Q 1 Given that

$$f(x)=\begin{cases}\dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & x\neq 2\\ k, & x=2\end{cases}$$

$k=?$

Here $f(2)=k$ (given)

$\because f(x)$ is continuous at $x=2$

$\therefore \lim\limits_{x\to 2} f(x) = f(2)$ $\left(\frac{0}{0}\right)$ form

$\Rightarrow \lim\limits_{x\to 2}\dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}=k$

$\Rightarrow \lim\limits_{x\to 2}\dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}\times\dfrac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}}=k$

$\Rightarrow \lim\limits_{x\to 2}\dfrac{(\sqrt{2x+5})^2-(\sqrt{x+7})^2}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}=k$

$\Rightarrow \lim\limits_{x\to 2}\dfrac{(2x+5)-(x+7)}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}=k$

$\Rightarrow \lim\limits_{x\to 2}\dfrac{2x+5-x-7}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}=k$

$\Rightarrow \lim\limits_{x\to 2}\dfrac{(x-2)}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}=k$

$\Rightarrow \lim\limits_{x\to 2}\dfrac{1}{\sqrt{2x+5}+\sqrt{x+7}}=k$

$\Rightarrow \dfrac{1}{\lim\limits_{x\to 2}\left[\sqrt{2x+5}+\sqrt{x+7}\right]}=k$

$\Rightarrow \dfrac{1}{\sqrt{2(2)+5}+\sqrt{2+7}}=k$

$\Rightarrow \dfrac{1}{\sqrt{9}+\sqrt{9}}=k \Rightarrow \dfrac{1}{3+3}=k \Rightarrow \dfrac{1}{6}=k$

$\boxed{k=\frac{1}{6}}$ Ans.

EXERCISE 1.5

Q 1 (i) $x^2+y^2=9$

$\Rightarrow y^2=9-x^2$

$\Rightarrow y=\pm\sqrt{9-x^2}$

$\Rightarrow y$ will be real if $9-x^2\geq 0$

$\Rightarrow 9\geq x^2 \Rightarrow x^2\leq 9$

$\Rightarrow \pm x\leq 3$

$\Rightarrow x\leq 3,\ -x\leq 3$

$\Rightarrow x\leq 3,\ x\geq -3$

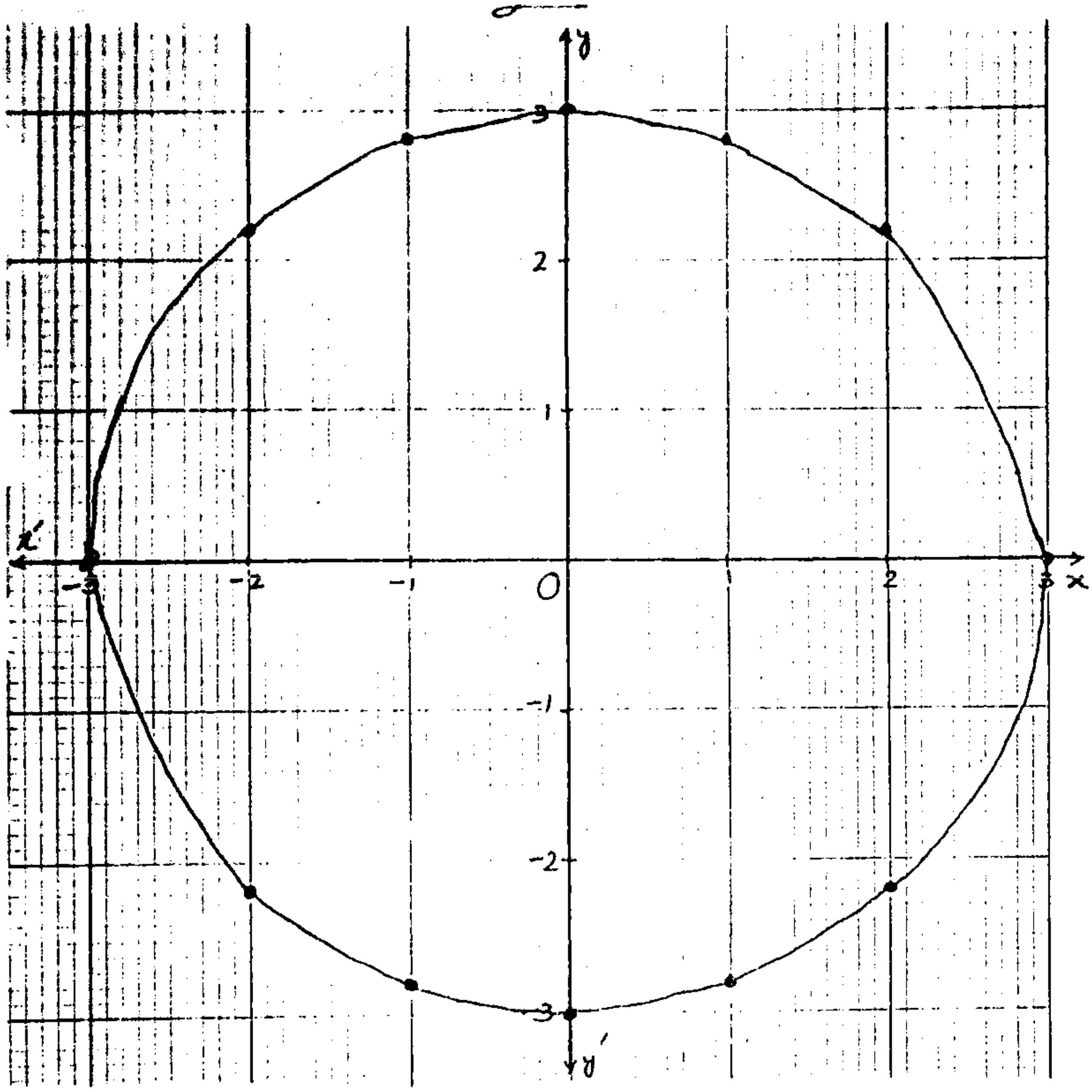
$\Rightarrow x\leq 3,\ -3\leq x$

$\Rightarrow -3\leq x\leq 3$

Table :

x	-3	-2	-1	0	1	2	3
y	0	± 2.2	± 2.8	± 3	± 2.8	± 2.2	0

Scale : One big square along x-axis = 1 units
One big square along y-axis = 1 unit



(ii) $\frac{x^2}{16}+\frac{y^2}{4}=1$

Multiplying by 16, we get

$x^2+4y^2=16 \Rightarrow 4y^2=16-x^2 \Rightarrow y^2=\frac{16-x^2}{4} \Rightarrow y=\pm\frac{\sqrt{16-x^2}}{2}$

$\Rightarrow y$ will be real if $16-x^2 \geq 0 \Rightarrow 16 \geq x^2 \Rightarrow x^2 \leq 16$

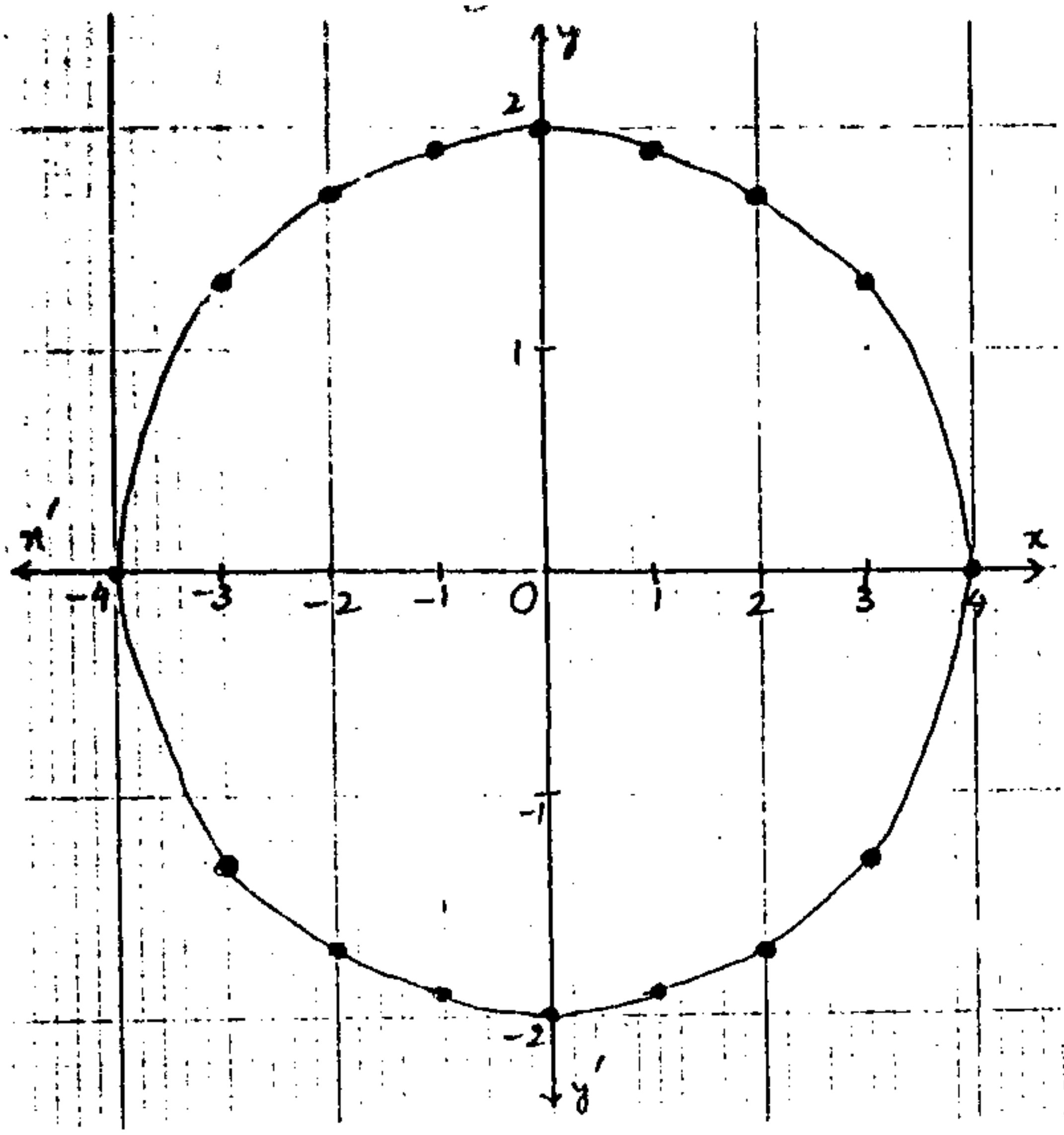
$\Rightarrow -4 \leq x \leq 4$

Table:

x	-4	-3	-2	-1	0	1	2	3	4
y	0	± 1.3	± 1.7	± 1.9	± 2	± 1.9	± 1.7	± 1.3	0

Scale: One big square along x-axis = 2 units

One big square along y-axis = 1 unit.



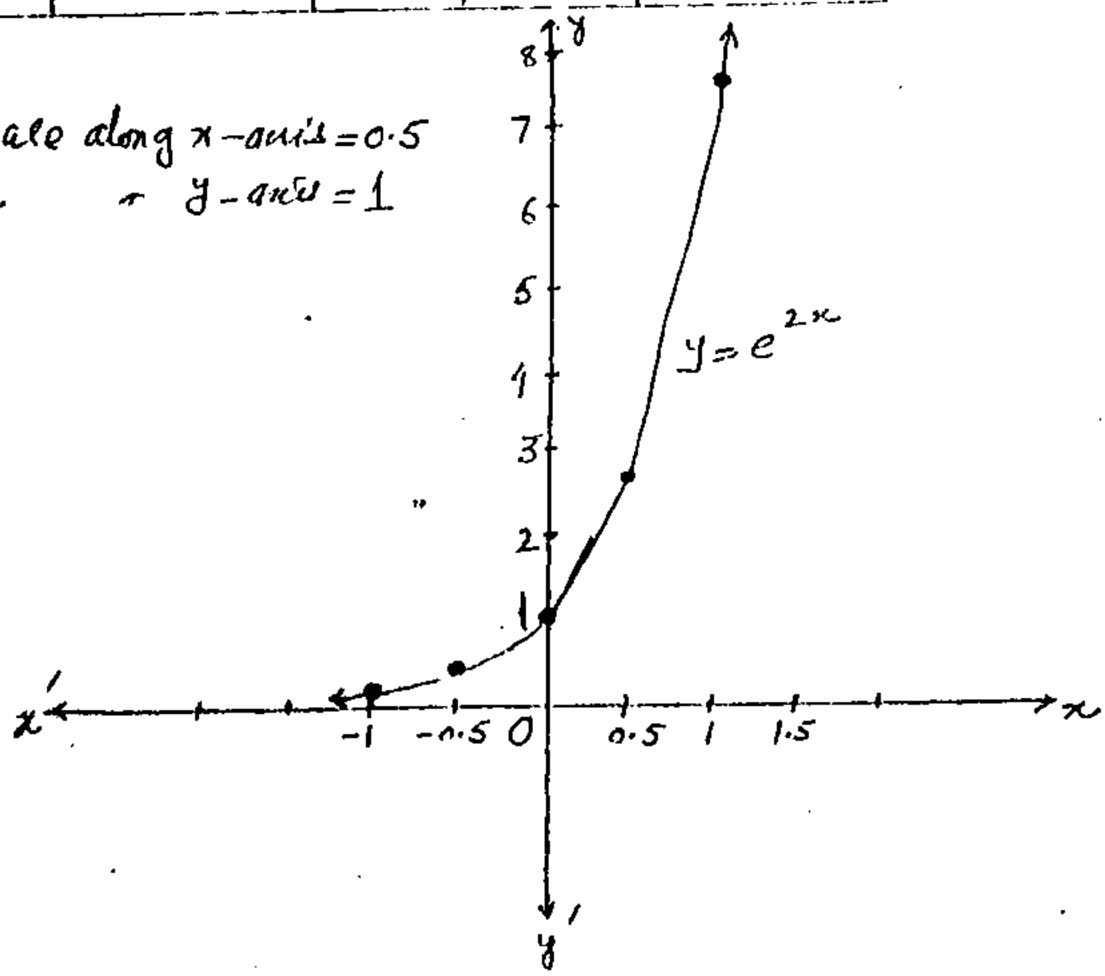
(iii) $y = e^{2x}$

x	-1	-0.5	0	0.5	1
y	0.1	0.4	1	2.7	7.4

Scales:-

One big square along x-axis = 0.5

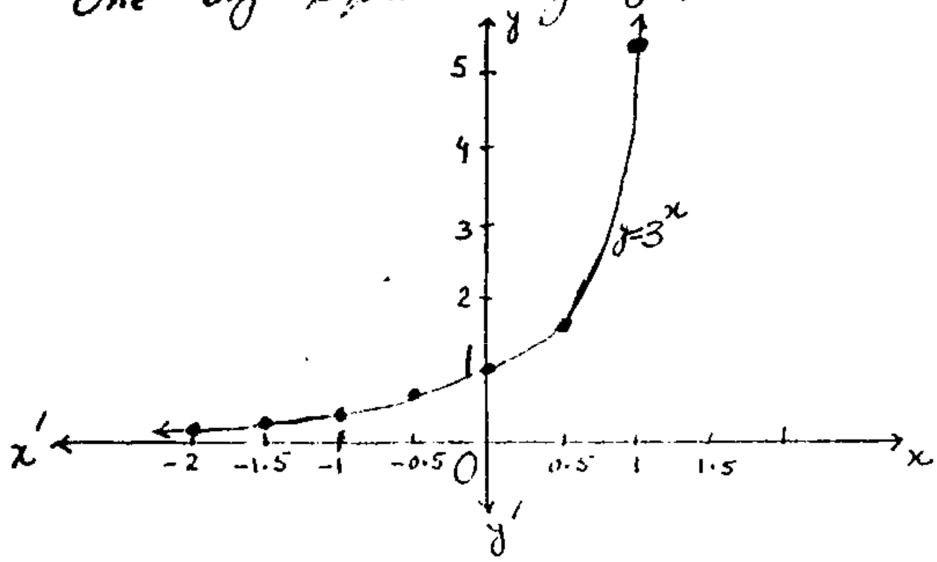
" " " " y-axis = 1



(iv) $y = 3^x$

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5
y	0.1	0.2	0.3	0.6	1	1.7	3	5.2

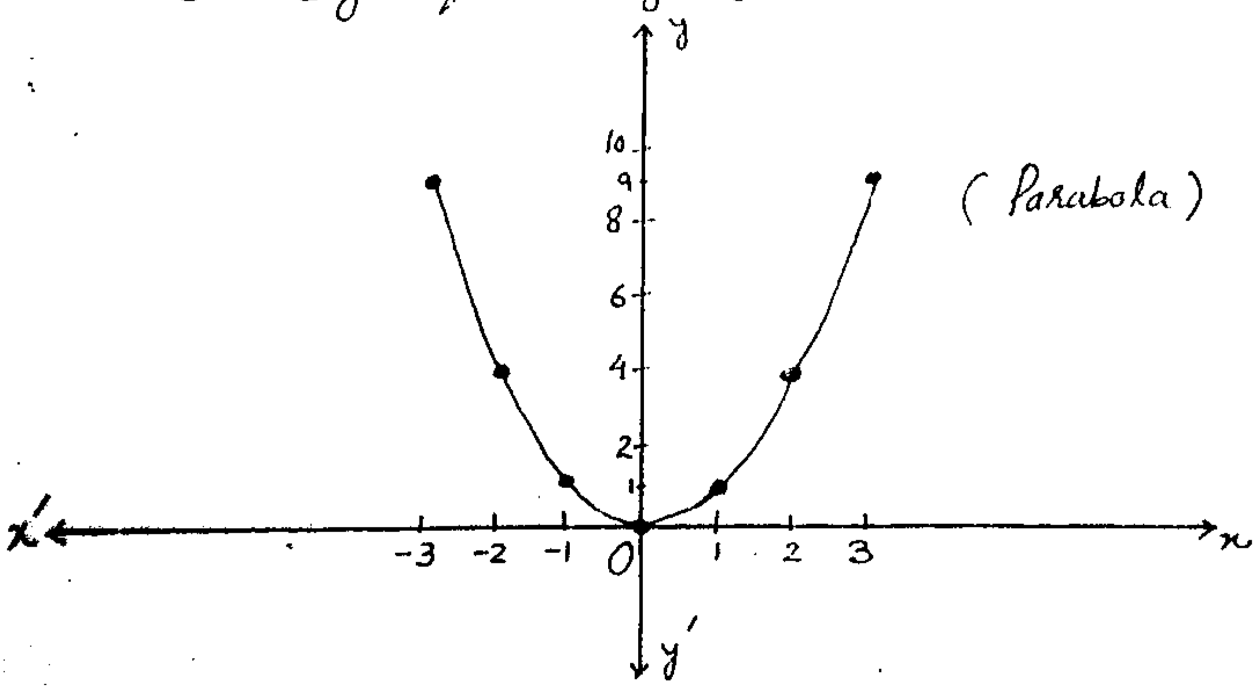
Scale: One big Square along x-axis = 0.5 units
One big Square along y-axis = 1 unit



② (i) $x = t,\quad y = t^2,\quad -3 \le t \le 3$

t	-3	-2	-1	0	1	2	3
x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

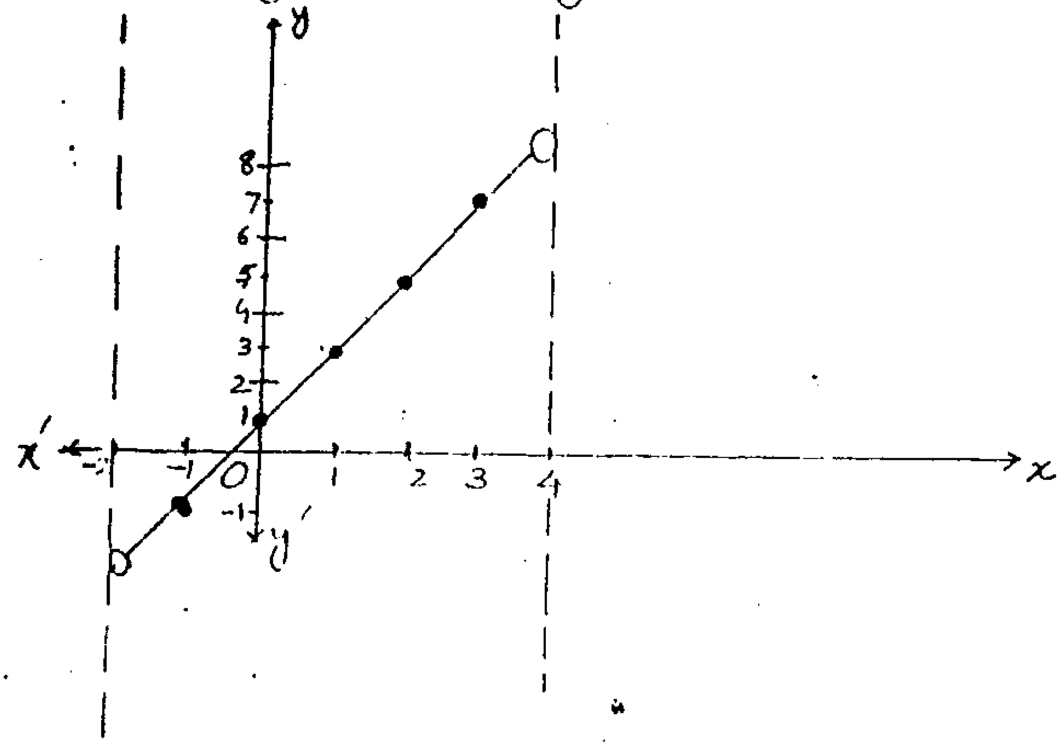
Scale: One big Square along x-axis = 1 unit
One big Square along y-axis = 2 units



(ii) $x = t-1,\quad y = 2t-1,\quad -1 < t < 5$

t	0	1	2	3	4
x	−1	0	1	2	3
y	−1	1	3	5	7

Scale: One big square along x-axis = 1 unit
One big square along y-axis = 2 units



(iii) $x = \sec\theta,\quad y = \tan\theta$, where θ is a parameter.

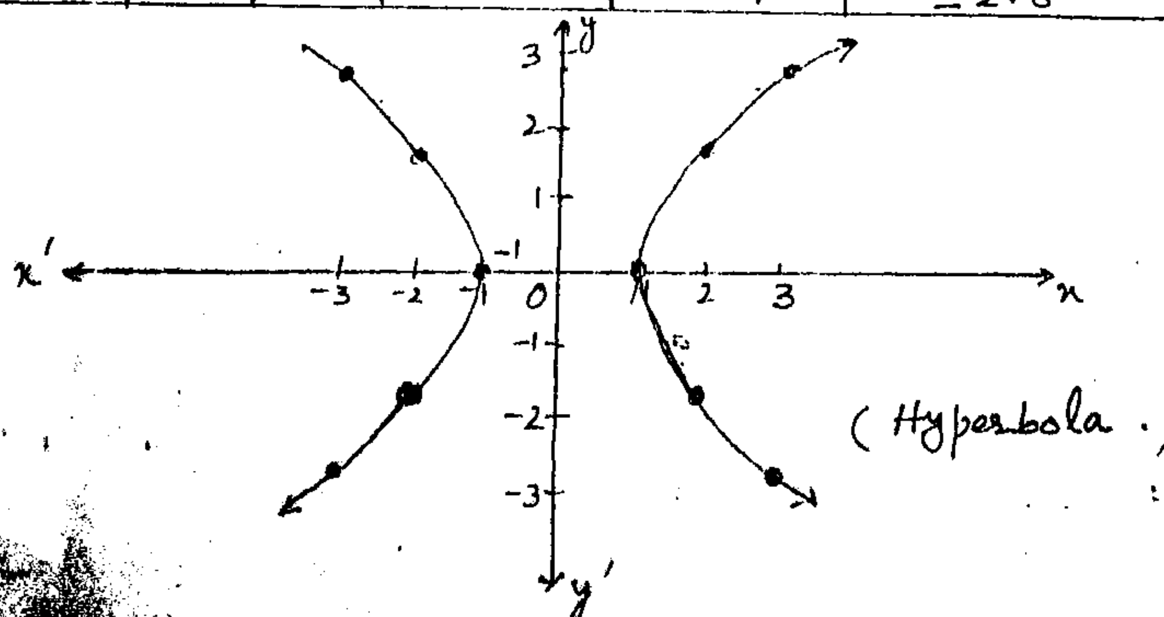
$\because\ 1+\tan^2\theta = \sec^2\theta \Rightarrow 1 = \sec^2\theta - \tan^2\theta$

$\Rightarrow 1 = x^2 - y^2 \Rightarrow y^2 = x^2-1 \Rightarrow y = \pm\sqrt{x^2-1}$

$\Rightarrow y$ will be real if $x^2-1 \geq 0 \Rightarrow x^2 \geq 1$

$\Rightarrow \pm x \geq 1 \Rightarrow x \geq 1,\ -x \geq 1 \Rightarrow x \geq 1,\ x \leq -1$

x	−3	−2	−1	1	2	3
y	±2.8	±1.7	0	0	±1.7	±2.8



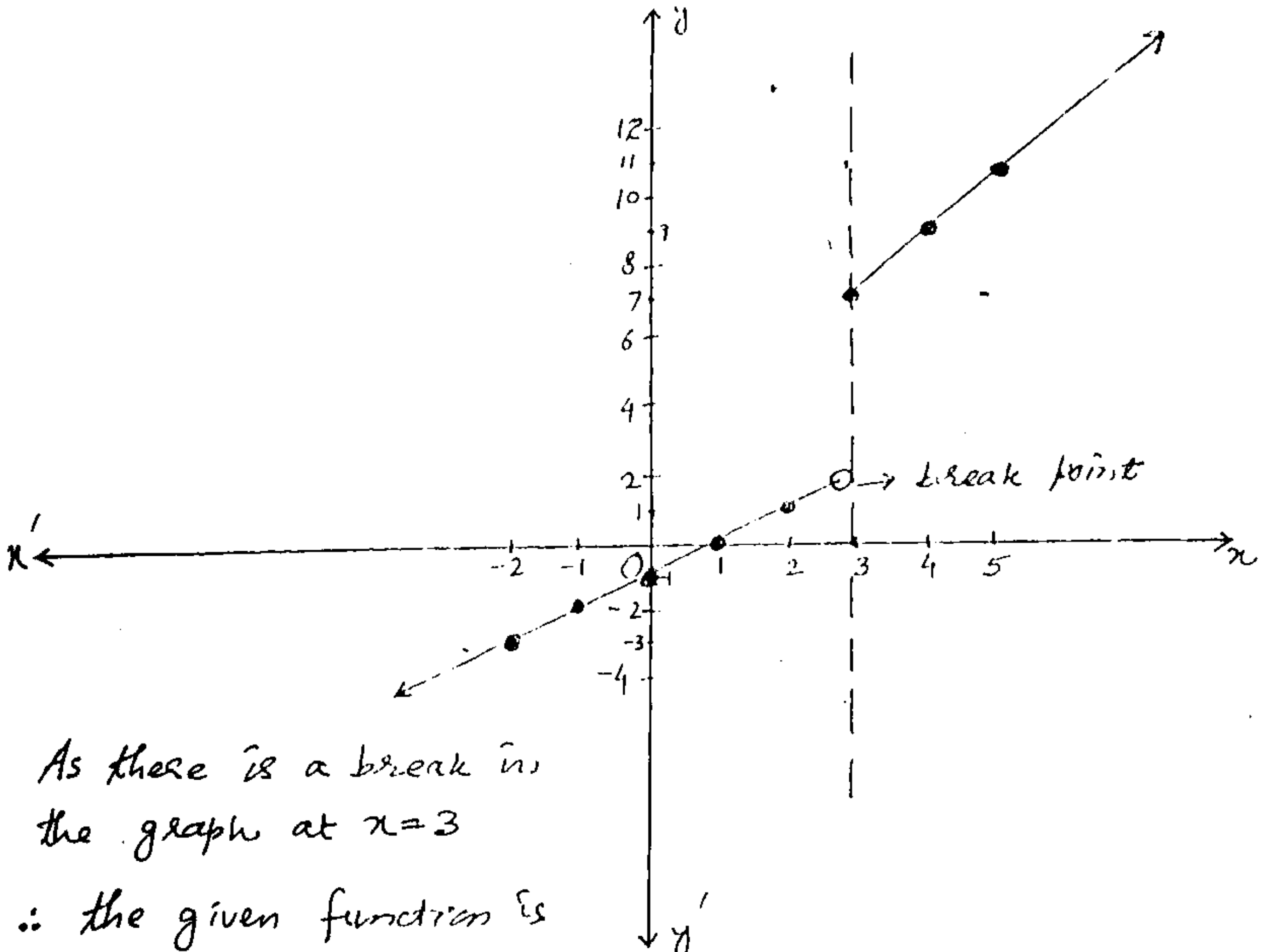
(3) (i) $y = \begin{cases} x-1 & \text{if } x < 3 \\ 2x+1 & \text{if } x \geq 3 \end{cases}$

Table for $y = x - 1$, $x < 3$ is

x	-2	-1	0	1	2
y	-3	-2	-1	0	1

Table for $y = 2x+1$, $x \geq 3$ is

x	3	4	5
y	7	9	11



As there is a break in the graph at $x=3$

$\therefore$ the given function is discontinuous at $x=3$

(ii) $y = \dfrac{x^2-4}{x-2}$, $x \neq 2$

$= \dfrac{(x-2)(x+2)}{x-2}$, $x \neq 2$

$= x+2$, $x \neq 2$

The given function is not defined at $x=2$

x	-3	-2	-1	0	1	3	4	5
y	-1	0	1	2	3	5	6	7

Scale: One big square along x-axis = 2 units
One big square along y-axis = 2 units.

Q. i) $x = \sin 2x$, $-\pi \le x \le \pi$

We draw the graphs of

$y = x$ and $y = \sin 2x$

For $y = \sin 2x$

x	-180°	-150°	-120°	-90°	-60°	-30°	0°	30°	60°	90°	120°	150°	180°
y	0	0.9	+0.9	0	-0.9	-0.9	0	0.9	0.9	0	-0.9	-0.9	0

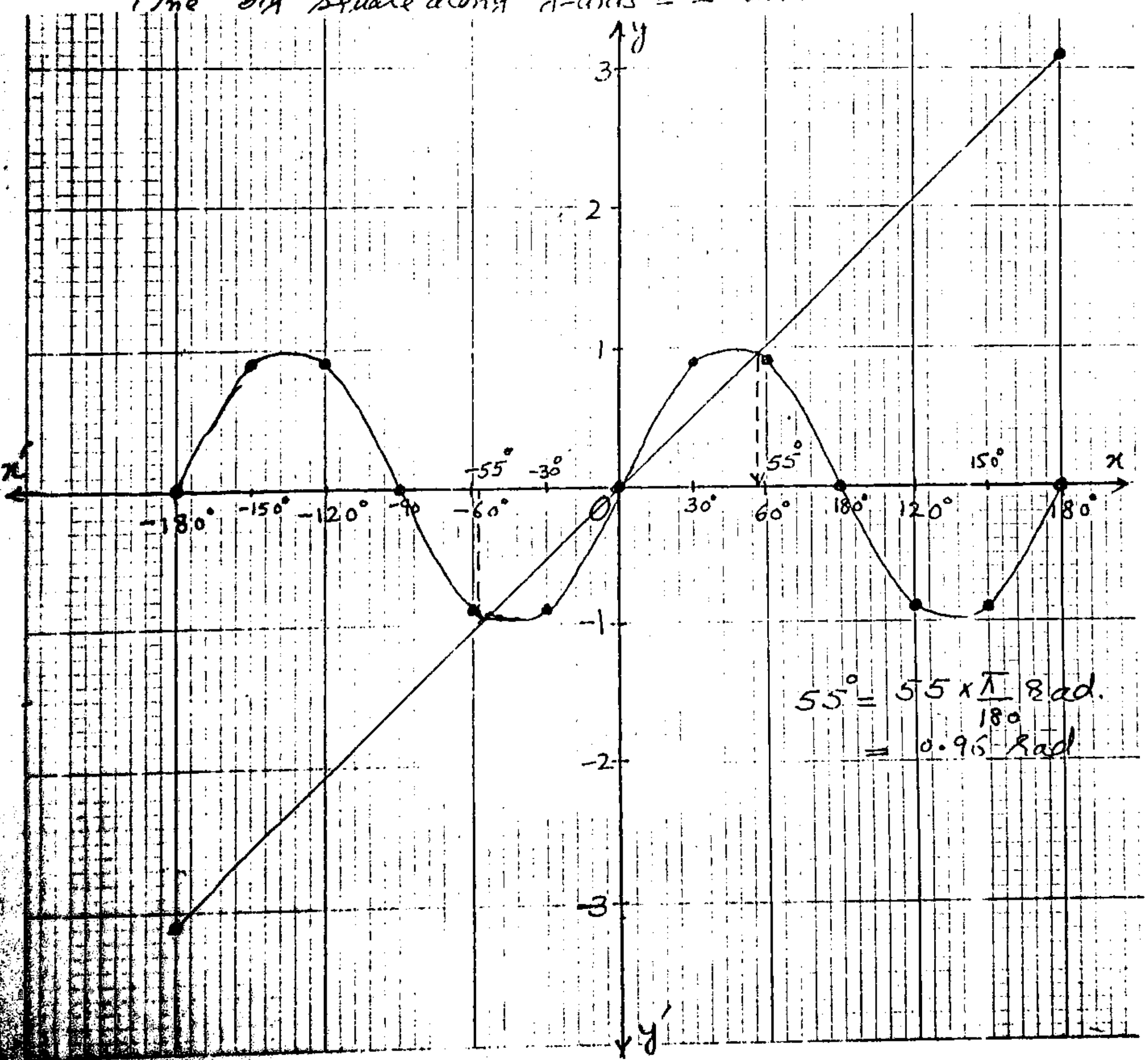
For $y = x$

x	-180°	0	180°
y	-3.1	0	3.1

$180^\circ = \pi$ Rad $= 3.1$ Rad.

Scale One big Square along x-axis $= 60^\circ$

One big Square along y-axis $= 1$ unit.



The line $y = x$ cuts the curve $y = \sin 2x$ at $x = -55^\circ, 0, 55^\circ$

i.e. S.S $= \{-55^\circ, 0, 55^\circ\} = \{-0.96, 0, 0.96\}$

Graph

(ii) $\frac{x}{2} = \cos x$, $-\pi \le x \le \pi$

We draw the graphs of $y = \frac{x}{2}$ and $y = \cos x$

For $y = \cos x$

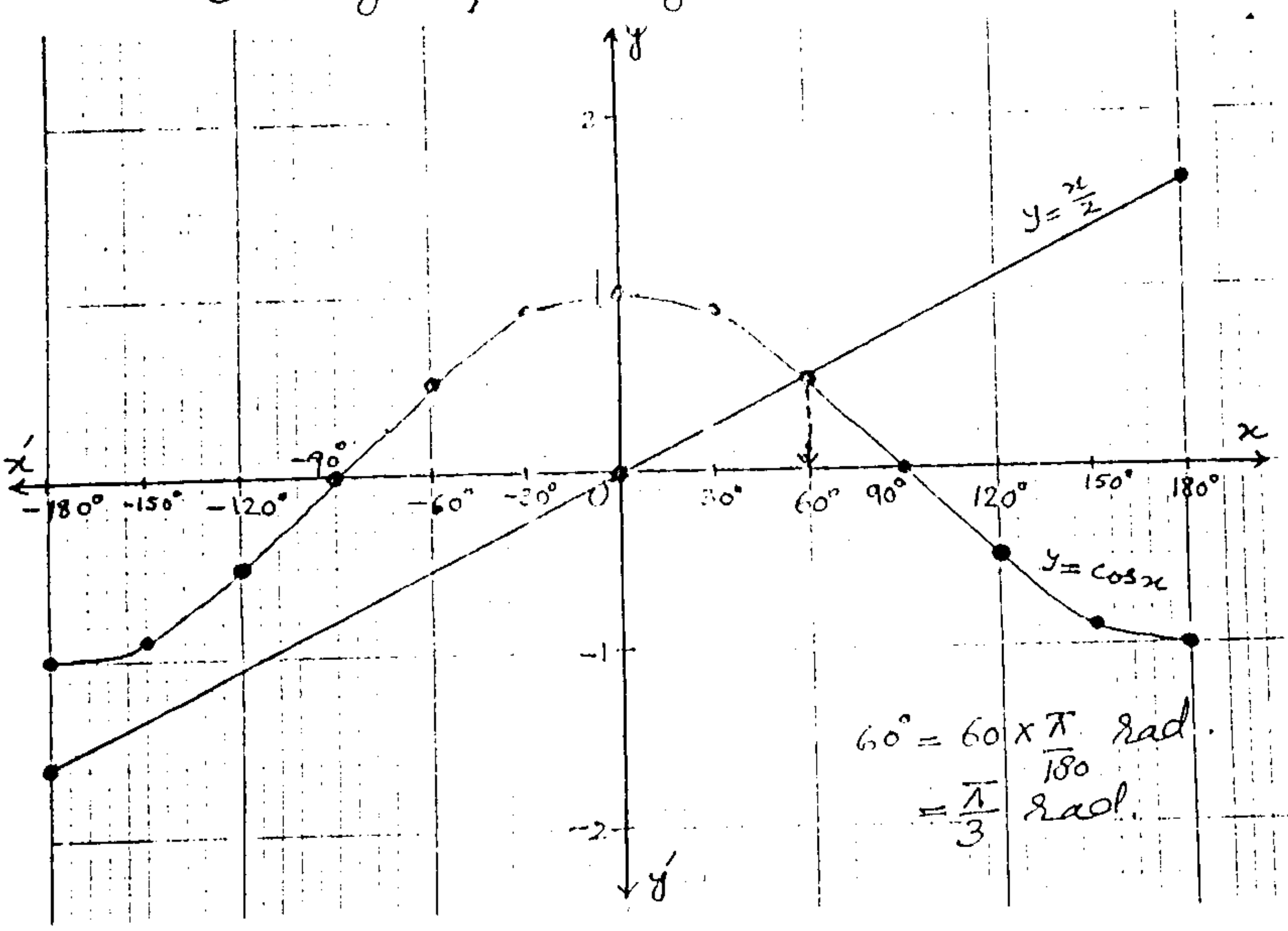
x	$-180°$	$-150°$	$-120°$	$-90°$	$-60°$	$-30°$	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$
y	-1	-0.9	-0.5	0	0.5	0.9	1	0.9	0.5	0	-0.5	-0.9	-1

For $y = \frac{x}{2}$

x	$-180°$	$0°$	$180°$
y	-1.6	0	1.6

Scale: One big square along x-axis = $60°$

One big square along y-axis = 1 unit.



The line $y = \frac{x}{2}$ cuts the curve $y = \cos x$ at $x = 60°$

$\therefore$ S.S. $= \{60°\} = \{\frac{\pi}{3}\}$ Ans.

(iii) $2x = \tan x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

We draw the graphs of $y = 2x$ and $y = \tan x$

For $y = \tan x$

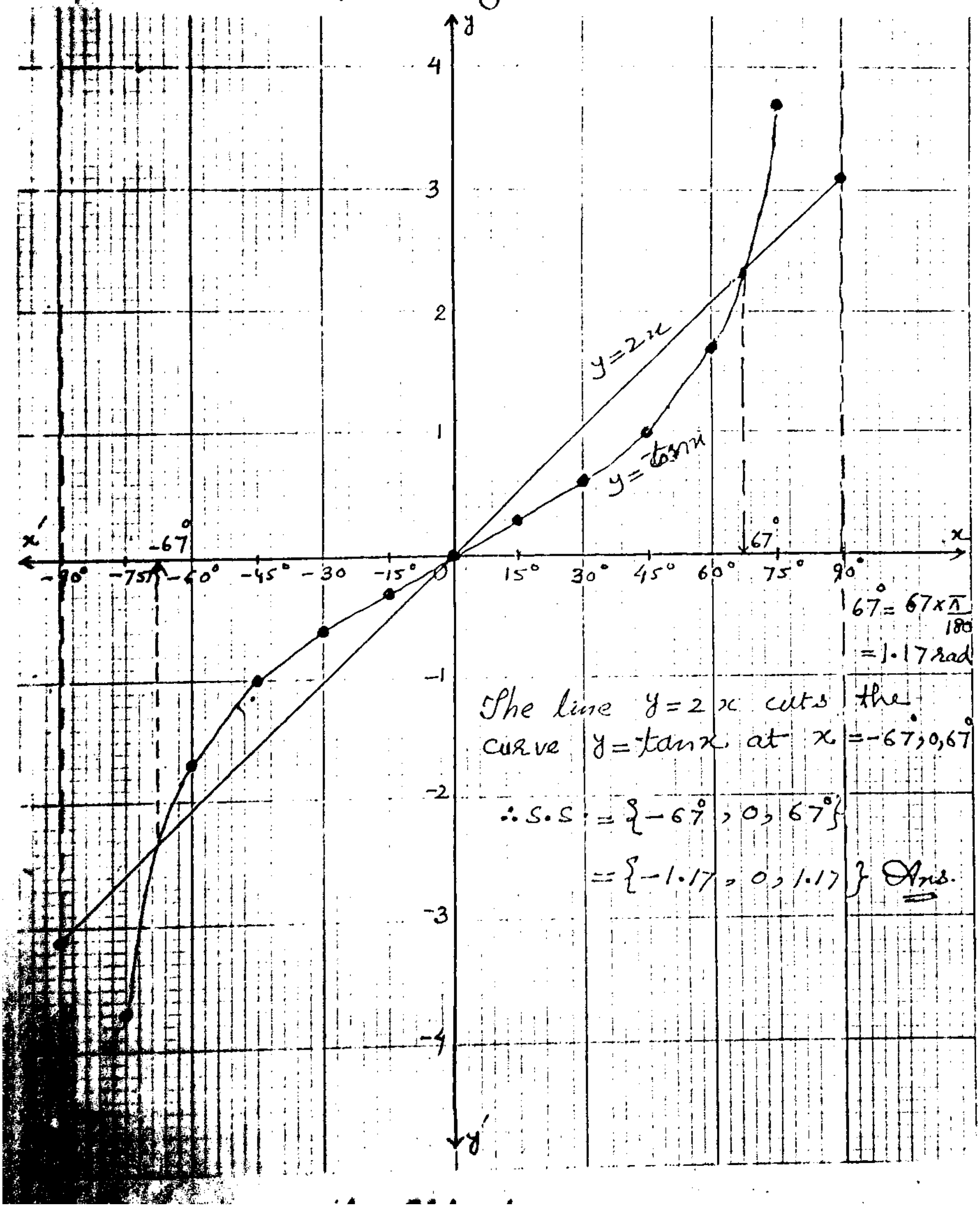
x	$-90°$	$-75°$	$-60°$	$-45°$	$-30°$	$-15°$	$0°$	$15°$	$30°$	$45°$	$60°$	$75°$	$90°$
y	∞	-3.7	-1.7	-1	-0.6	-0.3	0	0.3	0.6	1	1.7	3.7	∞

For $y = 2x$

x	-90°	0	90°
y	-3.1	0	3.1

Scale: One big Square along x-axis $= 30^\circ$

One big Square along y-axis $= 1$ unit



$67^\circ = 67 \times \frac{\pi}{180}$

$= 1.17$ rad

The line $y = 2x$ cuts the curve $y = \tan x$ at $x = -67^\circ, 0, 67^\circ$

$\therefore S.S = \{-67^\circ, 0, 67^\circ\}$

$= \{-1.17, 0, 1.17\}$ Ans.