Friedman two-way analysis of variance by ranks

Friedman's Two Way Analysis of Variance is a method to compare matched samples in multiple groups, where the data is nonparametric. It is the nonparametric equivalent of the Two Way Analysis of Variance, and similar to Wilcoxon's Matched Pair Ranks Test, except that there is usually more than 2 matched groups.

Nonparametric means the test doesn't assume your data comes from a particular distribution (like the normal distribution). Basically, it's used in place of the ANOVA test when you don't know the distribution of your data. Also in some cases only ranks may be available for analysis. We perform calculations on ranks which may be derived from observations measured on a higher scale or may be the original observations themselves. The objective is to determine if we may conclude from sample evidence that there is a difference in treatment effects.

Assumptions

- **a.** The data consist of b mutually independent samples (blocks). Rows represent the blocks and column are called treatments. The term treatment has a very general meaning; it may refer to a treatment in the usual sense of the word or it may refer to some other condition such as socioeconomic status or educational level.
- **b.** The variable of interest in continuous (data should be ordinal e.g. likert scale).
- **c.** There is no interaction between block and treatments.
- **d.** The observations within each block may be ranked in order of magnitude.

Hypotheses

$H_o: M_1 = M_2 = \cdots \ldots \ldots \ldots \ldots = M_k$ (Treatments all have identical effect)

$H_1: At\ least\ one\ equality\ is\ violated$

Test Statistic

The first step in calculating the test statistic is to convert the original observation to ranks. The observations within each block are ranked separately from smallest to larges, so each block contain a separate set of k ranks. The second step is to obtain the sum of the ranks in each column. The Friedman test statistic is defined as

$$FM = \frac{12}{bk(k+1)} \sum_{j=1}^{k} R^2 - 3b(k+1)$$

Example

Step 1: Sort your data into blocks (columns in a spreadsheet).for this example, we have 12 patients getting three different treatments.

Patient	Treatment 1	Treatment 2	Treatment 3
1	209	88	109
2	412	388	142
3	315	451	155
4	389	325	121
5	210	126	75
6	136	118	49
7	178	227	101
8	228	98	49
9	240	205	142
12	113	88	45
11	178	194	55
12	321	349	121

Step 2: Rank each column separately. The smallest score should get a rank of 1. I am ranking across rows here so each patient is being ranked a 1, 2, or 3 for each treatment.

	Rank		
Patient	Treatment 1	Treatment 2	Treatment 3
1	3	1	2
2	3	2	1
3	2	3	1
4	3	2	1
5	3	2	1
6	3	2	1
7	2	3	1
8	3	2	1
9	3	2	1
10	3	2	1
11	2	3	1
12	2	3	1

Step 3: Sum the ranks (find a total for each column).

	Rank		
Patient	Treatment 1	Treatment 2	Treatment 3
1	3	1	2
2	3	2	1
3	2	3	1
4	3	2	1
5	3	2	1
6	3	2	1
7	2	3	1
8	3	2	1
9	3	2	1
10	3	2	1
11	2	3	1
12	2	3	1
Totals	32	27	13

Step 4: Calculate the test statistic. You'll need:

1. n: the number of subjects/ blocks (12)
2. k: the number of treatments (3)
3. R: The total ranks for each of the three columns (32, 27, 13).

Insert these into the following formula and solve:

$$FM = \left[\frac{12}{(N * k * (k+1)}\right] * \Sigma R^2 - [3 * N * (k+1)]$$

$$FM = \left[\frac{12}{[12 * 3 * (3+1)]}\right] * 32^2 + 27^2 + 13^2 - [3 * 12 * (3+1)]$$

$$FM = \left[\frac{12}{144}\right] * [1024 + 729 + 169] - 144$$

$$FM = [.083 * 1922] - 144 = 15.526$$

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Step 5: Find the FM critical value from the table of critical values for Friedman (see table below).

Use the k=3 table (as that is how many treatments we have) and an alpha level of 5%. You could choose a higher or lower alpha level, but 5% if fairly common — so use the 5% table if you don't know your alpha level.

Looking up n-12 in that table, we find a **FM critical value of 6.17.**

Step 6: Compare the calculated FM test statistic (Step 4) to the FM critical value (Step 5). Reject the null hypothesis if the calculated F value is larger than the FM critical value.:

- Calculated FM Test Statistic = 15.526.
- FM Critical value from table = 6.17.

The calculated FM statistic is larger, so you would reject the null hypothesis.

Friedman's ANOVA by Ranks Critical Value Table

Three tables according by "k".

If your k is over 5, or your n (blocks) is over 13, use the chi square critical value table in Step 5 to get the critical value.

- **k=3**

N	**α <.10**	**α ≤.05**	**α <.01**
3	6.00	6.00	—
4	6.00	6.50	8.00
5	5.20	6.40	8.40
6	5.33	7.00	9.00
7	5.43	7.14	8.86
8	5.25	6.25	9.00
9	5.56	6.22	8.67
10	5.00	6.20	9.60
11	4.91	6.54	8.91
12	5.17	6.17	8.67
13	4.77	6.00	9.39
∞	4.61	5.99	9.21

- **k=4**

N	α <.10	α ≤.05	α <.01
2	6.00	6.00	—
3	6.60	7.40	8.60
4	6.30	7.80	9.60
5	6.36	7.80	9.96
6	6.40	7.60	10.00

7	6.26	7.80	10.37
8	6.30	7.50	10.35
∞	6.25	7.82	11.34

- **k=5**

N	α <.10	α ≤.05	α <.01
3	7.47	8.53	10.13
4	7.60	8.80	11.00
5	7.68	8.96	11.52
∞	7.78	9.49	13.28