cadmium that absorb neutrons very

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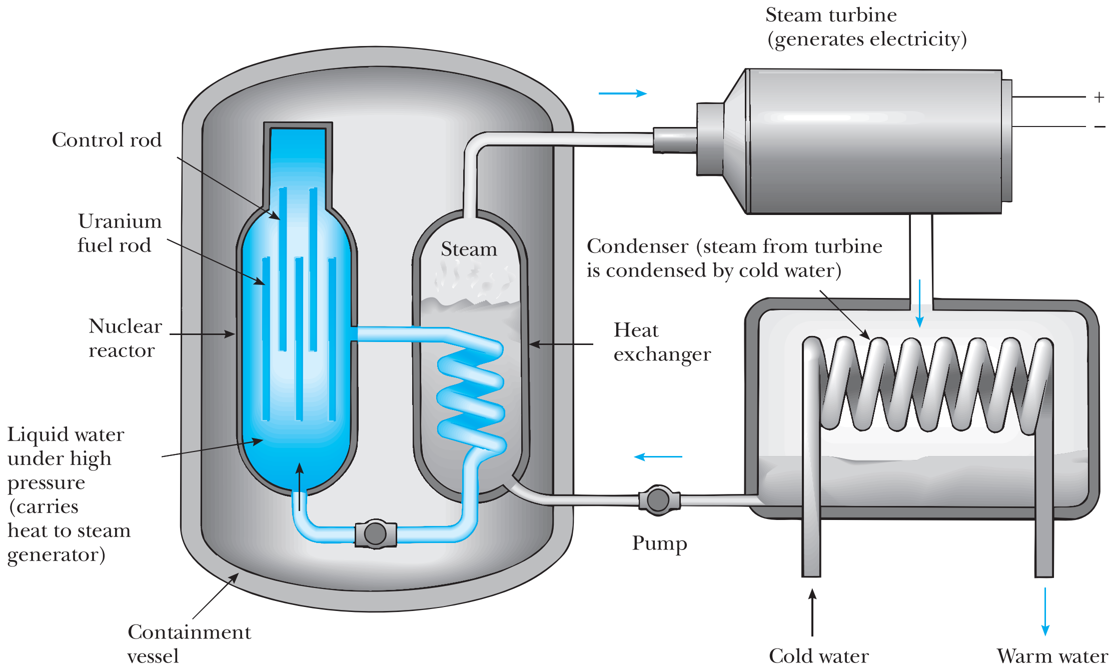

Figure 14.10 Main components of a pressurized-water reactor.

efficiently (see Fig. 14.3). By adjusting the number and position of these control rods in the reactor core, the K value can be varied and any power level within the design range of the reactor can be achieved.

Although there are several types of reactor that convert the kinetic energy of fission fragments to electrical energy, the most common type in use in the United States is the pressurized-water reactor (Fig. 14.10). Its main parts are common to all reactor designs. Fission events in the reactor core supply heat to the water contained in the primary (closed) loop, which is maintained at high pressure to keep it from boiling. This water also serves as the moderator. The hot water is pumped through a heat exchanger, and the heat is transferred to the water contained in the secondary loop. The hot water in the secondary loop is converted to steam, which drives a turbine-generator system to create electric power. Note that the water in the secondary loop is isolated from the water in the primary loop to prevent contamination of the secondary water and steam by radioactive nuclei from the reactor core.

Safety and Waste Disposal

The 1979 near-disaster at a nuclear power plant on Three Mile Island in Pennsylvania and the 1986 accident at the Chernobyl reactor in the Ukraine rightfully focused attention on reactor safety. The Three Mile Island accident was the result of inadequate control-room instrumentation and poor emergency-response training. There were no injuries or detectable health impacts from the event, even though more than one-third of the fuel melted. This unfortunately was not the case at Chernobyl, where the activity of the materials released immediately after the accident totaled approximately 1.2×10^{19} Bq and resulted in the evacu-

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ation of 135,000 people. Thirty individuals died during the accident or shortly thereafter and data from the Ukraine Radiological Institute suggest that more than 2,500 deaths could be attributed to the Chernobyl accident. In the period 1986–1997 there was a tenfold increase in the number of children contracting thyroid cancer from the ingestion of radioactive iodine in milk from cows that ate contaminated grass. One conclusion of an international conference studying the Ukraine accident was that the main causes of the Chernobyl accident were the coincidence of severe deficiencies in the reactor design and a violation of safety procedures. Most of these deficiencies have been addressed at plants of similar design in Russia and neighboring countries of the former Soviet Union.

Commercial reactors achieve safety through careful design, rigid operating protocol, and thorough emergency-response training of operators. It is only when these variables are compromised that reactors pose a danger. Radiation exposure and the potential health risks associated with such exposure are controlled by three layers of containment. The fuel and radioactive fission products are contained inside the reactor vessel. Should this vessel rupture, the reactor building acts as a second containment structure to prevent radioactive material from contaminating the environment. Finally, the reactor facilities must be in a remote location to protect the general public from exposure should radiation escape the reactor building.

A continuing concern about nuclear fission reactors is the safe disposal of radioactive material when the reactor core is replaced. Even when the uranium and plutonium are separated out and recycled, the remaining waste material contains long-lived, highly radioactive isotopes that must be stored over long time intervals in such a way that there is no chance of environmental contamination. At present, sealing radioactive wastes in waterproof containers and burying them in deep salt mines seems to be the most promising solution.

Transport of reactor fuel and reactor wastes poses additional safety risks. Accidents during transport of nuclear fuel could expose the public to harmful levels of radiation. To minimize these dangers, the Department of Energy requires stringent crash tests of all containers used to transport nuclear materials. Container manufacturers must demonstrate that their containers will not rupture even in high-speed collisions.

Despite these risks, there are advantages to the use of nuclear power to be weighed against the risks. For example, nuclear power plants do not produce air pollution and greenhouse gases as do fossil fuel plants, and the supply of uranium on the Earth is predicted to last longer than the supply of fossil fuels. For each source of energy, whether nuclear, hydroelectric, fossil fuel, wind, or solar, the risks must be weighed against the benefits and the regional availability of the energy source. Thus, thoughtful use of a variety of energy sources *and* increased emphasis on energy conservation methods appear to be logical components of a sensible energy policy.

14.6 NUCLEAR FUSION

In Chapter 13 we found that the binding energy for light nuclei (those having mass numbers less than 20) is much smaller than the binding energy for heavier nuclei. This suggests a process that is the reverse of fission, called **nuclear fusion.** Fusion occurs when two light nuclei combine to form a heavier nucleus. Because the mass of the final nucleus is less than the combined rest

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masses of the original nuclei, a loss of mass occurs, accompanied by a release of energy. The following are examples of such energy-liberating fusion reactions occuring in the Sun:

$$\begin{aligned} {}^1_1\text{H} + {}^1_1\text{H} &\longrightarrow {}^2_1\text{H} + {}^0_1\text{e}^+ + \nu \\ {}^1_1\text{H} + {}^2_1\text{H} &\longrightarrow {}^3_2\text{He} + \gamma \end{aligned} \qquad (14.10)$$

This second reaction is followed by one of the following reactions:

$$\begin{aligned} {}^1_1\text{H} + {}^3_2\text{H} &\longrightarrow {}^4_2\text{He} + {}^0_1\text{e}^+ + \nu \\ {}^3_2\text{He} + {}^3_2\text{He} &\longrightarrow {}^4_2\text{He} + {}^1_1\text{H} + {}^1_1\text{H} \end{aligned}$$

These are the basic reactions in what is called the **proton–proton cycle,** believed to be one of the basic cycles by which energy is generated in the Sun and other stars that have an abundance of hydrogen. Most of the energy production takes place in the Sun's interior, where the temperature is approximately 1.5×10^7 K. As we will see later, such high temperatures are required to drive these reactions that they are called **thermonuclear fusion reactions.** The hydrogen (fusion) bomb, which was first exploded in 1952, is an example of an uncontrolled thermonuclear fusion reaction. All of the reactions in the proton–proton cycle are exothermic—that is, they involve a release of energy. An overall view of the proton–proton cycle is that four protons combine to form an alpha particle and two positrons, with the release of 25 MeV of energy.

Thermonuclear reactions

Fusion Reactions

The enormous amount of energy released in fusion reactions suggests the possibility of harnessing this energy for useful purposes here on Earth. A great deal of effort is currently directed toward developing a sustained and controllable thermonuclear reactor—a fusion power reactor. Controlled fusion is often called the ultimate energy source because of the availability of its fuel source: water. For example, if deuterium were used as the fuel, 0.12 g of it could be extracted from 1 gal of water at a cost of about 4 cents. Such rates would make the fuel costs of even an inefficient reactor almost insignificant. An additional advantage of fusion reactors is that comparatively few radioactive by-products are formed. For the proton–proton cycle described earlier in this section, the end product of the fusion of hydrogen nuclei is safe, nonradioactive helium. Unfortunately, a thermonuclear reactor that can deliver a net power output spread out over a reasonable time interval is not yet a reality, even though research has been in progress since the 1950s. Many difficulties must be resolved before a successful device is constructed.

We have seen that the Sun's energy is based, in part, upon a set of reactions in which hydrogen is converted to helium. Unfortunately, the proton–proton interaction is not suitable for use in a fusion reactor, because this reaction requires very high pressures and densities. The process works in the Sun only because of the extremely high density of protons in the Sun's interior.

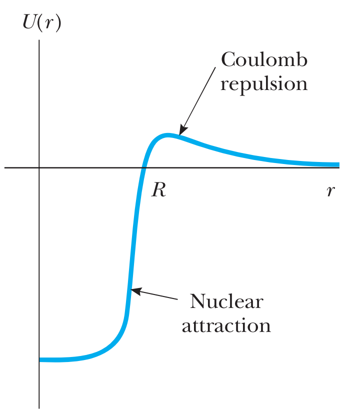

Figure 14.11 Potential energy as a function of separation between two deuterons. The Coulomb repulsive force is dominant at long range, whereas the nuclear attractive force is dominant at short range, where R is of the order of several fermi.

The fusion reactions that appear most promising for a terrestrial fusion power reactor involve deuterium (${}^2_1\text{H}$) and tritium (${}^3_1\text{H}$):

$$\begin{aligned} {}^2_1\text{H} + {}^2_1\text{H} &\longrightarrow {}^3_2\text{He} + {}^1_0\text{n} \qquad Q = 3.27\text{ MeV} \\ {}^2_1\text{H} + {}^2_1\text{H} &\longrightarrow {}^3_1\text{H} + {}^1_1\text{H} \qquad Q = 4.03\text{ MeV} \\ {}^2_1\text{H} + {}^3_1\text{H} &\longrightarrow {}^4_2\text{He} + {}^1_0\text{n} \qquad Q = 17.59\text{ MeV} \end{aligned} \qquad (14.11)$$

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As noted earlier, deuterium is available in almost unlimited quantities from our lakes and oceans and is very inexpensive to extract. Tritium, however, is radioactive ($T_{1/2}$ = 12.3 yr) and undergoes beta decay to ^{3}He. As a result, tritium does not occur naturally to any great extent and must be artificially produced.

One of the major problems in obtaining energy from any fusion reaction is the fact that the Coulomb repulsion force between two charged nuclei must be overcome before they can fuse. The potential energy as a function of particle separation for two deuterons (each with charge $+e$) is shown in Figure 14.11. The potential energy is positive in the region $r > R$, where the Coulomb repulsive force dominates, and negative in the region $r < R$, where the strong nuclear force dominates. The fundamental problem, then, is to give the two nuclei enough kinetic energy to overcome this repulsive potential barrier. This can be accomplished by heating the fuel to extremely high temperatures (about 10^8 K, far greater than the interior temperature of the Sun). At these high temperatures, the atoms are ionized and the system consists of a collection of electrons and nuclei, commonly referred to as a **plasma.**

High temperatures are required to overcome the large Coulomb barrier

EXAMPLE 14.6 The Fusion of Two Deuterons

The separation between two deuterons must be about 1.0×10^{-14} m for the attractive nuclear force to overcome the repulsive Coulomb force. (a) Calculate the height of the potential barrier due to the repulsive force.

Solution The potential energy associated with two charges separated by a distance r is

$$U = k\frac{q_1 q_2}{r}$$

where k is the Coulomb constant. For the case of two deuterons, $q_1 = q_2 = +e$, so

$$U = k\frac{e^2}{r} = \left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(1.60 \times 10^{-19}\ \text{C})^2}{1.0 \times 10^{-14}\ \text{m}}$$
$$= 2.3 \times 10^{-14}\ \text{J} = 0.14\ \text{MeV}$$

(b) Estimate the effective temperature required for a deuteron to overcome the potential barrier, assuming an energy of $\frac{3}{2}k_BT$ per deuteron (where k_B is Boltzmann's constant).

Solution Since the total Coulomb energy of the pair of deuterons is 0.14 MeV, the Coulomb energy per deuteron is 0.07 MeV = 1.1×10^{-14} J. Setting this equal to the average thermal energy per deuteron gives

$$\tfrac{3}{2}k_BT = 1.1 \times 10^{-14}\ \text{J}$$

where k_B is equal to 1.38×10^{-23} J/K. Solving for T gives

$$T = \frac{2 \times (1.1 \times 10^{-14}\ \text{J})}{3 \times (1.38 \times 10^{-23}\ \text{J/K})} = 5.3 \times 10^8\ \text{K}$$

Example 14.6 suggests that deuterons must be heated to about 5×10^8 K to achieve fusion. This estimate of the required temperature is too high, however, because the particles in the plasma have a Maxwellian speed distribution, and therefore some fusion reactions are caused by particles in the high-energy "tail" of this distribution. Furthermore, even the particles without enough energy to overcome the barrier have some probability of tunneling through the barrier. When these effects are taken into account, a temperature of "only" 4×10^8 K appears adequate to fuse two deuterons.

The temperature at which the power generation rate exceeds the loss rate (due to mechanisms such as radiation losses) is called the **critical ignition temperature.** This temperature for the deuterium–deuterium (D–D) reaction is 4×10^8 K. According to $E \cong k_BT$, this temperature is equivalent to approximately 35 keV. It turns out that the critical ignition temperature for the deuterium–tritium (D–T) reaction is about 4.5×10^7 K, or only 4 keV.

Critical ignition temperature

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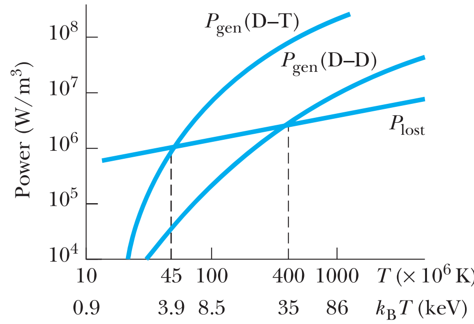

Figure 14.12 Power generated (or lost) versus temperature for the deuterium–deuterium and deuterium–tritium fusion reactions. When the generation rate P_{gen} exceeds the loss rate P_{lost}, ignition takes place.

Figure 14.12 is a plot of the power generated by fusion, P_{gen}, versus temperature for the two reactions. The straight line represents the power lost, via the radiation mechanism known as **bremsstrahlung,** versus temperature. This is the principal mechanism of energy loss, in which radiation (primarily x-ray) is emitted as the result of electron–ion collisions within the plasma.[2] The intersections of the P_{lost} line with the P_{gen} curves give the critical ignition temperatures.

In addition to the high temperature requirements, there are two other critical parameters that determine whether or not a thermonuclear reactor will be successful: the **ion density,** n, and **confinement time,** τ. **The confinement time is the period for which the interacting ions are maintained at a temperature equal to or greater than the ignition temperature.** The British physicist J. D. Lawson has shown that the ion density and confinement time must both be large enough to ensure that more fusion energy is released than is required to heat the plasma. In particular, **Lawson's criterion** states that a net energy output is possible under the following conditions:

Confinement time

Lawson's criterion

$$n\tau \geq 10^{14}\ \text{s/cm}^3 \qquad (\text{D–T})$$
$$n\tau \geq 10^{16}\ \text{s/cm}^3 \qquad (\text{D–D}) \qquad (14.12)$$

Figure 14.13 is a graph of $n\tau$ versus the so-called **kinetic temperature** $k_B T$ for the D–T and D–D reactions.

Lawson arrived at his criterion by comparing the energy required to heat the plasma with the energy generated by the fusion process. The energy E_h required to heat the plasma is proportional to the ion density n; that is, $E_h = D_1 n$. The energy generated by the fusion process, E_{gen}, is proportional to $n^2\tau$, or $E_{gen} = D_2 n^2 \tau$. This can be understood by realizing that the fusion energy released is proportional to both the rate at which interacting ions collide, n^2, and the confinement time, τ. Net energy is produced when the energy generated by fusion, E_{gen}, exceeds E_h. When the constants D_1 and D_2 are

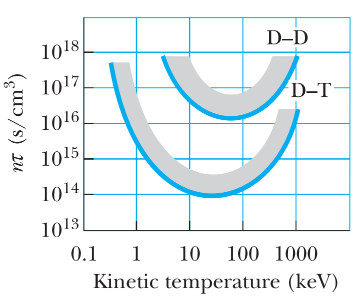

Figure 14.13 The Lawson number $n\tau$ at which net energy output is possible versus temperature for the D–T and D–D fusion reactions. The regions above the curves represent favorable conditions for fusion.

[2]Cyclotron radiation is another loss mechanism; it is especially important in the case of the D–D reaction.

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