Lab-12

1.1 Develop the elemental Stiffness Matrix of a beam

Truss elements carry only axial forces. Beam elements carry shear forces and bending moments. Frame elements carry shear forces, bending moments, and axial forces. This document presents the development of beam element stiffness matrices in local coordinates.

Divide the beam into finite elements and arbitrarily identify each element and its nodes. Use a number written in a circle for a node and a number written in a square for a member. Usually an element extends between points of support, points of concentrated loads, and joints, or to points where internal loadings or displacements are to be determined. Also, E and I for the elements must be constants

Specify the near and far ends of each element symbolically by directing an arrow along the element, with the head directed toward the far end.

At each nodal point specify numerically the y and z code numbers. In all cases use the lowest code numbers to identify all the unconstrained degrees of freedom, followed by the remaining or highest numbers to identify the degrees of freedom that are constrained.

1.2 Member and Node Identification.

In order to apply the stiffness method to beams, we must first determine how to subdivide the beam into its component finite elements. In general, each element must be free from load and have a prismatic cross section. For this reason the nodes of each element are located at a support or at points where members are connected together, where an external force is applied, where the cross-sectional area suddenly changes, or where the vertical or rotational displacement at a point is to be determined. For example, consider the beam in Figure **Error! No text of specified style in document.**-1. Using the same scheme as that for trusses, four nodes are specified numerically within a circle, and the three elements are identified numerically within a square. Also, notice that the “near” and “far” ends of each element are identified by the arrows written alongside each element.

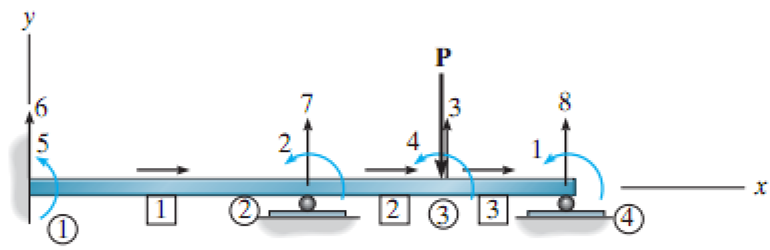

Figure **Error! No text of specified style in document.**-1

1.3 Global and Member Coordinates.

The global coordinate system will be identified using x, y, z axes that generally have their origin

at a node and are positioned so that the nodes at other points on the beam all have positive coordinates, Figure **Error! No text of specified style in document.**-1. The local or member x', y', z' coordinates have their origin at the "near" end of each element, and the positive x' axis is directed towards the "far" end. Figure **Error! No text of specified style in document.**-2 shows these coordinates for element 2.

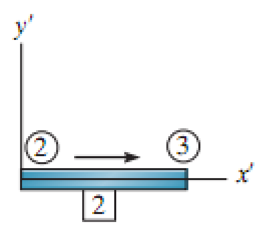

Figure **Error! No text of specified style in document.**-2

In both cases we have used a right-handed coordinate system, so that if the fingers of the right hand are curled from the x(x') axis towards the y(y') axis, the thumb points in the positive direction of the z(z') axis, which is directed out of the page. Notice that for each beam element the x and axes will be collinear, and the global and member coordinates will all be parallel. Therefore, unlike the case for trusses, here we will not need to develop transformation matrices between these coordinate systems.

1.4 Beam Member Stiffness Metrix

In this section we will develop the stiffness matrix for a beam element or member having a constant cross-sectional area and referenced from the local x', y', z' coordinate system, Figure **Error! No text of specified style in document.**-3. The origin of the coordinates is placed at the "near" end N, and the positive x' axis extends toward the "far" end F. There are two reactions at each end of the element, consisting of shear forces $q_{Ny'}$ and q_{Fy} bending moments $q_{Nz'}$ and $q_{Fz'}$ These loadings all act in the positive coordinate directions. In particular, the moments $q_{Nz'}$ and $q_{Fz'}$ are positive counter clockwise, since by the right-hand rule the moment vectors are then directed along the positive z'axis, which is out of the page. Linear and angular displacements associated with these loadings also follow this same positive sign convention. We will now impose each of these displacements separately and then determine the loadings acting

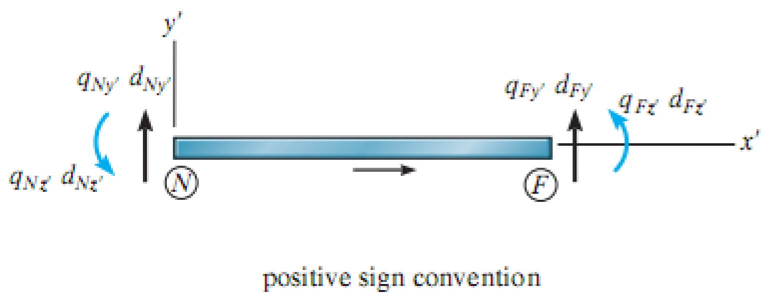

Figure **Error! No text of specified style in document.**-3

1.5 y' Displacements.

When a positive displacement $d_{Ny'}$ is imposed while other possible displacements are prevented, the resulting shear forces and bending moments that are created are shown in Figure **Error! No text of specified style in document.**-4a. In particular, the moment has been developed in Sec. 11–2 as Eq. 11–5 of Structural Analysis by RC Hibbler. Likewise, when $d_{Fy'}$ is imposed, the required shear forces and bending moments are given in Figure **Error! No text of specified style in document.**-4b.

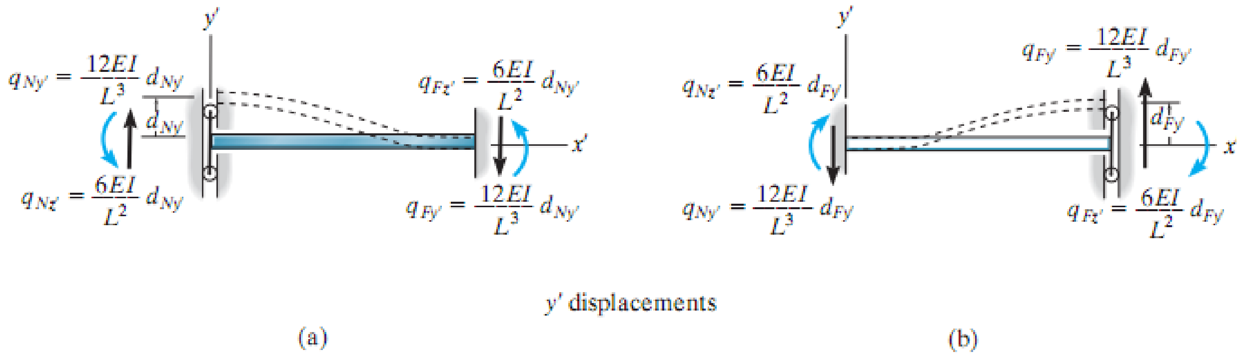

Figure **Error! No text of specified style in document.**-4

1.6 Z' Rotations.

If a positive rotation $d_{NZ'}$ is imposed while all other possible displacements are prevented, the required shear forces and moments necessary for the deformation are shown in Figure **Error! No text of specified style in document.**-5a. In particular, the moment results have been developed in Sec. 11–2 as Eqs. 11–1 and 11–2. Of Structural Analysis by RC Hibbler. Likewise, when $d_{Fz'}$ is imposed, the resultant loadings are shown in Figure **Error! No text of specified style in document.**-5b.

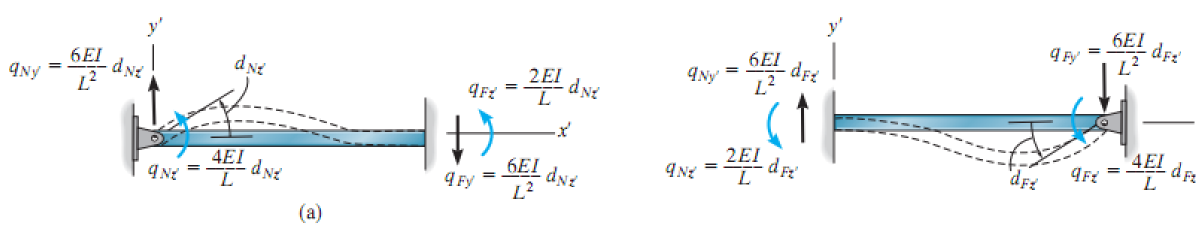

Figure **Error! No text of specified style in document.**-5

By superposition, if the above results in Figure **Error! No text of specified style in document.**-4 and Figure **Error! No text of specified style in document.**-5 are added, the resulting four load-displacement relations for the member can be expressed in matrix form as

$$\begin{bmatrix} q_{Ny'} \\ q_{Nz'} \\ q_{Fy'} \\ q_{Fz'} \end{bmatrix} = EI \begin{bmatrix} \frac{12}{L^3} & \frac{6}{L^2} & -\frac{12}{L^3} & \frac{6}{L^2} \\ \frac{6}{L^2} & \frac{4}{L} & -\frac{6}{L^2} & \frac{2}{L} \\ -\frac{12}{L^3} & -\frac{6}{L^2} & \frac{12}{L^3} & -\frac{6}{L^2} \\ \frac{6}{L^2} & \frac{2}{L} & -\frac{6}{L^2} & \frac{4}{L} \end{bmatrix} \begin{bmatrix} d_{Ny'} \\ d_{Nz'} \\ d_{Fy'} \\ d_{Fz'} \end{bmatrix}$$

These equations can also be written in abbreviated form as

$$q = kD$$

The symmetric matrix k in above Equation. is referred to as the member stiffness matrix. The 16 influence coefficients k_{ij} that comprise it account for the shear-force and bending-moment displacements of the member. Physically these coefficients represent the load on the member when the member undergoes a specified unit displacement.

Now Write a program for Elemental stiffness matrix of the beam

$$L = x_2 - x_1$$

$$K = EI \begin{bmatrix} 12/L^3 & 6/L^2 & -12/L^3 & 6/L^2 \\ 6/L^2 & 4/L & -6/L^2 & 2/L \\ -12/L^3 & -6/L^2 & 12/L^3 & -6/L^2 \\ 6/L^2 & 2/L & -6/L^2 & 4/L \end{bmatrix}$$