

Theorem on Anti-Derivatives

i) $\int cf(x)dx = c \int f(x)dx$ where c is constant.

ii) $\int [f(x) \pm g(x)] dx = \int f(x)dx \pm \int g(x)dx$

Important Integral

Since $\frac{d}{dx} x^{n+1} = (n+1)x^n$

Taking integral w.r.t x

$$\int \frac{d}{dx} x^{n+1} dx = \int (n+1)x^n dx$$

$$\Rightarrow x^{n+1} = (n+1) \int x^n dx$$

$$\Rightarrow \boxed{\int x^n dx = \frac{x^{n+1}}{n+1}} \quad \text{where } n \neq -1$$

If $n = -1$ then

$$\int x^{-1} dx = \int \frac{1}{x} dx \quad (\text{here } x \neq 0)$$

Since $\frac{d}{dx} \ln x = \frac{1}{x}$

Therefore $\boxed{\int \frac{1}{x} dx = \ln|x| + c}$

Note: Since log of zero and negative numbers does not exist therefore in above formula mod assure that we are taking a log of +ive quantity.

Question # 1

Evaluate the following indefinite integrals.

(i) $\int (3x^2 - 2x + 1) dx$

(ii) $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx, (x > 0)$

(iii) $\int x(\sqrt{x} + 1) dx, (x > 0)$

(iv) $\int (2x+3)^{\frac{1}{2}} dx$

(v) $\int (\sqrt{x} + 1)^2 dx, (x > 0)$

(vi) $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 dx, (x > 0)$

(vii) $\int \frac{3x+2}{\sqrt{x}} dx, (x > 0)$

(viii) $\int \frac{\sqrt{y}(y+1)}{y} dy, (y > 0)$

(ix) $\int \frac{(\sqrt{\theta} - 1)^2}{\sqrt{\theta}} d\theta, (\theta > 0)$

(x) $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx, (x > 0)$

(xi) $\int \frac{e^{2x} + e^x}{e^x} dx$

Solution

(i) $\int (3x^2 - 2x + 1) dx = 3 \int x^2 dx - 2 \int x dx + \int dx$
 $= 3 \cdot \frac{x^{2+1}}{2+1} - 2 \cdot \frac{x^{1+1}}{1+1} + x + c$
 $= 3 \cdot \frac{x^3}{3} - 2 \cdot \frac{x^2}{2} + x + c$
 $= x^3 - x^2 + x + c$

(ii) $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx = \int \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}} \right) dx$
 $= \int x^{\frac{1}{2}} dx + \int x^{-\frac{1}{2}} dx$
 $= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$
 $= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$
 $= \frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$

(iii) $\int x(\sqrt{x} + 1) dx = \int x \left(x^{\frac{1}{2}} + 1 \right) dx$
 $= \int \left(x^{\frac{3}{2}} + x \right) dx$
 $= \int x^{\frac{3}{2}} dx + \int x dx$
 $= \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{x^{1+1}}{1+1} + c$
 $= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^2}{2} + c$

Important Integral

Since $\frac{d}{dx} (ax+b)^{n+1} = (n+1)(ax+b)^n \cdot a$

Taking integral

$$\int \frac{d}{dx} (ax+b)^{n+1} dx = \int (n+1)(ax+b)^n \cdot a dx$$

$$\Rightarrow (ax+b)^{n+1} = (n+1) \cdot a \int (ax+b)^n dx$$

$$\Rightarrow \boxed{\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1) \cdot a}}$$

$$(iv) \int (2x+3)^{\frac{1}{2}} dx = \frac{(2x+3)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right) \cdot 2} + c$$

$$= \frac{(2x+3)^{\frac{3}{2}}}{\left(\frac{3}{2}\right) \cdot 2} + c$$

$$= \frac{1}{3} (2x+3)^{\frac{3}{2}} + c$$

$$(v) \int (\sqrt{x}+1)^2 dx = \int ((\sqrt{x})^2 + 2\sqrt{x} + 1) dx$$

$$= \int \left(x + 2(x)^{\frac{1}{2}} + 1 \right) dx$$

$$= \int x dx + 2 \int (x)^{\frac{1}{2}} dx + \int dx$$

$$= \frac{x^{1+1}}{1+1} + 2 \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + x + c$$

$$= \frac{x^2}{2} + 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + x + c$$

$$= \frac{x^2}{2} + \frac{4x^{\frac{3}{2}}}{3} + x + c$$

$$(vi) \int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2 dx = \int \left(x + \frac{1}{x} - 2 \right) dx$$

$$= \int x dx + \int \frac{1}{x} dx - 2 \int dx$$

$$= \frac{x^2}{2} + \ln|x| - 2x + c$$

$$(vii) \int \frac{3x+2}{\sqrt{x}} dx = \int \frac{3x+2}{x^{1/2}} dx$$

$$= \int \frac{3x}{x^{1/2}} + \frac{2}{x^{1/2}} dx$$

$$= \int (3x^{1/2} + 2x^{-1/2}) dx$$

$$= 3 \int x^{1/2} dx + 2 \int x^{-1/2} dx$$

Now do yourself.

$$(viii) \int \frac{\sqrt{y}(y+1)}{y} dy$$

$$= \int \frac{\sqrt{y}(y+1)}{(\sqrt{y})^2} dy = \int \frac{(y+1)}{\sqrt{y}} dy$$

$$= \int \left(\frac{y}{\sqrt{y}} + \frac{1}{\sqrt{y}} \right) dy = \int \left(y^{\frac{1}{2}} + y^{-\frac{1}{2}} \right) dy$$

$$= \int y^{\frac{1}{2}} dy + \int y^{-\frac{1}{2}} dy$$

$$= \frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{y^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c = \frac{y^{\frac{3}{2}}}{\frac{3}{2}} + \frac{y^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{2}{3} y^{\frac{3}{2}} + 2y^{\frac{1}{2}} + c$$

$$(ix) \int \frac{(\sqrt{\theta}-1)^2}{\sqrt{\theta}} d\theta = \int \frac{\theta - 2\sqrt{\theta} + 1}{\sqrt{\theta}} d\theta$$

$$= \int \left(\frac{\theta}{\sqrt{\theta}} - \frac{2\sqrt{\theta}}{\sqrt{\theta}} + \frac{1}{\sqrt{\theta}} \right) d\theta$$

$$= \int \left(\theta^{\frac{1}{2}} - 2 + \theta^{-\frac{1}{2}} \right) d\theta$$

$$= \frac{\theta^{\frac{1}{2}+1}}{\frac{1}{2}+1} - 2\theta + \frac{\theta^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$= \frac{\theta^{\frac{3}{2}}}{\frac{3}{2}} - 2\theta + \frac{\theta^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{2}{3} \theta^{\frac{3}{2}} - 2\theta + 2\theta^{\frac{1}{2}} + c \quad \text{Ans}$$

$$(x) \int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx = \int \frac{1 - 2\sqrt{x} + x}{\sqrt{x}} dx$$

$$= \int \left(\frac{1}{\sqrt{x}} - \frac{2\sqrt{x}}{\sqrt{x}} + \frac{x}{\sqrt{x}} \right) dx$$

$$= \int \left(x^{-\frac{1}{2}} - 2 + x^{\frac{1}{2}} \right) dx$$

$$= \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - 2x + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - 2x + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= 2x^{\frac{1}{2}} - 2x + \frac{2}{3}x^{\frac{3}{2}} + c \quad \text{Ans}$$

Important Integral

We know $\frac{d}{dx} e^{ax} = a \cdot e^{ax}$

Taking integral

$$\int \frac{d}{dx} e^{ax} dx = \int a \cdot e^{ax} dx$$

$$\Rightarrow e^{ax} = a \int e^{ax} dx$$

$$\Rightarrow \boxed{\int e^{ax} dx = \frac{e^{ax}}{a}}$$

Also note that $\int e^{(ax+b)} dx = \frac{e^{(ax+b)}}{a}$

$$\begin{aligned} \text{(xi)} \int \frac{e^{2x} + e^x}{e^x} dx &= \int \left(\frac{e^{2x}}{e^x} + \frac{e^x}{e^x} \right) dx \\ &= \int (e^x + 1) dx \\ &= \int e^x dx + \int dx \\ &= e^x + x + c \quad \text{Ans} \end{aligned}$$

Question # 2

Evaluate

$$\text{(i)} \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} \quad \begin{matrix} (x+a > 0) \\ (x+b > 0) \end{matrix}$$

$$\text{(ii)} \int \frac{1-x^2}{1+x^2} dx$$

$$\text{(iii)} \int \frac{dx}{\sqrt{x+a} + \sqrt{x}}, (x > 0, a > 0)$$

$$\text{(iv)} \int (a-2x)^{\frac{3}{2}} dx$$

$$\text{(v)} \int \frac{(1+e^x)^3}{e^x} dx$$

$$\text{(vi)} \int \sin(a+b)x dx$$

$$\text{(vii)} \int \sqrt{1-\cos 2x} dx, (1-\cos 2x > 0)$$

$$\text{(viii)} \int \ln x \times \frac{1}{x} dx, (x > 0)$$

$$\text{(ix)} \int \sin^2 x dx$$

$$\text{(x)} \int \frac{1}{1+\cos x} dx, \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

$$\text{(xi)} \int \frac{ax+b}{ax^2+2bx+c} dx$$

$$\text{(xii)} \int \cos 3x \sin 2x dx$$

$$\text{(xiii)} \int \frac{\cos 2x-1}{1+\cos 2x} dx, (1+\cos 2x \neq 0)$$

$$\text{(xiv)} \int \tan^2 x dx$$

Solution

$$\begin{aligned} \text{(i)} \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} &= \int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} \cdot \frac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{x+a} - \sqrt{x+b}} \\ &= \int \frac{\sqrt{x+a} - \sqrt{x+b}}{x+a-x-b} dx \\ &= \int \frac{(x+a)^{\frac{1}{2}} - (x+b)^{\frac{1}{2}}}{a-b} dx \\ &= \frac{1}{a-b} \left[\int (x+a)^{\frac{1}{2}} dx - \int (x+b)^{\frac{1}{2}} dx \right] \\ &= \frac{1}{a-b} \left[\frac{(x+a)^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{(x+b)^{\frac{1}{2}}}{\frac{1}{2}+1} \right] + c \\ &= \frac{1}{a-b} \left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}} \right] + c \\ &= \frac{2}{3(a-b)} \left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}} \right] + c \quad \text{Ans.} \end{aligned}$$

Important Integral

Since $\frac{d}{dx} \tan^{-1}x = \frac{1}{1+x^2}$

Also $\frac{d}{dx} (-\cot^{-1}x) = \frac{1}{1+x^2}$

Therefore $\int \frac{1}{1+x^2} dx = \tan^{-1}x$ or $-\cot^{-1}x$

Similarly $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}x$ or $-\cos^{-1}x$

$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}x$ or $-\csc^{-1}x$

$$\begin{aligned} \text{(ii)} \int \frac{1-x^2}{1+x^2} dx &= \int \frac{-1+x^2}{1+x^2} dx \\ &= \int \left(-1 + \frac{2}{1+x^2} \right) dx \\ &= -\int dx + 2 \int \frac{1}{1+x^2} dx \\ &= -x + 2 \tan^{-1}x + c \end{aligned}$$

$$\begin{aligned} \text{(iii)} \int \frac{dx}{\sqrt{x+a} + \sqrt{x}} &= \int \frac{dx}{\sqrt{x+a} + \sqrt{x}} \cdot \frac{\sqrt{x+a} - \sqrt{x}}{\sqrt{x+a} - \sqrt{x}} \\ &= \int \frac{\sqrt{x+a} - \sqrt{x}}{\sqrt{x+a} + \sqrt{x}} \cdot \frac{\sqrt{x+a} - \sqrt{x}}{\sqrt{x+a} - \sqrt{x}} dx \end{aligned}$$

$$\begin{aligned}
 &= \int \frac{\sqrt{x+a} - \sqrt{x}}{x+a-x} dx \\
 &= \int \frac{(x+a)^{\frac{1}{2}} - (x)^{\frac{1}{2}}}{a} dx \\
 &= \frac{1}{a} \left[\int (x+a)^{\frac{1}{2}} dx - \int (x)^{\frac{1}{2}} dx \right] \\
 &= \frac{1}{a} \left[\frac{(x+a)^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{(x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right] + c \\
 &= \frac{1}{a} \left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x)^{\frac{3}{2}}}{\frac{3}{2}} \right] + c \\
 &= \frac{2}{3a} \left[(x+a)^{\frac{3}{2}} - x^{\frac{3}{2}} \right] + c \text{ Ans.}
 \end{aligned}$$

(iv) $\int (a-2x)^{\frac{3}{2}} dx$

$$\begin{aligned}
 &= \frac{(a-2x)^{\frac{3}{2}+1}}{\left(\frac{3}{2}+1\right) \cdot (-2)} + c \\
 &= \frac{(a-2x)^{\frac{5}{2}}}{\left(\frac{5}{2}\right) \cdot (-2)} + c \\
 &= -\frac{(a-2x)^{\frac{5}{2}}}{5} + c
 \end{aligned}$$

(v) $\int \frac{(1+e^x)^3}{e^x} dx = \int \frac{(1+3e^x+3e^{2x}+e^{3x})}{e^x} dx$

$$\begin{aligned}
 &= \int \left(\frac{1}{e^x} + \frac{3e^x}{e^x} + \frac{3e^{2x}}{e^x} + \frac{e^{3x}}{e^x} \right) dx \\
 &= \int (e^{-x} + 3 + 3e^x + e^{2x}) dx \\
 &= \frac{e^{-x}}{-1} + 3x + 3e^x + \frac{e^{2x}}{2} + c \\
 &= -e^{-x} + 3x + 3e^x + \frac{1}{2}e^{2x} + c
 \end{aligned}$$

Important Integrals

We know $\frac{d}{dx} \cos ax = -a \sin ax$

Taking integral

$$\begin{aligned}
 \int \frac{d}{dx} \cos ax dx &= -\int a \sin ax dx \\
 \Rightarrow \cos ax &= -a \int \sin ax dx \\
 \Rightarrow \int \sin ax dx &= -\frac{\cos ax}{a}
 \end{aligned}$$

Also $\frac{d}{dx} \sin ax = a \cdot \cos ax$

$$\therefore \int \cos ax dx = \frac{\sin ax}{a}$$

Similarly

$$\int \sec^2 ax dx = \frac{\tan ax}{a}$$

$$\int \operatorname{cosec}^2 ax dx = -\frac{\cot ax}{a}$$

$$\int \sec ax \tan ax dx = \frac{\sec ax}{a}$$

$$\int \operatorname{csc} ax \cot ax dx = -\frac{\operatorname{csc} ax}{a}$$

Also note that

$$\int \sin(ax+b) dx = -\frac{\cos(ax+b)}{a}$$

$$\int \cos(ax+b) dx = \frac{\sin(ax+b)}{a} \text{ and so on.}$$

(vi) $\int \sin(a+b)x dx = -\frac{\cos(a+b)x}{a+b} + c$
Do yourself

(vii) $\int \sqrt{1-\cos 2x} dx$

$$\begin{aligned}
 &= \int \sqrt{2 \sin^2 x} dx \quad \because \sin^2 x = \frac{1-\cos 2x}{2} \\
 &= \sqrt{2} \int \sin x dx = \sqrt{2} (-\cos x) + c \\
 &= -\sqrt{2} \cos x + c
 \end{aligned}$$

Important Formula

$$\therefore \frac{d}{dx} [f(x)]^{n+1} = (n+1)[f(x)]^n \frac{d}{dx} f(x)$$

$$\Rightarrow \frac{d}{dx} [f(x)]^{n+1} = (n+1)[f(x)]^n f'(x)$$

Taking integral

$$\int \frac{d}{dx} [f(x)]^{n+1} dx = \int (n+1)[f(x)]^n f'(x) dx$$

$$\Rightarrow [f(x)]^{n+1} = (n+1) \int [f(x)]^n f'(x) dx$$

$$\Rightarrow \int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{(n+1)} \quad ; \quad n \neq -1$$

Also $\frac{d}{dx} \ln |f(x)| = \frac{1}{f(x)} \cdot f'(x)$

Taking integral

$$\ln |f(x)| = \int \frac{f'(x)}{f(x)} dx$$

i.e. $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$

(viii) Let $I = \int \ln x \times \frac{1}{x} dx$

Put $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$

So $I = \int [f(x)] f'(x) dx$
 $= \frac{[f(x)]^{1+1}}{1+1} + c = \frac{[f(x)]^2}{2} + c$
 $= \frac{(\ln x)^2}{2} + c$

(ix) $\int \sin^2 x dx = \int \left(\frac{1 - \cos 2x}{2} \right) dx$
 $= \int \left(\frac{1}{2} - \frac{1}{2} \cos 2x \right) dx$
 $= \frac{1}{2} \int dx - \frac{1}{2} \int \cos 2x dx$
 $= \frac{1}{2} x - \frac{1}{2} \frac{\sin 2x}{2} + c$
 $= \frac{1}{2} x - \frac{1}{4} \sin 2x + c$

(x) $\int \frac{1}{1 + \cos x} dx$
 $= \int \frac{1}{2 \cos^2 \frac{x}{2}} dx \quad \because \cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$
 $= \frac{1}{2} \int \sec^2 \frac{x}{2} dx = \frac{1}{2} \frac{\tan \frac{x}{2}}{1/2} + c = \tan \frac{x}{2} + c$

Alternative

$\int \frac{1}{1 + \cos x} dx = \int \frac{1}{1 + \cos x} \times \frac{1 - \cos x}{1 - \cos x} dx$
 $= \int \frac{1 - \cos x}{1 - \cos^2 x} dx$
 $= \int \frac{1 - \cos x}{\sin^2 x} dx$
 $= \int \left(\frac{1}{\sin^2 x} - \frac{\cos x}{\sin^2 x} \right) dx$
 $= \int \left(\operatorname{cosec}^2 x - \frac{\cos x}{\sin x \cdot \sin x} \right) dx$
 $= \int \operatorname{cosec}^2 x dx - \int \operatorname{cosec} x \cot x dx$
 $= -\cot x - (-\operatorname{cosec} x) + c$
 $= \operatorname{cosec} x - \cot x + c$

(xi) Let $I = \int \frac{ax + b}{ax^2 + 2bx + c} dx$

Put $f(x) = ax^2 + 2bx + c$

$\Rightarrow f'(x) = 2ax + 2b$

$\Rightarrow f'(x) = 2(ax + b) \Rightarrow \frac{1}{2} f'(x) = ax + b$

So $I = \int \frac{\frac{1}{2} f'(x)}{f(x)} dx$
 $= \frac{1}{2} \int \frac{f'(x)}{f(x)} dx = \frac{1}{2} \ln|f(x)| + c_1$
 $= \frac{1}{2} \ln|ax^2 + 2bx + c| + c_1$

Review

- $2 \sin \alpha \cos \beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$
- $2 \cos \alpha \sin \beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$
- $2 \cos \alpha \cos \beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$
- $-2 \sin \alpha \sin \beta = \cos(\alpha + \beta) - \cos(\alpha - \beta)$

(xii) $\int \cos 3x \sin 2x dx$

$= \frac{1}{2} \int 2 \cos 3x \sin 2x dx$
 $= \frac{1}{2} \int [\sin(3x + 2x) - \sin(3x - 2x)] dx$
 $= \frac{1}{2} \int [\sin 5x - \sin x] dx$
 $= \frac{1}{2} \left[-\frac{\cos 5x}{5} - (-\cos x) \right] + c$
 $= -\frac{1}{2} \left[\frac{\cos 5x}{5} - \cos x \right] + c$

(xiii) $\int \frac{\cos 2x - 1}{1 + \cos 2x} dx$

$= -\int \frac{1 - \cos 2x}{1 + \cos 2x} dx \quad \because \sin^2 x = \frac{1 - \cos 2x}{2}$
 $= -\int \frac{2 \sin^2 x}{2 \cos^2 x} dx \quad \cos^2 x = \frac{1 + \cos 2x}{2}$
 $= -\int \tan^2 x dx = -\int (\sec^2 x - 1) dx$
 $= -\int \sec^2 x dx + \int dx$
 $= -\tan x + x + c$

(xiv) $\int \tan^2 x dx = \int (\sec^2 x - 1) dx$
 $= \int \sec^2 x dx - \int dx$
 $= \tan x - x + c$

Important Integral

$$\text{Since } \frac{d}{dx} \ln|ax+b| = \frac{1}{ax+b} \cdot \frac{d}{dx}(ax+b)$$

$$\Rightarrow \frac{d}{dx} \ln|ax+b| = \frac{1}{ax+b} \cdot a$$

On Integrating

$$\Rightarrow \ln|ax+b| = a \int \frac{1}{ax+b} dx$$

$$\Rightarrow \int \frac{1}{ax+b} dx = \frac{\ln|ax+b|}{a}$$

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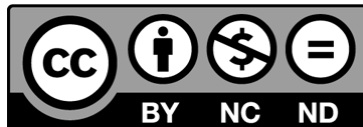
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Book: Exercise 3.2 (Page 130)

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