14

Nuclear Physics Applications

Chapter Outline

This chapter is concerned with nuclear reactions in which particles and nuclei collide and change into other nuclei and particles. We also consider the two means by which energy can be derived from nuclear reactions: *fission*, in which a large nucleus splits, or fissions, into two smaller nuclei, and *fusion*, in which two small nuclei fuse to form a larger one. In either case, a release of energy occurs that can then be used either destructively through bombs or constructively through production of electric power. Finally, we examine the interaction of radiation with matter and several devices for detecting radiation.

14.1 NUCLEAR REACTIONS

It is possible to change the structure of nuclei by bombarding them with energetic particles. Such collisions that change the identities of the target nuclei are called **nuclear reactions.** Rutherford was the first to observe them in 1919, using naturally occurring radioactive sources for the bombarding particles. Since then, thousands of nuclear reactions have been observed, especially after the development of charged-particle accelerators in the 1930s. With today's technology in particle accelerators, it is possible to achieve particle energies of 1000 GeV = 1 TeV and higher. These high-energy particles are used to create new particles whose properties are helping to solve the mystery of the nucleus.

Consider a reaction in which a target nucleus X is bombarded by a particle a, resulting in a nucleus Y and a particle b:

Nuclear reaction

$$\mathrm{a} + \mathrm{X} \longrightarrow \mathrm{Y} + \mathrm{b} \tag{14.1}$$

Sometimes this reaction is written in the more compact form

$$\mathrm{X(a, b)Y}$$

As an example, consider the reaction $^7\mathrm{Li}(\mathrm{p}, \alpha)^4\mathrm{He}$, or

$$^1_1\mathrm{H} + {}^7_3\mathrm{Li} \longrightarrow {}^4_2\mathrm{He} + {}^4_2\mathrm{He}$$

Cockroft and Walton were first to observe this reaction in 1932, using protons accelerated to 600 keV in an accelerator they had designed and built. A nuclear reaction such as this, and in fact any reaction, can occur only if it satisfies certain *conservation laws*. The conservation laws for nuclear reactions are

- *Conservation of mass number, A*. The total number of nucleons must be the same after the reaction as before. For the reaction under discussion, $A_{\text{before}} = 1 + 7 = A_{\text{after}} = 4 + 4$.
- *Conservation of charge, q*. Here the charged nuclear particles are protons, and $q_{\text{before}} = 1 + 3 = q_{\text{after}} = 2 + 2$.
- *Conservation of energy, linear momentum, and angular momentum*. These quantities are conserved because a nuclear reaction involves only internal forces between a target nucleus and a bombarding nucleus, and there are no external forces to upset these conservation principles.

Let us apply the conservation of energy to a reaction of the form of Equation 14.1 to compute the total kinetic energy released (or absorbed) in the reaction, which is called the **reaction energy,** Q. Assume that the target nucleus X is originally at rest, the bombarding particle a has kinetic energy K_a, and the reaction products b and Y have kinetic energies K_b and K_Y. Conserving energy,

$$M_\mathrm{X}c^2 + K_\mathrm{a} + M_\mathrm{a}c^2 = M_\mathrm{Y}c^2 + K_\mathrm{Y} + M_\mathrm{b}c^2 + K_\mathrm{b}$$

As the total kinetic energy released in the reaction, Q, is equal to the difference between the kinetic energy of the final particles and that of the initial particle, we find

***Q* of a reaction**

$$Q = (K_\mathrm{Y} + K_\mathrm{b}) - K_\mathrm{a} = (M_\mathrm{X} + M_\mathrm{a} - M_\mathrm{Y} - M_\mathrm{b})c^2 \tag{14.2}$$

A reaction for which Q is positive converts nuclear mass to kinetic energy of the products Y and b and is called an **exothermic reaction.** A reaction for which Q is negative requires some minimum input kinetic energy from the bombarding particle in order to occur. Such a reaction is called **endothermic.**

For an endothermic reaction to proceed, the incident particle must have a minimum kinetic energy called the **threshold energy,** K_{th}. Since K_{th} must not only supply $|Q|$, the excess mass–energy of the products, but also supply some kinetic energy to the products to conserve momentum, K_{th} is greater than $|Q|$.

For low-energy reactions, where the kinetic energies of all the interacting particles are small compared to their rest energies, we can apply the nonrela-

Table 14.1 *Q* Values for Nuclear Reactions Involving Light Nuclei

Reaction[a]	Measured *Q* Value (MeV)
$^{2}H(n, \gamma)^{3}H$	6.257 ± 0.004
$^{2}H(d, p)^{3}H$	4.032 ± 0.004
$^{6}Li(p, \alpha)^{3}H$	4.016 ± 0.005
$^{6}Li(d, p)^{7}Li$	5.020 ± 0.006
$^{7}Li(p, n)^{7}Be$	-1.645 ± 0.001
$^{7}Li(p, \alpha)^{4}He$	17.337 ± 0.007
$^{9}Be(n, \gamma)^{10}Be$	6.810 ± 0.006
$^{9}Be(\gamma, n)^{8}Be$	-1.666 ± 0.002
$^{9}Be(d, p)^{10}Be$	4.585 ± 0.005
$^{9}Be(p, \alpha)^{6}Li$	2.132 ± 0.006
$^{10}B(n, \alpha)^{7}Li$	2.793 ± 0.003
$^{10}B(p, \alpha)^{7}Be$	1.148 ± 0.003
$^{12}C(n, \gamma)^{13}C$	4.948 ± 0.004
$^{13}C(p, n)^{13}N$	-3.003 ± 0.002
$^{14}N(n, p)^{14}C$	0.627 ± 0.001
$^{14}N(n, \gamma)^{15}N$	10.833 ± 0.007
$^{18}O(p, n)^{18}F$	-2.453 ± 0.002
$^{19}F(p, \alpha)^{16}O$	-8.124 ± 0.007

[a]The symbols n, p, d, α, and γ denote the neutron, proton, deuteron, alpha particle, and photon, respectively. From C. W. Li, W. Whaling, W. A. Fowler, and C. C. Lauritsen, *Phy. Rev.,* 83:512, 1951.

tivistic expressions $K = \frac{1}{2}mv^2$ and $p = mv$ to find the threshold energy. It is left as a problem (Problem 9) to show that when momentum and energy are conserved in a low-energy negative Q reaction, the threshold energy is

$$K_{\text{th}} = -Q\left(1 + \frac{M_{\text{a}}}{M_{\text{X}}}\right) \tag{14.3}$$

Finally, we have included a selected list of measured Q values for reactions involving light nuclei in Table 14.1.

EXAMPLE 14.1

(a) Calculate the Q value for the reaction observed by Cockcroft and Walton.

Solution (a) The reaction is

$$^{1}_{1}H + ^{7}_{3}Li \longrightarrow ^{4}_{2}He + ^{4}_{2}He$$

Using $Q = (M_{\text{Li}} + M_{\text{H}} - 2M_{\text{He}})c^2$ and substituting atomic mass values from Appendix B, we find

$$Q = (7.016\,003\text{ u} + 1.007\,825\text{ u} - 2(4.002\,603\text{ u}))(931.50\text{ MeV/u}) = 17.3\text{ MeV}$$

(b) Find the kinetic energy of the products if 600-keV protons are incident.

Solution (b) Since $Q = K_{\text{products}} - K_{\text{incident particle}}$,

$$K_{\text{products}} = Q + K_{\text{incident particle}} = 17.3\text{ MeV} + 0.6\text{ MeV} = 17.9\text{ MeV}$$

This means that the two alpha particles share 17.9 MeV of kinetic energy.

14.2 REACTION CROSS SECTION

Since we deal in this chapter with the interactions between nuclei and matter, it is useful to introduce a quantity called the **cross section, which is a measure of the probability that a particular nuclear reaction will occur.**

When a beam of particles is incident on a target in the form of a thin foil, not every particle interacts with a target nucleus. The probability that an interaction will occur depends on the ratio of the "effective" area of the target nucleus to the area of the foil. The situation is analogous to throwing darts at a large wall upon which many inflated balloons are hanging. If the darts are thrown at random in the direction of the wall and the balloons are spread out so as not to touch each other, there is some chance that you will hit a balloon on any given throw. Furthermore, if you throw darts at a rate R_0, the rate R at which balloons burst will be less than R_0. In fact, the probability of hitting a balloon will equal R/R_0. The ratio R/R_0 will depend on the number of balloons N on the wall, the area σ of each balloon, and the area A of the wall. Since the total cross-sectional area of the balloons is $N\sigma$, the probability R/R_0 equals the ratio $N\sigma/A$.

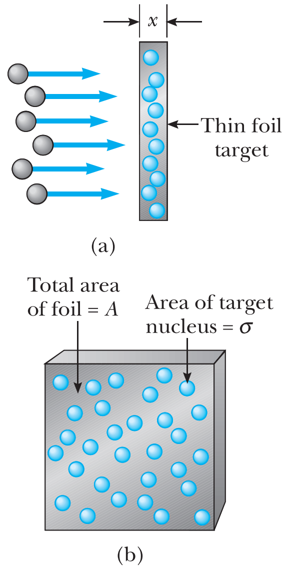

Figure 14.1 (a) A beam of particles incident on a thin foil target of thickness x. The view shown is of an edge of the target. (b) A front view of the target, where the circles represent the target nuclei.

With this analogy, we can now understand the concept of cross section as it pertains to nuclear events. Suppose a beam of particles is incident upon a thin-foil target, as in Figure 14.1a. Each target nucleus X has an effective area σ called the **cross section.** You can think of σ as an *effective* area of the nucleus at right angles to the direction of motion of the bombarding particles, as in Figure 14.1b, but note that the reaction cross reaction σ can be greater than, equal to, or less than the actual geometrical cross section of the target nucleus. It is assumed that the reaction X(a, b)Y will occur only if the incident particle strikes the area σ. Therefore, the probability that a collision will occur is proportional to σ. That is, the probability increases as σ increases. In the general case, the size of σ for a specific reaction may also depend on the energy of the incident particle.

Let us consider the concept of cross section in more detail. In what follows, we take the foil thickness to be x and its area to be A. Furthermore, we use the following notation.

R_0 = Rate at which incident particles strike the foil (particles/s)
R = Rate at which reaction events occur (reactions/s)
n = Number of target nuclei per unit volume (particles/m^3)

Since the total number of target nuclei in the foil is nAx, the total area exposed to the incident beam must be σnAx. It is assumed that the foil is sufficiently thin that nuclei are not "hidden" behind others. The ratio of the rate of interactions to the rate of incident particles, R/R_0, must equal the ratio of the area σnAx to the total area A of the foil, in analogy with the dart–balloon collision events. That is,

Reaction rate is proportional to cross section and target density

$$\frac{R}{R_0} = \frac{\sigma nAx}{A} = \sigma nx \tag{14.4}$$

This result shows that the probability that a nuclear reaction will occur is proportional to the cross section σ, the density of target nuclei n, and the thickness of the target, x. A value for σ for a specific reaction can therefore be obtained by measuring R, R_0, n, and x and using Equation 14.4.

We can use the same reasoning to arrive at an expression for the number of particles that penetrate a foil without undergoing reaction. Suppose that N_0

particles are incident on a foil of thickness dx and dN is the number of particles that interact with the target nuclei (Fig. 14.2). The ratio of the number of interacting particles to the number of incident particles, dN/N, equals the ratio of the total target cross section, $nA\sigma\, dx$, to the total foil area A. That is,

$$-\frac{dN}{N} = \frac{nA\sigma\, dx}{A} = n\sigma\, dx$$

where the minus sign indicates that particles are being removed from the beam. Integrating this expression and taking $N = N_0$ at $x = 0$,

$$\int_{N_0}^{N} \frac{dN}{N} = -n\sigma \int_0^x dx$$

$$\ln\left(\frac{N}{N_0}\right) = -n\sigma x$$

$$N = N_0 e^{-n\sigma x} \qquad (14.5)$$

Number of particles transmitted through a target of thickness x

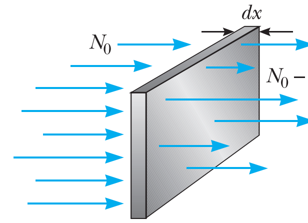

Figure 14.2 If N_0 is the number of particles incident on a target of thickness dx in some time interval, the number emerging from the target is $N_0 - dN$.

That is, if N_0 is the number of incident particles, the number that emerge from the slab, N, decreases exponentially with target thickness.

Nuclear cross sections, which have dimensions of area, are typically of the order of the square of the nuclear radius, which is about 10^{-28} m^2. For this reason, it is common to use the unit of 10^{-28} m^2 for measuring nuclear cross sections. This small unit is known, strangely enough, as the **barn** (b)[1] and is defined as

The barn

$$1 \text{ barn} = 10^{-28} \text{ m}^2 \qquad (14.6)$$

In reality, the concept of cross section in nuclear and atomic physics has little to do with the actual geometric area of target nuclei. The model we have used is simply a convenient one for describing the probability of occurrence of any nuclear reaction. In fact, cross sections vary with both the specific reaction considered and with the incident particle's kinetic energy over much more than several times the target nucleus's geometrical area. For example, the cross section for an antineutrino to interact with a proton via the nuclear weak interaction is only about 10^{-19} b in the reaction

$$\bar{\nu} + \text{p} \longrightarrow \text{e}^+ + \text{n} \qquad (\sigma = 10^{-19} \text{ b})$$

The cross sections for inelastic scattering of neutrons from iodine and xenon via the nuclear strong interaction, however, are about 4 barns in the following reactions:

$$\text{n} + {}^{127}\text{I} \longrightarrow {}^{127}\text{I}^* + \text{n} \qquad (\sigma = 4 \text{ b})$$

$$\text{n} + {}^{129}\text{Xe} \longrightarrow {}^{129}\text{Xe}^* + \text{n} \qquad (\sigma = 4 \text{ b})$$

(An inelastic scattering reaction is one in which the incident particle loses energy to the target nucleus, emerging from the reaction with less kinetic energy but leaving the target in an excited state, here denoted by the asterisk.)

[1]"Barn" was introduced by the American physicists M. G. Holloway and C. P. Baker in 1942 in a humorous twist. It served the purpose of a code word in concealing war work on reaction probabilities and was appropriate because a cross section of 10^{-28} m^2 really is "as big as the broad side of a barn" for nuclear processes.