## Lecture. 4

## Measures of averages - Mean - median - mode - geometric mean - harmonic mean computation of the above statistics for raw and grouped data - merits and demerits measures of location - percentiles - quartiles - computation of the above statistics for raw and grouped data

In the study of a population with respect to one in which we are interested we may get a large number of observations. It is not possible to grasp any idea about the characteristic when we look at all the observations. So it is better to get one number for one group. That number must be a good representative one for all the observations to give a clear picture of that characteristic. Such representative number can be a central value for all these observations. This central value is called a measure of central tendency or an average or a measure of locations. There are five averages. Among them mean, median and mode are called simple averages and the other two averages geometric mean and harmonic mean are called special averages.

## Arithmetic mean or mean

Arithmetic mean or simply the mean of a variable is defined as the sum of the observations divided by the number of observations. It is denoted by the symbol $\bar{x}$ If the variable x assumes n values $\mathrm{x} 1, \mathrm{x} 2 \ldots \mathrm{xn}$ then the mean is given by

$$
\begin{aligned}
\bar{x} & =\frac{x_{1}+x_{2}+x_{3}+\ldots+x_{n}}{n} \\
& =\frac{1}{n} \sum_{i=1}^{n} x_{i}
\end{aligned}
$$

This formula is for the ungrouped or raw data.

## Example 1

Calculate the mean for pH levels of soil $6.8,6.6,5.2,5.6,5.8$

## Solution

$$
\bar{x}=\frac{6.8+6.6+5.2+5.6+5.8}{5}=\frac{30}{5}=6
$$

## Grouped Data

The mean for grouped data is obtained from the following formula:
$\bar{x}=\frac{\sum f x}{n}$
Where $x=$ the mid-point of individual class
$f=$ the frequency of individual class
$n=$ the sum of the frequencies or total frequencies in a sample.

## Short-cut method

$\bar{x}=A+\frac{\sum f d}{n} x c$
Where $d=\frac{x-A}{c}$
$\mathrm{A}=$ any value in x
$n=$ total frequency
$\mathrm{c}=$ width of the class interval

## Example 2

Given the following frequency distribution, calculate the arithmetic mean
$\begin{array}{lllllll}\text { Marks } & : & 64 & 63 & 62 & 61 & 60\end{array} 59$
Number of
Students $\}$ :
6
Solution

| $\mathbf{X}$ | $\mathbf{F}$ | $\mathbf{F x}$ | $\mathbf{D}=\mathbf{x}-\mathbf{A}$ | $\mathbf{F d}$ |
| :---: | :---: | :---: | :---: | :---: |
| 64 | 8 | 512 | 2 | 16 |
| 63 | 18 | 1134 | 1 | 18 |
| $\mathbf{6 2}$ | 12 | 744 | 0 | 0 |
| 61 | 9 | 549 | -1 | -9 |
| 60 | 7 | 420 | -2 | -14 |
| 59 | 6 | 354 | -3 | -18 |
|  | 60 | 3713 |  | -7 |

## Direct method

$\bar{x}=\frac{\sum f x}{n}$
$\bar{x}=\frac{3713}{60}=61.88$

## Short-cut method

$\bar{x}=A+\frac{\sum f d}{n} x c$
Here $\mathrm{A}=62$
$\bar{x}=62-\frac{7}{60} \times 1=61.88$

## Example 3

For the frequency distribution of seed yield of seasamum given in table, calculate the mean yield per plot.

| Yield per <br> plot in(in <br> $\mathrm{g})$ | $64.5-84.5$ | $84.5-104.5$ | $104.5-124.5$ | $124.5-144.5$ |
| :--- | :---: | :---: | :---: | :---: |
| No of <br> plots | 3 | 5 | 7 | 20 |

## Solution

| Yield (in g) | No of Plots (f) | Mid X | $d=\frac{x-A}{c}$ | Fd |
| :---: | :---: | :---: | :---: | :---: |
| $64.5-84.5$ | 3 | 74.5 | -1 | -3 |
| $84.5-104.5$ | 5 | 94.5 | 0 | 0 |
| $104.5-124.5$ | 7 | 114.5 | 1 | 7 |
| $124.5-144.5$ | 20 | 134.5 | 2 | 40 |
| Total | $\mathbf{3 5}$ |  |  | $\mathbf{4 4}$ |

$\mathrm{A}=94.5$
The mean yield per plot is
Direct method:

$$
\begin{aligned}
\bar{x}=\frac{\sum f x}{n} & =\frac{(74.5 \times 3)+(94.5 \times 5)+(114.5 \times 7)+(134.5 \times 20)}{35} \\
& =\frac{4187.5}{35}=119.64 \mathrm{gms}
\end{aligned}
$$

## Shortcut method

$\bar{x}=A+\frac{\sum f d}{n} x c$

$$
\bar{x}=94.5+\frac{44}{35} \times 20=119.64 \mathrm{~g}
$$

## Merits and demerits of Arithmetic mean

## Merits

1. It is rigidly defined.
2. It is easy to understand and easy to calculate.
3. If the number of items is sufficiently large, it is more accurate and more reliable.
4. It is a calculated value and is not based on its position in the series.
5. It is possible to calculate even if some of the details of the data are lacking.
6. Of all averages, it is affected least by fluctuations of sampling.
7. It provides a good basis for comparison.

## Demerits

1. It cannot be obtained by inspection nor located through a frequency graph.
2. It cannot be in the study of qualitative phenomena not capable of numerical measurement i.e. Intelligence, beauty, honesty etc.,
3. It can ignore any single item only at the risk of losing its accuracy.
4. It is affected very much by extreme values.
5. It cannot be calculated for open-end classes.
6. It may lead to fallacious conclusions, if the details of the data from which it is computed are not given.

## Median

The median is the middle most item that divides the group into two equal parts, one part comprising all values greater, and the other, all values less than that item.

## Ungrouped or Raw data

Arrange the given values in the ascending order. If the number of values are odd, median is the middle value
If the number of values are even, median is the mean of middle two values.
By formula
When n is odd, Median $=\mathrm{Md}=\left(\frac{n+1}{2}\right)^{\text {th }}$ value

When n is even, Average of $\left(\frac{n}{2}\right)$ and $\left(\frac{n}{2}+1\right)^{\text {th }}$ value

## Example 4

If the weights of sorghum ear heads are $45,60,48,100,65 \mathrm{gms}$, calculate the median

## Solution

Here $\mathrm{n}=5$
First arrange it in ascending order
$45,48,60,65,100$
Median $=\left(\frac{n+1}{2}\right)^{\text {th }}$ value

$$
=\left(\frac{5+1}{2}\right)=3^{\text {rd }} \quad \text { value }=60
$$

## Example 5

If the sorghum ear- heads are $5,48,60,65,65,100$ gms, calculate the median.

## Solution

Here $\mathrm{n}=6$
Meadian $=$ Average of $\left(\frac{n}{2}\right)$ and $\left(\frac{n}{2}+1\right)^{\text {th }}$ value
$\left(\frac{n}{2}\right)=\frac{6}{2}=3^{\text {rd }}$ value $=60$ and $\left(\frac{n}{2}+1\right)=\frac{6}{2}+1=4^{\text {th }}$ value $=65$
Median $=\frac{60+65}{2}=62.5 \mathrm{~g}$

## Grouped data

In a grouped distribution, values are associated with frequencies. Grouping can be in the form of a discrete frequency distribution or a continuous frequency distribution. Whatever may be the type of distribution, cumulative frequencies have to be calculated to know the total number of items.

## Cumulative frequency (cf)

Cumulative frequency of each class is the sum of the frequency of the class and the frequencies of the pervious classes, ie adding the frequencies successively, so that the last cumulative frequency gives the total number of items.

## Discrete Series

Step1: Find cumulative frequencies.
Step2: Find $\left(\frac{n}{2}+1\right)$
Step3: See in the cumulative frequencies the value just greater than $\left(\frac{n}{2}+1\right)$
Step4: Then the corresponding value of x is median.

## Example 6

The following data pertaining to the number of insects per plant. Find median number of insects per plant.

| Number of insects per plant (x) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| No. of plants(f) | 1 | 3 | 5 | 6 | 10 | 13 | 9 | 5 | 3 | 2 | 2 | 1 |

Solution
Form the cumulative frequency table

| $\mathbf{x}$ | $\mathbf{f}$ | $\mathbf{c f}$ |
| :---: | :---: | :---: |
| 1 | 1 | 1 |
| 2 | 3 | 4 |
| 3 | 5 | 9 |
| 4 | 6 | 15 |
| 5 | 10 | 25 |
| 6 | 13 | 38 |
| 7 | 9 | 47 |
| 8 | 5 | 52 |
| 9 | 3 | 55 |
| 10 | 2 | 57 |
| 11 | 2 | 59 |
| 12 | 1 | 60 |
|  | 60 |  |

Median $=$ size of $\left(\frac{n+1}{2}\right)^{\text {th }}$ item
Here the number of observations is even. Therefore median = average of $(\mathrm{n} / 2)$ th item and ( $\mathrm{n} / 2+1$ )th item.

$$
=\left(30^{\text {th }} \text { item }+31^{\text {st }} \text { item }\right) / 2=(6+6) / 2=6
$$

Hence the median size is 6 insects per plant.

## Continuous Series

The steps given below are followed for the calculation of median in continuous series.
Step1: Find cumulative frequencies.
Step2: Find $\left(\frac{n}{2}\right)$
Step3: See in the cumulative frequency the value first greater than $\left(\frac{n}{2}\right)$, Then the corresponding class interval is called the Median class. Then apply the formula

Median $=l+\frac{\frac{n}{2}-m}{f} \times c$
where $\quad l=$ Lower limit of the medianal class
$\mathrm{m}=$ cumulative frequency preceding the medianal class
$\mathrm{c}=$ width of the class
$\mathrm{f}=$ frequency in the median class.
$n=$ Total frequency.

## Example 7

For the frequency distribution of weights of sorghum ear-heads given in table below. Calculate the median.

| Weights of ear <br> heads (in g) | No of ear <br> heads (f) | Less than <br> class | Cumulative <br> frequency (m) |
| :---: | :---: | :---: | :---: |
| $60-80$ | 22 | $<80$ | 22 |
| $80-100$ | 38 | $<100$ | 60 |
| $100-120$ | 45 | $<120$ | 105 |
| $120-140$ | 35 | $<140$ | 140 |
| $140-160$ | 24 | $<160$ | 164 |
| Total | 164 |  |  |

## Solution

Median $=l+\frac{\frac{n}{2}-m}{f} \times c$
$\left(\frac{n}{2}\right)=\left(\frac{164}{2}\right)=82$
It lies between 60 and 105. Corresponding to 60 the less than class is 100 and corresponding to 105 the less than class is 120 . Therefore the medianal class is $100-120$. Its lower limit is 100 .

Here $l=100, \mathrm{n}=164, \mathrm{f}=45, \mathrm{c}=20, \mathrm{~m}=60$
Median $=100+\frac{82-60}{45} \times 20=109.78 \mathrm{gms}$

## Merits of Median

1. Median is not influenced by extreme values because it is a positional average.
2. Median can be calculated in case of distribution with open-end intervals.
3. Median can be located even if the data are incomplete.

## Demerits of Median

1. A slight change in the series may bring drastic change in median value.
2. In case of even number of items or continuous series, median is an estimated value other than any value in the series.
3. It is not suitable for further mathematical treatment except its use in calculating mean deviation.
4. It does not take into account all the observations.

## Mode

The mode refers to that value in a distribution, which occur most frequently. It is an actual value, which has the highest concentration of items in and around it. It shows the centre of concentration of the frequency in around a given value. Therefore, where the purpose is to know the point of the highest concentration it is preferred. It is, thus, a positional measure.

Its importance is very great in agriculture like to find typical height of a crop variety, maximum source of irrigation in a region, maximum disease prone paddy variety. Thus the mode is an important measure in case of qualitative data.

## Computation of the mode

## Ungrouped or Raw Data

For ungrouped data or a series of individual observations, mode is often found by mere inspection.

## Example 8

Find the mode for the following seed weight
$2,7,10,15,10,17,8,10,2$ gms
$\therefore$ Mode $=10$
In some cases the mode may be absent while in some cases there may be more than one mode.

## Example 9

(1) $12,10,15,24,30$ (no mode)
(2) $7,10,15,12,7,14,24,10,7,20,10$
the modal values are 7 and 10 as both occur 3 times each.

## Grouped Data

For Discrete distribution, see the highest frequency and corresponding value of $x$ is mode.
Example:
Find the mode for the following

| Weight of sorghum in <br> gms (x) | No. of ear head(f) |
| :---: | :---: |
| 50 | 4 |
| 65 | 6 |
| 75 | 16 |
| 80 | 8 |
| 95 | 7 |
| 100 | 4 |

## Solution

The maximum frequency is 16 . The corresponding $x$ value is 75 .
$\therefore$ mode $=75$ gms.

## Continuous distribution

Locate the highest frequency the class corresponding to that frequency is called the modal class.
Then apply the formula.

Mode $=l+\frac{f_{s}}{f_{p}+f_{s}} \times c$
Where $l=$ lower limit of the model class
$f_{p}=$ the frequency of the class preceding the model class
$f_{s}=$ the frequency of the class succeeding the model class
and $\mathrm{c}=$ class interval

## Example 10

For the frequency distribution of weights of sorghum ear-heads given in table below. Calculate the mode

| Weights of ear <br> heads (g) | No of ear <br> heads (f) |  |
| :---: | :---: | :---: |
| $60-80$ | 22 |  |
| $80-100$ | 38 | $f_{p}$ |
| $100-120$ | 45 | f |
| $120-140$ | 35 | $f_{s}$ |
| $140-160$ | 20 |  |
| Total | $\mathbf{1 6 0}$ |  |

## Solution

Mode $=l+\frac{f_{s}}{f_{p}+f_{s}} \times c$
Here $l=100, \mathrm{f}=45, \mathrm{c}=20, \mathrm{~m}=60, f_{p}=38, f_{s}=35$
Mode $=100+\frac{35_{s}}{38+35} \times 20$
$=100+\frac{35_{s}}{73} \times 20$
$=109.589$

## Geometric mean

The geometric mean of a series containing $n$ observations is the nth root of the product of the values.

If $\mathrm{x} 1, \mathrm{x} 2 \ldots, \mathrm{xn}$ are observations then
$\mathrm{G} . \mathrm{M}=\sqrt[n]{x_{1}, x_{2} \ldots x_{n}}$

$$
=\left(x_{1}, x_{2} \ldots x_{n}\right)^{1 / n}
$$

$\log \mathrm{GM}=\frac{1}{n} \log \left(x_{1}, x_{2} \ldots x_{n}\right)$

$$
\begin{aligned}
& =\frac{1}{n}\left(\log x_{1}+\log x_{2}+\ldots+\log x_{n}\right) \\
& =\frac{\sum \log x_{i}}{n} \\
& \text { GM }=\text { Antilog } \frac{\sum \log x_{i}}{n}
\end{aligned}
$$

For grouped data
$\mathrm{GM}=\operatorname{Antilog}\left[\frac{\sum f \log x_{i}}{n}\right]$
GM is used in studies like bacterial growth, cell division, etc.

## Example 11

If the weights of sorghum ear heads are $45,60,48,100,65 \mathrm{gms}$. Find the Geometric mean for the following data

| Weight of ear <br> head $\mathbf{x}(\mathbf{g})$ | $\log \mathbf{x}$ |
| :--- | :--- |
| 45 | 1.653 |
| 60 | 1.778 |
| 48 | 1.681 |
| 100 | 2.000 |
| 65 | 1.813 |
| Total | $\mathbf{8 . 9 2 5}$ |

## Solution

Here $\mathrm{n}=5$

$$
\begin{aligned}
\mathrm{GM} & =\text { Antilog } \frac{\sum \log x_{i}}{n} \\
& =\text { Antilog } \frac{8.925}{5} \\
& =\text { Antilog } 1.785
\end{aligned}
$$

$$
=60.95
$$

## Grouped Data

## Example 12

Find the Geometric mean for the following

| Weight of sorghum (x) | No. of ear head(f) |
| :---: | :---: |
| 50 | 4 |
| 65 | 6 |
| 75 | 16 |
| 80 | 8 |
| 95 | 7 |
| 100 | 4 |

## Solution

| Weight of <br> sorghum (x) | No. of ear <br> head(f) | $\log \mathrm{x}$ | $\mathrm{fx} \log \mathrm{x}$ |
| :---: | :---: | :---: | :---: |
| 50 | 5 | 1.699 | 8.495 |
| 63 | 10 | 10.799 | 17.99 |
| 65 | 5 | 1.813 | 9.065 |
| 130 | 15 | 2.114 | 31.71 |
| 135 | 15 | 2.130 | 31.95 |
| Total | $\mathbf{5 0}$ | $\mathbf{9 . 5 5 5}$ | $\mathbf{9 9 . 2 1}$ |

Here $\mathrm{n}=50$
$\mathrm{GM}=$ Antilog $\left[\frac{\sum f \log x_{i}}{n}\right]$
$=$ Antilog $\left[\frac{99.21}{50}\right]$
$=$ Antilog $1.9842=96.43$

## Continuous distribution

## Example 13

For the frequency distribution of weights of sorghum ear-heads given in table below.
Calculate the Geometric mean

| Weights of ear <br> heads (in $\mathbf{g}$ ) | No of ear <br> heads (f) |
| :---: | :---: |
| $60-80$ | 22 |
| $80-100$ | 38 |
| $100-120$ | 45 |


| $120-140$ | 35 |
| :---: | :---: |
| $140-160$ | 20 |
| Total | $\mathbf{1 6 0}$ |

## Solution

| Weights of ear <br> heads (in g) | No of ear <br> heads (f) | Mid x | $\log \mathbf{x}$ | f $\log \mathbf{x}$ |
| :---: | :---: | :---: | :---: | :---: |
| $60-80$ | 22 | 70 | 1.845 | 40 |
| $80-100$ | 38 | 90 | 1.954 | 74.25 |
| $100-120$ | 45 | 110 | 2.041 | 91.85 |
| $120-140$ | 35 | 130 | 2.114 | 73.99 |
| $140-160$ | 20 | 150 | 2.176 | 43.52 |
| Total | $\mathbf{1 6 0}$ |  |  | $\mathbf{3 2 4 . 2}$ |

Here $n=160$

$$
\begin{aligned}
\text { GM } & =\text { Antilog }\left[\frac{\sum f \log x_{i}}{n}\right] \\
& =\text { Antilog }\left[\frac{324.2}{160}\right] \\
& =\text { Antilog }[2.02625] \\
& =106.23
\end{aligned}
$$

## Harmonic mean (H.M)

Harmonic mean of a set of observations is defined as the reciprocal of the arithmetic average of the reciprocal of the given values. If $\mathrm{x} 1, \mathrm{x} 2 \ldots . . \mathrm{xn}$ are n observations,

$$
\mathrm{H} . \mathrm{M}=\frac{n}{\sum_{i=n}^{n}\left(\frac{1}{x_{i}}\right)}
$$

For a frequency distribution

$$
\mathrm{H} . \mathrm{M}=\frac{n}{\sum_{i=n}^{n} f\left(\frac{1}{x_{i}}\right)}
$$

H.M is used when we are dealing with speed, rates, etc.

## Example 13

From the given data 5, 10, 17,24,30 calculate H.M.

| X | $\frac{1}{x}$ |
| :---: | :---: |
| 5 | 0.2000 |
| 10 | 0.1000 |
| 17 | 0.0588 |
| 24 | 0.0417 |
| 30 | 0.4338 |

$H . M=\frac{5}{0.4338}=11.526$

## Example 14

Number of tomatoes per plant are given below. Calculate the harmonic mean.

| Number of tomatoes per plant | 20 | 21 | 22 | 23 | 24 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Number of plants | 4 | 2 | 7 | 1 | 3 | 1 |

## Solution

| Number of <br> tomatoes per <br> plant (x) | No of <br> plants(f) | $\frac{1}{x}$ | $f\left(\frac{1}{x}\right)$ |
| :---: | :---: | :---: | :---: |
| 20 | 4 | 0.0500 | 0.2000 |
| 21 | 2 | 0.0476 | 0.0952 |
| 22 | 7 | 0.0454 | 0.3178 |
| 23 | 1 | 0.0435 | 0.0435 |
| 24 | 3 | 0.0417 | 0.1251 |
| 25 | 1 | 0.0400 | 0.0400 |
|  | 18 |  | 0.8216 |

H.M $=\frac{n}{\sum f\left(\frac{1}{x_{i}}\right)}=\frac{18}{0.1968}=21.91$

## Merits of H.M

1. It is rigidly defined.
2. It is defined on all observations.
3. It is amenable to further algebraic treatment.
4. It is the most suitable average when it is desired to give greater weight to smaller observations and less weight to the larger ones.

## Demerits of H.M

1. It is not easily understood.
2. It is difficult to compute.
3. It is only a summary figure and may not be the actual item in the series
4. It gives greater importance to small items and is therefore, useful only when small items have to be given greater weightage.
5. It is rarely used in grouped data.

## Percentiles

The percentile values divide the distribution into 100 parts each containing 1 percent of the cases. The $\mathrm{x}^{\text {th }}$ percentile is that value below which x percent of values in the distribution fall. It may be noted that the median is the $50^{\text {th }}$ percentile.

For raw data, first arrange the n observations in increasing order. Then the $\mathrm{x}^{\text {th }}$ percentile is given by

$$
P_{x}=\left(\frac{x(n+1)}{100}\right)^{t h} \text { item }
$$

For a frequency distribution the $\mathrm{x}^{\text {th }}$ percentile is given by

$$
P_{x}=l+\left(\frac{(x . n / 100)-c f}{f} \times C\right)
$$

Where
$l=$ lower limit of the percentile calss which contains the $\mathrm{x}^{\text {th }}$ percentile value (x. $\mathrm{n} / 100$ )
$c f=$ cumulative frequency uotp $l$
$f=$ frequency of the percentile class
$\mathrm{C}=$ class interval
$\mathrm{N}=$ total number of observations

## Percentile for Raw Data or Ungrouped Data

## Example 15

The following are the paddy yields ( $\mathrm{kg} / \mathrm{plot}$ ) from 14 plots:
$30,32,35,38,40.42,48,49,52,55,58,60,62$, and 65 ( after arranging in ascending order). The computation of $25^{\text {th }}$ percentile $\left(\mathrm{Q}_{1}\right)$ and $75^{\text {th }}$ percentile $\left(\mathrm{Q}_{3}\right)$ are given below:

$$
\begin{aligned}
& P_{25}\left(\text { or } Q_{1}\right)=\left(\frac{25(14+1)}{100}\right)^{\text {th }} \text { item } \\
& =\left(3 \frac{3}{4}\right)^{\text {th }} \text { item } \\
& \quad=3^{\text {rd }} \text { item }+\left(4^{\text {th }} \text { item }-3^{\text {rd }} \text { item }\right)\left(\frac{3}{4}\right) \\
& \quad=35+(38-35)\left(\frac{3}{4}\right) \\
& \quad=35+3\left(\frac{3}{4}\right)=37.25 \mathrm{~kg} \\
& \begin{aligned}
& P_{75}(\text { or }\left.Q_{3}\right)=\left(\frac{75(14+1)}{100}\right)^{\text {th }} \text { item } \\
&=\left(11 \frac{1}{4}\right)^{\text {th }} \text { item } \\
& \quad=11^{\text {th }} \text { item }+\left(12^{\text {th }} \text { item }-11^{\text {th }} \text { item }\right)\left(\frac{1}{4}\right) \\
& \quad=55+(58-55)\left(\frac{1}{4}\right) \\
& \quad=55+3\left(\frac{1}{4}\right)=55.75 \mathrm{~kg}
\end{aligned}
\end{aligned}
$$

## Example 16

The frequency distribution of weights of 190 sorghum ear-heads are given below. Compute $25^{\text {th }}$ percentile and $75^{\text {th }}$ percentile.

| Weight of ear- <br> heads (in g) | No of ear <br> heads |
| :---: | :---: |
| $40-60$ | 6 |
| $60-80$ | 28 |
| $80-100$ | 35 |
| $100-120$ | 55 |
| $120-140$ | 30 |
| $140-160$ | 15 |
| $160-180$ | 12 |
| $180-200$ | 9 |
| Total | $\mathbf{1 9 0}$ |

## Solution

| Weight of ear- <br> heads (in g) | No of ear heads | Less than class | Cumulative <br> frequency |
| :---: | :---: | :---: | :---: |
| $40-60$ | 6 | $<60$ | 6 |
| $60-80$ | 28 | $<80$ | 34 |
| $80-100$ | 35 | $<100$ | 69 |
| $100-120$ | 55 | $<120$ | 124 |
| $120-140$ | 30 | $<140$ | 154 |
| $140-160$ | 15 | $<160$ | 169 |
| $160-180$ | 12 | $<180$ | 181 |
| $180-200$ | 9 | $<200$ | 190 |
| Total | $\mathbf{1 9 0}$ |  | 142.5 |

or $\mathrm{P}_{25}$, first find out $\left(\frac{25(190)}{100}\right)$, and for $P_{75},\left(\frac{75(190)}{100}\right)$, and proceed as in the case of median.
For $\mathrm{P}_{25}$, we have $\left(\frac{25(190)}{100}\right)=47.5$.
The value 47.5 lies between 34 and 69 . Therefore, the percentile class is $80-100$. Hence,

$$
\begin{aligned}
P_{25}=Q_{1} & =l+\left(\frac{(25 . n / 100)-c f}{f} \times C\right) \\
& =80+\left(\frac{(47.5)-34}{35} \times 20\right) \\
& =80+\left(\frac{(13.5)}{35} \times 20\right) \\
& =80+7.71 \text { or } 87.71 \mathrm{~g} .
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
P_{75}=l+ & \left(\frac{(75 . n / 100)-c f}{f} \times C\right) \text { Class } \\
& =120+\left(\frac{(142.5)-121}{30} \times 20\right) \\
& =120+\left(\frac{(21.5)}{30} \times 20\right) \\
& =120+14.33=134.33 \mathrm{~g} .
\end{aligned}
$$

## Quartiles

The quartiles divide the distribution in four parts. There are three quartiles. The second quartile divides the distribution into two halves and therefore is the same as the median. The first (lower).quartile (Q1) marks off the first one-fourth, the third (upper) quartile (Q3) marks off the three-fourth. It may be noted that the second quartile is the value of the median and $50^{\text {th }}$ percentile.

## Raw or ungrouped data

First arrange the given data in the increasing order and use the formula for Q1 and Q3 then quartile deviation, Q.D is given by

$$
Q \cdot D=\frac{Q_{3}-Q_{1}}{2}
$$

Where $Q_{1} .=\left(\frac{n+1}{4}\right)^{\text {th }}$ item and $Q_{3} .=3\left(\frac{n+1}{4}\right)^{\text {th }}$ item

## Example 18

Compute quartiles for the data given below (grains/panicles) $25,18,30,8,15,5,10,35,40,45$

## Solution

$5,8,10,15,18,25,30,35,40,45$

$$
\begin{aligned}
Q_{1} & =\left(\frac{n+1}{4}\right)^{\text {th }} \\
& =\left(\frac{10+1}{4}\right)^{\text {th }} \\
& =(2.75)^{\text {th }} \text { item } \\
& =2^{\text {nd }} \text { item }+\left(\frac{3}{4}\right)\left(3^{\text {rd }} \text { item }-2^{\text {nd }} \text { item }\right) \\
& =8+\frac{3}{4}(10-8) \\
& =8+\frac{3}{4} \times 2
\end{aligned}
$$

$$
\begin{aligned}
& =8+1.5 \\
& =9.5
\end{aligned}
$$

$$
\begin{aligned}
Q_{3} . & =3\left(\frac{n+1}{4}\right)^{\text {th }} \\
& =3 \times(2.75)^{\text {th }} \text { item } \\
& =(8.75)^{\text {th }} \text { item } \\
= & 8^{\text {th }} \text { item }+\left(\frac{1}{4}\right)\left(9^{\text {th }} \text { item }-8^{\text {th }} \text { item }\right) \\
= & 35+\frac{1}{4}(40-35) \\
& =35+1.25 \\
= & 36.25
\end{aligned}
$$

## Discrete Series

Step1: Find cumulative frequencies.
Step2: Find $\left(\frac{n+1}{4}\right)$
Step3: See in the cumulative frequencies, the value just greater than $\left(\frac{n+1}{4}\right)$, then the corresponding value of $x$ is Q 1
Step4: Find $3\left(\frac{n+1}{4}\right)$
Step5: See in the cumulative frequencies, the value just greater than $3\left(\frac{n+1}{4}\right)$, then the corresponding value of $x$ is Q3

## Example 19

Compute quartiles for the data given bellow (insects/plant).

| X | 5 | 8 | 12 | 15 | 19 | 24 | 30 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| f | 4 | 3 | 2 | 4 | 5 | 2 | 4 |

## Solution

| $\mathbf{x}$ | $\mathbf{f}$ | $\mathbf{c f}$ |
| :---: | :---: | :---: |
| 5 | 4 | 4 |
| 8 | 3 | 7 |
| 12 | 2 | 9 |
| 15 | 4 | 13 |
| 19 | 5 | 18 |
| 24 | 2 | 20 |

$Q_{1}=\left(\frac{n+1}{4}\right)^{\text {th }}$ item $=\left(\frac{24+1}{4}\right)=\left(\frac{25}{4}\right)=6.25^{\text {th }}$ item
$Q_{3}=3\left(\frac{n+1}{4}\right)^{\text {th }}$ item $=3\left(\frac{24+1}{4}\right)=18.75$ th item $\therefore \mathrm{Q} 1=8 ; \mathrm{Q} 3=24$

## Continuous series

Step1: Find cumulative frequencies
Step2: Find $\left(\frac{n}{4}\right)$
Step3: See in the cumulative frequencies, the value just greater than $\left(\frac{n}{4}\right)$, then the corresponding class interval is called first quartile class.
Step4: Find $3\left(\frac{n}{4}\right)$ See in the cumulative frequencies the value just greater than $3\left(\frac{n}{4}\right)$ then the corresponding class interval is called $3^{\text {rd }}$ quartile class. Then apply the respective formulae
$Q_{1}=l_{1}+\frac{\frac{n}{4}-m_{1}}{f_{1}} \times c_{1}$
$Q_{3}=l_{3}+\frac{3\left(\frac{n}{4}\right)-m_{3}}{f_{3}} \times c_{3}$
Where $l_{1}=$ lower limit of the first quartile class
$f_{1}=$ frequency of the first quartile class
$c_{1}=$ width of the first quartile class
$m_{1}=$ c.f. preceding the first quartile class
$l_{3}=$ lower limit of the $3^{\text {rd }}$ quartile class
$f_{3}=$ frequency of the $3^{\text {rd }}$ quartile class
$c_{3}=$ width of the $3^{\text {rd }}$ quartile class
$m_{3}=$ c.f. preceding the $3^{\text {rd }}$ quartile class

Example 20: The following series relates to the marks secured by students in an examination.

| Marks | No. of Students |
| :---: | :---: |
| $0-10$ | 11 |
| $10-20$ | 18 |
| $20-30$ | 25 |
| $30-40$ | 28 |
| $40-50$ | 30 |
| $50-60$ | 33 |
| $60-70$ | 22 |
| $70-80$ | 15 |
| $80-90$ | 12 |
| $90-100$ | 10 |

Find the quartiles

## Solution

| C.I | f | cf |
| :---: | :---: | :---: |
| $0-10$ | 11 | 11 |
| $10-20$ | 18 | 29 |
| $20-30$ | 25 | 54 |
| $30-40$ | 28 | 82 |
| $40-50$ | 30 | 112 |
| $50-60$ | 33 | 145 |
| $60-70$ | 22 | 167 |
| $70-80$ | 15 | 182 |
| $80-90$ | 12 | 194 |
| $90-100$ | 10 | 204 |
|  | 204 |  |

$$
\left(\frac{n}{4}\right)=\left(\frac{204}{4}\right)=51
$$

$$
\begin{aligned}
3\left(\frac{n}{4}\right) & =153 \\
Q_{1} & =l_{1}+\frac{\frac{n}{4}-m_{1}}{f_{1}} \times c_{1} \\
& =20+\frac{51-29_{1}}{25_{1}} \times 10=20+8.8=28.8 \\
Q_{3} & =l_{3}+\frac{3\left(\frac{n}{4}\right)-m_{3}}{f_{3}} \times c_{3} \\
& =60+\frac{153-145}{22} \times 12=60+4.36=64.36
\end{aligned}
$$

## Questions

1. The middle value of an ordered series is called
a)2nd quartile
b) 5th decile
c) 50th percentile
d) all the above

## Ans: all the above

2. For a set of values the model value can be
a) Unimodel
b) bimodal
c) Trimodel
d) All of these
d) Ans: all the above
3. Mode is suitable for qualitative data.

## Ans: True

4. Decile divides the group in to ten equal parts.

Ans: True
5. Mean is affected by extreme values.

Ans: True
6. Geometric mean can be calculated for negative values.

Ans: False
7. Define mean and median
8. For what type of data mode can be calculated.
9. Explain how to calculate the arithmetic mean for raw and grouped data.
10. Explain how to calculate median and mode for grouped data.

