# 3rd lecture

Sample space of tossing 1 Coin

$S = \{H, T\}$

for 2 Coin.

| | H | T |
|---|---|---|
| H | HH | HT |
| T | TH | TT |

$S = \{HH, HT, TH, TT\}$

Sample Space for 1 die :-

$S = \{1, 2, 3, 4, 5, 6\}$

2 fair dices

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1,6) |
| 2 | (2,1) | (2,2) | (2,3) | (2,4) | (2,5) | (2,6) |
| 3 | (3,1) | (3,2) | (3,3) | (3,4) | (3,5) | (3,6) |
| 4 | (4,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4,6) |
| 5 | (5,1) | (5,2) | (5,3) | (5,4) | (5,5) | (5,6) |
| 6 | (6,1) | (6,2) | (6,3) | (6,4) | (6,5) | (6,6) |

Sum of dots:

for two dices:

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

Question no: 1

If a die is thrown, what is the probability of getting

i) a Six
ii) a four
iii) two and
iv) five

Solution:

Single die so

$S = \{1, 2, 3, 4, 5, 6\} = n(S) = 6$

i) a Six

Let A = getting Six so

$A = \{6\} = n(A) = 1$

$P(A) = \frac{n(A)}{n(S)}$

$P(A) = \frac{n(A)}{n(S)} = 1/6$

$P(A) = \frac{1}{2}$

Example:-

Two uniform coins are tossed. Find the following probabilities

i) both are heads
ii) both faces are same
iii) Only One is head
iv) at least one is head
v) head on the first coin.

Solution:-

When two coins are tossed,
Sample space

$S = \{(HH), (TH), (HT), (TT)\}$
$n(S) = 4$

i) Let A denot the event that both the coins give head
then $A = \{(HH)\}$ $n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$$

ii) Let B be the event that both faces are same

$B = \{(HH), (TT)\}$ $= n(B) = 2$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

iii) Let C denote the event the only one is head

$C = \{(HT), (TH)\}$. $n(C) = 2$

$$P(C) = \frac{n(C)}{n(S)} = \frac{2}{4} = \frac{1}{2}.$$

iv) let D denote the event that at least one is head. it means any number of head

$$D = \{(HH), (HT), (TH)\}$$
$$= n(D) = 3$$
$$P(D) = \frac{n(D)}{n(S)} = \frac{3}{4}$$

v) head on the first coin.

# do it yourself. 1/4

#2 A pair of dice are thrown. Find the probability of getting

Lec # 5

a) a double six
b) a sum of 8
c) a sum of 5
d) a sum of 3.

Solution:-

The sample space

$$n(S) = 36$$

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$$
$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)$$
$$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$$
$$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$$
$$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$$
$$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

a) Let A represent the event that a double six appear. So

$$A = \{(6,6)\} = n(A) = 1$$

$$n(S) = 36$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{36}$$

b) Let B represent the event that a sum of 8.

$$B = \{(6,2), (5,3), (4,4), (3,5), (2,6)\}$$

$$n(B) = 5$$

$$n(S) = 36$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$$

c) Let c represent the event Sum of 5

$$C = \{(1,4), (2,3), (3,2), (4,1)\}$$

$$= n(C) = 4$$

$$n(S) = 36$$

$$P(C) = 4/36$$

d) D = Sum of three.

# do it yourself $\frac{2}{36} = \frac{1}{18}$

# n(d) =

Q#2 A man tosses two fair dice. what is the prob of

a) Sum is greater than 6

b) the two dice had the same outcome.

(8)

Solution.

a) the sum is greater than 6

Let A be the event that sum is greater than 6

$A = \{(1,6), (2,5), (2,6), (3,4), (3,5), (3,6), (4,3), (4,4), (4,5), (4,6), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$n(A) = 21$

$n(S) = 36$

$P(A) = \frac{21}{36}$

b) two dice had same out come

$B = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$n(B) = 6$

$n(S) = 36$

$P(B) = \frac{6}{36} = \frac{1}{6}$

## Laws of Addition

i) Addition law for mutually exclusive events

ii) Addition law for not mutually Exclusive Events

i) Mutually Exclusive Events

$$P(A \cup B) = P(A) + P(B)$$

~~$P(A \cup B) =$~~

ii) Not mutually Exclusive

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Independent Events — 0-32

If $P(A \cap B) = P(A) \cdot P(B)$

then Events are called independent ~~Events~~. Events.

A die is rolled. Find the probability that the face is a prime number or is even

Solution:

When die is tossed there are 6 possible outcomes

$S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

Let A represent the event "face is prime"

i) $A = \{2, 3, 5\} = P(A) = 3/6$

Let B represent the event face is even

$B = \{2, 4, 6\} = P(B) = 3/6$

$A \cap B = \{2\} = P(A \cap B) = 1/6$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = 5/6$

ii) Four



B = face is 4

$B = \{4\} = n(B) = 1$

$n(S) = 6$

$P(B) = \frac{n(B)}{n(S)} = 1/6$

(iii)

C = event is 4 / face is 4 2

~~B =~~

$C = \{2\} = n(C) = 1$

$n(S) = 6$ so

$P(C) = \frac{n(C)}{n(S)} = \frac{1}{6}$

iv) face is number 5.

# do it yourself

Q# 2.

A fair coin is tossed. what is ~~that~~ the probability that

i) head appear

ii) tail appear.

Solution.

$S = \{H, T\}$ ⇒ Sample Space

i) head appear

$S = \{H, T\}$ $\quad n(S) = 2$

(10)

Let A = head appear

$A = \{H\} \quad = n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$$

ii) tail

B = face is tail

$B = \{T\}$

$n(B) = 1, \quad n(S) = 2.$

So

$$P(B) = \frac{1}{2}.$$

Question no: 3 (assignment)

If a die is rolled. What is the prob of getting

i) Even numbers

ii) Odd numbers

Questio

A die is rolled. Find the following Probabilities

i) The face is multiple of 3 or multiple of 5

Solution

$S = \{1, 2, 3, 4, 5, 6\}$

Let A represent the event the face is multiple of 3

$A = \{3, 6\} = P(A) = \frac{2}{6}$

Let B represent the event that face is multiple of 5

$B = \{5\} = P(B) = \frac{1}{6}$

$P(A \cup B) = P(A \text{ or } B) = P(A) + P(B)$

$= \frac{2}{6} + \frac{1}{6}$

$= \frac{3}{6} = \frac{1}{2}$

Two dice are cast $E_1$ is the event that a 6 appears on at least one die $E_2$ is the event that the same number appear on both dice

i) Are $E_1$ and $E_2$ independent?

Solution.

$$E_1 = \{(1,6),(2,6),(3,6),(4,6),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$$E_2 = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$$

$P(A \cap B) = P(A)\,P(B)$ then events are independent.

So

$P(E_1) = 11/36$

$P(E_2) = \frac{6}{36}$

~~AA~~ $E_1 \cap E_2 = \{(6,6)\}$

$P(E_1 \cap E_2) = 1/36$

$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$

$\frac{1}{36} = \frac{(11)}{36} \cdot \frac{(6)}{36}$

$\frac{1}{36} = \frac{66}{1296}$

$P(E_1 \cap E_2) \neq P(E_1) \cdot P(E_2)$ so Events are not independent

# Home Assignment

A digit is selected at random from the first ten natural numbers. Find the probability that the selected digit is

i) greater than 6
ii) a complete square
iii) multiple of three
iv) prime number less than 3