AC BRIDGES AND THEIR APPLICATIONS

CONDITIONS FOR BRIDGE BALANCE

• An ac bridge consists of four bridge arms, a source of excitation and a null detector. The power source is an ac source and the null detector is usually a headphone.

• An ac bridge having impedences Z_1, Z_2, Z_3 and Z_4 in its arms is shown in the Fig. (1) below.

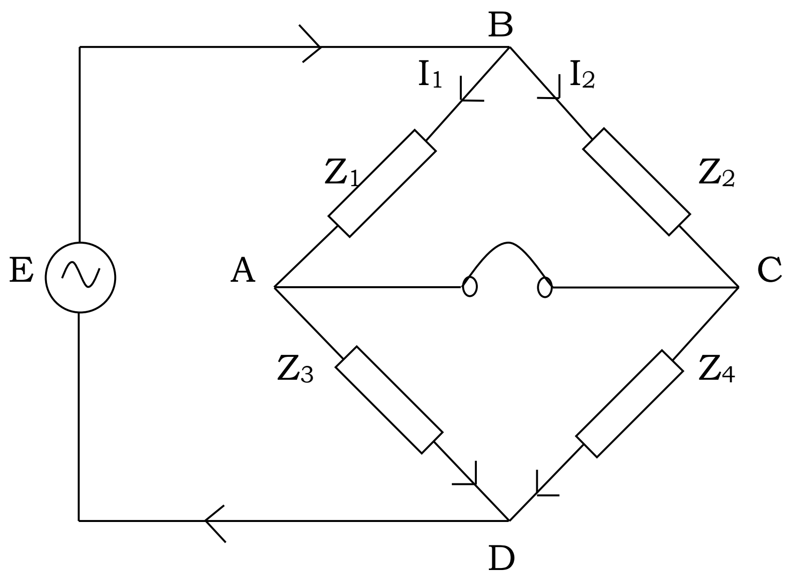

Fig. (1)

• When balance of the bridge is obtained the terminals A and C are said to be at same potential with respect to point B i.e.,

$$E_{AB} = E_{BC} \tag{1}$$

• For this condition, we can write,

$$I_1Z_1 = I_2Z_2 \tag{2}$$

Where

$$I_1 = \frac{E}{Z_1+Z_3} \tag{3}$$

and

$$I_2 = \frac{E}{Z_2+Z_4} \quad (4)$$

- Substituting the values of I_1 and I_2 from equations (3) and (4) into (2), we get

$$\frac{E}{Z_1+Z_3} Z_1 = \frac{E}{Z_2+Z_4} Z_2$$

$$\frac{Z_1}{Z_1+Z_3} = \frac{Z_2}{Z_2+Z_4}$$

therefore, $Z_1(Z_2+Z_4) = Z_2(Z_1+Z_3)$

$$Z_1 Z_4 = Z_2 Z_3 \quad (5)$$

- Equation (5) is called the general equation for balance of ac bridge. Similarly, we can also write this equation in terms of admittances(Y) instead of impedences as

$$Y_1 Y_4 = Y_2 Y_3 \quad (6)$$

- If the impedence is written in the form of $Z = \angle{Z}\ \theta$, then equation (5) can be written as

$$(Z_1\angle{\theta_1})\ (Z_4\angle{\theta_4}) = (Z_2\angle{\theta_2})\ (Z_3\angle{\theta_3})$$

$$Z_1\ Z_4\angle{(\theta_1+\theta_4)} = Z_2\ Z_3\angle{(\theta_2+\theta_3)} \quad (7)$$

- Equation (7) shows two balance conditions. The first condition is that the magnitudes of the impedances satisfy the relationship shown by equation (5).
- It states that “The products of the magnitudes of impedances of the opposite arms must be equal”.
- The second condition requires that the phase angles of the impedances satisfy the relationship

$$\angle{\theta_1} + \angle{\theta_4} = \angle{\theta_2} + \angle{\theta_3} \quad (8)$$

- It states that “The sum of the phase angles of the opposite arms must be equal”.

MAXWELL BRIDGE

• The Maxwell bridge is used to measure an unknown inductance in terms of a known capacitance. The circuit for the Maxwell Bridge is shown in Fig. (1) below.

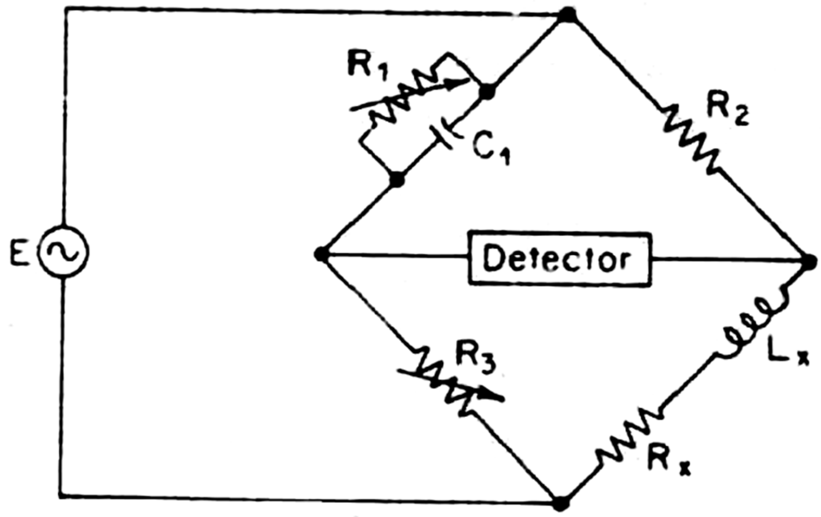

Fig. (1)

• As shown in Fig. (1), the ratio arm has parallel combination of resistance R_1 and a capacitance C_1.

• The unknown arm contains unknown inductance L_x and resistance R_x.

• The balance condition of the bridge can be given by

$$Z_1 Z_X = Z_2 Z_3$$

$$Z_X = Z_2 Z_3 Y_1 \qquad (1)$$

where $Y_1 = 1/Z_1$ = admittance

• The bridge is balanced by first adjusting R_3 for inductive balance and then by adjusting R_1 for resistive balance.

• Adjustment of R_1 disturbs the inductive balance and R_3 needs to be modified and accordingly R_1 also is modified.

• The process gives a slow convergence and the balance slowly shifts towards the actual null point. This type of balance is called the 'sliding balance'. The actual null point is obtained after few adjustments.
• From Fig. (1) we can write the impedence of the arms as

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1} + \frac{j}{X_1} = \frac{1}{R_1} + j\omega C_1 \qquad (2)$$

$$Z_2 = R_2 \ ; \quad Z_3 = R_3 \qquad (3)$$

$$Z_x = R_x + jX_L = R_x + j\omega L_x \qquad (4)$$

$$R_x + j\omega L_X = R_2 R_3 \left(\frac{1}{R_1} + j\omega C_1\right)$$

$$R_x + j\omega L_X = \left(\frac{R_2 R_3}{R_1} + j\omega C_1 R_1 R_2\right)$$

• Equating the real parts of the above equation we get

$$R_x = \frac{R_2 R_3}{R_1}$$

• Equating the imaginary parts we get

$$L_x = R_2 R_3 C_1$$

• **Limitation of the Maxwell bridge** is that it is not suitable for the determination of the self-inductance of coils having quality factor $Q > 10$ or $Q < 1$. It is suitable for the coils having Q values in the range $1<Q<10$.
• This is because for high Q coils the phase angle is very nearly 90^o(positive). This requires a capacitor having phase angle very nearly equal to 90^o (positive).
• This situation demands that R_1 should be very large, which is impractical.

HAY BRIDGE

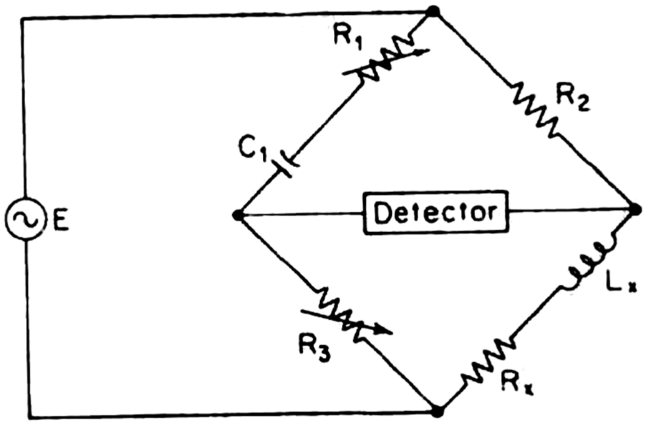

Fig. (1)

• Hay bridge is usually used to determine the unknown inductance of coils having higher Q values(>10).

• It consists of resistor R_1 in series with standard capacitor C_1 as shown in Fig. (1).

• The balance condition for the bridge is given by

$$Z_1 Z_x = Z_2 Z_3 \tag{1}$$

• The impedances of the arms of the bridge can be given by

$$Z_1 = R_1 - jX_{C1} = R_1 - \frac{j}{\omega C_1}\ ; \quad Z_2 = R_2\ ; \quad Z_3 = R_3\ ; \tag{2}$$

$$Z_x = R_x + jX_{Lx} = R_x + j\omega L_x \tag{3}$$

• Substituting the values from equations (2) and (3) into equation (1), we get

$$\left(R_1 - \frac{j}{\omega C_1}\right)(R_x + j\omega L_x) = R_2 R_3$$

It can be expanded as

$$R_1 R_x + \frac{L_x}{C_1} - \frac{jR_x}{\omega C_1} + j\omega L_x R_1 = R_2 R_3 \tag{4}$$

• Separating the real and imaginary terms on both the sides of equation (4) we get

$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3 \tag{5}$$

$$-\frac{jR_x}{\omega C_1} + j\omega L_x R_1 = 0$$

$$\frac{jR_x}{\omega C_1} = j\omega L_x R_1 \quad \text{or} \quad \frac{R_x}{\omega C_1} = \omega L_x R_1 \tag{6}$$

• Here both equations (5) and (6) contain L_x and R_x and to find their values we must solve them. The solution gives the following values of L_x and R_x.

$$R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1+\omega^2 C_1^2 R_1^2} \tag{7}$$

And $$L_x = \frac{R_2 R_3 C_1}{1+\omega^2 C_1^2 R_1^2} \tag{8}$$

• Now the impedance triangles for the inductive and capacitive arms can be represented as in Fig. (2).

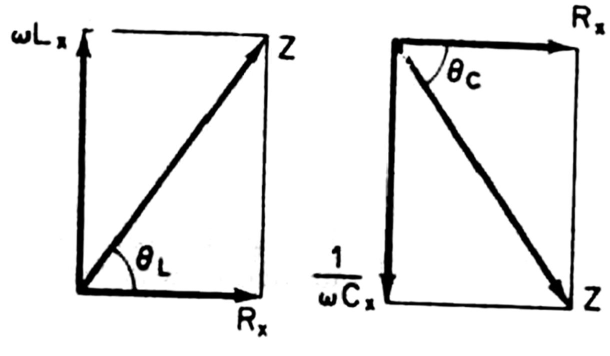

Fig. (2)

• In Fig. (2) θ_L and θ_C and the inductive and capacitive phase angles respectively. Then from the Fig. we can write

$$\tan\theta_L = \frac{X_L}{R} = \frac{\omega L_x}{R_x} = Q$$

$$\text{and } \tan\theta_C = \frac{X_C}{R} = \frac{1}{\omega C_1 R_1}$$

• When the two phase angles are equal, we have

$$\tan\theta_L = \tan\theta_C$$

$$Q = \frac{1}{\omega C_1 R_1} \tag{9}$$

• Then L_x can be given by the equation

$$L_x = \frac{R_2 R_3 C_1}{1+\left(\frac{1}{Q}\right)^2}$$

•So for Q > 10, (1/100) <<< 1 and hence we have

$$L_x = R_2 R_3 C_1$$

• Thus for Q>10 Hay bridge is the most suitable bridge, however, for Q<10, (1/Q)2 is not negligible and hence Maxwell bridge is more suitable.

SCHERRING BRIDGE

• A Scherring Bridge is generally used to determine the unknown capacitance of a capacitor. However, it is sometimes also used to measure the insulating or dielectric properties of materials.

• The circuit arrangements for the bridge are given below in Fig. (1).

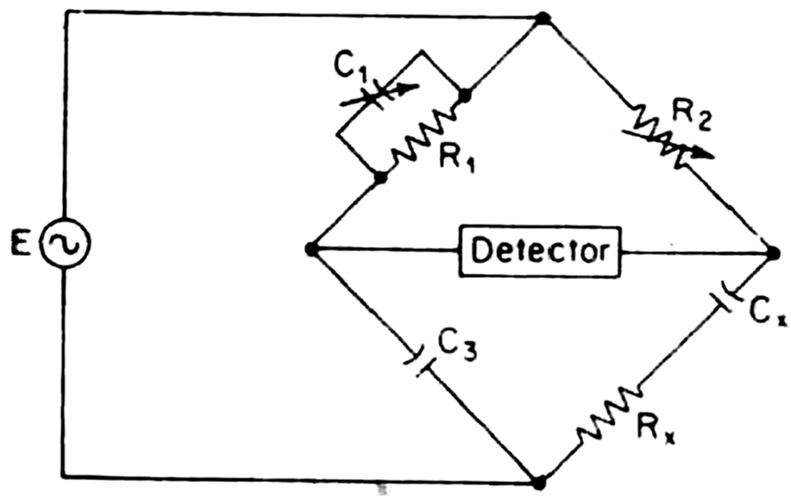

Fig. (1)

• Here, the ratio arm has the capacitor C_1 in parallel with resistor R_1.

• The balance condition for the bridge is given by the equation

$$Z_x = Z_2 Z_3 Y_1 \tag{1}$$

• The impedances of the arms are given by

$$\frac{1}{Z_1} = Y_1 = \frac{1}{R_1} + \frac{j}{X_{C1}} = \frac{1}{R_1} + j\omega C_1 \quad ; \qquad Z_2 = R_2 \; ; \tag{2}$$

$$Z_3 = -jX_{C3} = \frac{-j}{\omega C_3} \; ; \qquad Z_x = R_x - jX_{Cx} = R_x - \frac{j}{\omega C_X} \tag{3}$$

• Substituting the values from equations (2) and (3) into equation (1), we get

$$R_x - \frac{j}{\omega C_X} = R_2 \left(\frac{-j}{\omega C_X}\right)\left(\frac{1}{R_1} + j\omega C_1\right)$$

$$R_x - \frac{j}{\omega C_X} = \frac{R_2 C_1}{C_3} - \frac{jR_2}{\omega C_3 R_1}$$

• Then equating the real and imaginary parts in the above equation, we get

$$R_x = R_2C_1/C_3 \quad \text{and} \quad C_x = C_3R_1/R_2 \qquad (4)$$

• The power factor for the series RC combination is defined as the cosine of the phase angle of the circuit. It is given by

$$PF = R_x/X_x = \omega C_x R_x \qquad (5)$$

• Similarly, the dissipation factor of a series RC circuit is defined as the cotangent of the phase angle of the circuit. It is given by

$$D = \omega C_1 R_1 \qquad (6)$$

WIEN BRIDGE

• A Wien bridge is generally used to measure the frequency of the source. It is also extensively used in other applications like, harmonic distortion analyser, in audio and HF oscillators as the frequency determining element, etc.

• As shown in Fig. (1) below, the Wien bridge consists of a series RC combination in one arm and a parallel RC combination in the adjoining arm.

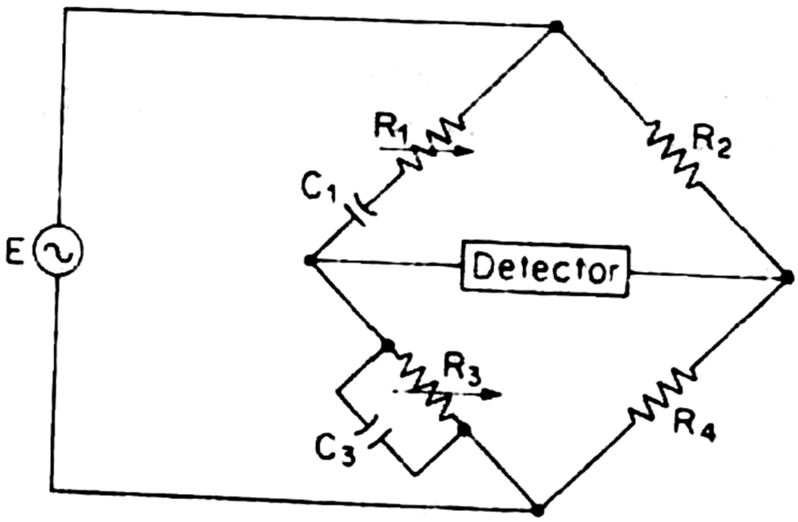

Fig. (1)

• The balance condition for the bridge is given by

$$Z_2Z_3 = Z_1Z_4$$

$$Z_2 = Z_1Z_4(1/Z_3) = Z_1Z_4Y_3 \qquad (1)$$

• Now the impedances of the arms are given by the following equations

$$Z_1 = R_1 - jX_{C1} = R_1 - \frac{j}{\omega C_1} \quad ; \qquad Z_2 = R_2 \; ; \qquad (2)$$

$$1/Z_3 = Y_3 = \frac{1}{R_3} + \frac{j}{X_{C3}} = \frac{1}{R_3} + j\omega C_3 \quad ; \qquad Z_4 = R_4 \qquad (3)$$

• Substituting the values from equations (2) and (3) into equation (1), we get

$$R_2 = \left(R_1 - \frac{j}{\omega C_1}\right) R_4 \left(\frac{1}{R_3} + j\omega C_3\right)$$

$$R_2 = \frac{R_1 R_4}{R_3} + j\omega C_3 R_1 R_4 - \frac{jR_4}{\omega C_1 R_3} + \frac{R_4 C_3}{C_1} \qquad (4)$$

• Equating the real parts we get,

$$R_2 = \frac{R_1 R_4}{R_3} + \frac{R_4 C_3}{C_1}$$

•Dividing by R_4 on both the sides we get

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1} \qquad (5)$$

• Now equating the imaginary parts we get,

$$j\omega C_3 R_1 R_4 = \frac{jR_4}{\omega C_1 R_3} \qquad (6)$$

$$\omega^2 = \frac{1}{C_1 C_3 R_1 R_3} \qquad \text{where } \omega = 2\pi f$$

$$f = \frac{1}{2\pi\sqrt{C_1 C_3 R_1 R_3}} \qquad (7)$$

• If $R_1 = R_3 = R$ and $C_1 = C_3 = C$, then we get $R_2 = 2R_4$ and

$$f = \frac{1}{2\pi RC} \qquad (8)$$

• Using the above equation, we can determine the frequency of the source. Also the bridge can be calibrated directly in terms of frequency.