

to support it. It is important to understand what we mean by the terms *reject* and *accept* in hypothesis testing. The *rejection* of a hypothesis is to declare it false. The *acceptance* of a hypothesis is to declare it sufficient evidence to reject it. Acceptance does not necessarily mean that the hypothesis is true.

The basic concepts associated with hypothesis testing are discussed below:

(i) 16.1.1. Null and Alternative Hypothesis. A null hypothesis, H_0 is any hypothesis which is to be tested for possible rejection under the assumption that it is true. Today the term is used for any hypothesis that is being tested. The word null in the term *null hypothesis* implies that usually H_0 is the hypothesis of no effect. A null hypothesis should always be precise such as "the given coin is unbiased" or "a drug is ineffective in curing a particular disease" or "the difference between the two teaching methods is null or zero." The hypothesis is usually assigned a numerical value. For example, suppose we think that the average height of students in all colleges is 62". This

statement is taken as a hypothesis and is written symbolically as $H_0 : \mu = 62''$. In other words, we hypothesize that $\mu = 62''$.

An alternative hypothesis is any other hypothesis which we accept when the null hypothesis H_0 is rejected. It is customarily denoted by H_1 or H_A . A null hypothesis H_0 is thus tested against an alternative hypothesis H_1 . For example, if our null hypothesis is $H_0 : \mu = 62''$, then our alternative hypothesis may be $H_1 : \mu \neq 62''$ or $H_1 : \mu > 62''$ or

$$H_1 : \mu < 62''.$$

16.1.2.

Simple and Composite Hypotheses. A simple

hypothesis is one in which all parameters of the distribution are specified. For example, if the heights of college students are normally distributed with $\sigma^2 = 4$, the hypothesis that its mean μ is, say, $62''$

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or H_A . A null hypothesis H_0 is rejected. It is customarily denoted by H_0 and the alternative hypothesis which we are testing against is denoted by H_1 . For example, if our null hypothesis is $H_0 : \mu = 62''$, then our alternative hypothesis may be $H_1 : \mu \neq 62''$ or $H_1 : \mu > 62''$ or $H_1 : \mu < 62''$.

16.1.2. Simple and Composite Hypotheses. A simple hypothesis is one in which all parameters of the distribution are specified. For example, if the heights of college students are normally distributed with $\sigma^2 = 4$, the hypothesis that its mean μ is, say, $62''$, is $H : \mu = 62$; we have stated a simple hypothesis completely. A simple hypothesis, in general, states that $\theta = \theta_0$ where θ_0 is the specified value of a parameter θ , (θ may represent $\mu, p, \mu_1 - \mu_2$, etc.).

A hypothesis which is not simple (i.e. in which not all of the parameters are specified) is called a *composite hypothesis*. For instance if we hypothesize that $H : \mu > 62$ (and $\sigma^2 = 4$) or $H : \mu = 62$ and $\sigma^2 < 4$ the hypothesis becomes a composite hypothesis because we cannot know the exact distribution of the population in either case. Obviously, the parameters $\mu > 62''$ and $\sigma^2 < 4$ have more than one value and no specified values are being assigned. The general form of a composite hypothesis is $\theta \leq \theta_0$ or $\theta \geq \theta_0$, that is the parameter θ does not exceed or

... not fall short of a specified value θ_0 . The concept of simple and composite hypotheses applies to both null hypothesis and alternative hypothesis.

Hypotheses may also be classified as exact and inexact. A hypothesis is said to be an exact hypothesis if it selects a unique value for the parameter such as $H : \mu = 62$ or $p = 0.5$. A hypothesis is called an inexact hypothesis when it indicates more than one possible values for the parameter such as $H : \mu \neq 62$ or $H : p > 0.5$. A simple hypothesis must be an exact one while an exact hypothesis is not necessarily a simple hypothesis. An inexact hypothesis is a composite hypothesis.

★ 16.1.3. Test-statistic. A sample statistic which provides a basis for testing a null hypothesis, is called a test-statistic. Every test-statistic has a probability (sampling) distribution which gives the probability of obtaining a specified value of the test-statistic when the null hypothesis is true. It is important to remember that a test-statistic does not prove the hypothesis to be correct but it furnishes an evidence against the

hypothesis. The sampling distributions of the most commonly used test-statistics are normal, t , chi-square or F .

16.1.4. Acceptance and Rejection Regions.

All possible values which a test-statistic may assume can be divided into two mutually exclusive groups: (one group consisting of values which appear to be consistent with the null hypothesis) and the other having values which are unlikely to occur if H_0 is true. The first group is called the acceptance region and the second set of values is known as the rejection region for a test. The rejection region is also called the critical region. The value(s) that separates the critical region from the acceptance region, is called the critical value(s). The critical value which can be in the same units as the parameter or in the standardized units, is to be decided by the experimenter keeping in view the degree of confidence he (she) is willing to have in the null hypothesis.

16.1.5. Type I and Type II Errors.

When we perform a hypothesis test, we derive the evidence from the sample in the form of a test-statistic. There is a possibility that the sample evidence may lead us to make a wrong decision. We may reject a null hypothesis H_0 , when it is, in fact, true or we may accept a null hypothesis H_0 , when it is actually false. The former type is called an error of the first kind or a Type I-error, while the latter, an error of the second kind or a Type II-error. The decision and the corresponding two types of error may be displayed in a tabular form as below:

True Situation	DECISION	
	Accept H_0	Reject H_0 (or accept H_1)
H_0 is true	Correct decision (No error)	Wrong decision (Type-I error)
H_0 is false	Wrong decision = β (Type-II error)	Correct decision (No error)

Then Type I error (α) is away from the null hypothesis. Type II error (β) is staying in the null hypothesis.

The shaded area in the lower diagram represents power. This probability corresponds to the rejection region of the distribution under H_0 . The power generally increases with an increase in the sample size. A test for which β is small, is defined to be a powerful test.

A curve giving the probabilities of making Type II errors for various parameteric values under alternative hypotheses, is called an *Operating Characteristic Curve* or simply the *OC curve*. The *Power curve* which may be regarded as the complement of the *OC curve*, shows the probabilities of rejecting the null hypothesis H_0 for various values of the parameter θ . It is defined as the rejection of the null hypothesis.

16.1.7. The Significance Level of a test is the probability used as a standard for rejecting a null hypothesis H_0 when H_0 is assumed to be true. This probability is equal to some small pre-assigned value, conventionally denoted by α . The value α is also known as the size of the *critical region*. It is noteworthy that the significance level and the probability of Type I error are equivalent. The most frequently used values of α , the significance level, are 0.05 and 0.01, i.e. 5 percent and 1 percent but occasionally 0.10 or 0.001 is used. By $\alpha=5\%$, we mean that there are about 5 chances in 100 of incorrectly rejecting a true null hypothesis. To put it in another way, we say that we are 95% confident in making the correct decision.

A test of significance is a rule or

16.1.9. One-tailed and Two-tailed Tests. A test for which the entire rejection region is located in only one of the two tails—either in the right tail or in the left tail—of the sampling distribution of the test-statistic, is called a *One-tailed test* or *One-sided test*. For example, if Z is a test-statistic, then the rejection region consists of all z -values which are greater than $+z_\alpha$ or less than $-z_\alpha$ where α is the size of

