Basic Probability Theory

- Basic probability concepts
- Conditional probability
- Discrete Random Variables and Probability Distributions
- Continuous Random Variables and Probability

Probability

- Chance behavior is unpredictable in the short run, but has a regular and predictable pattern in the long run.
- The probability of any out come of a random phenomenom is the proportion of times the outcome would occur in a very long series of repetitions

- Sample Space - the set of all possible outcomes of a random phenomenon
- Event - any set of outcomes of interest
- Probability of an event - the relative frequency of this set of outcomes over an infinite number of trials
- Pr(A) is the probability of event A

Example

Suppose we roll two die and take their sum

$$S = \{2, 3, 4, 5, .., 11, 12\}$$

$$\Pr(sum = 5) = \frac{4}{36}$$

Because we get the sum of two die to be 5 if we roll a (1,4),(2,3),(3,2) or (4,1).

Notation

Let A and B denote two events.

- $A \cup B$ is the event that either A or B or both occur.
- $A \cap B$ is the event that both A and B occur simultaneously.
- The complement of A is denoted by $\overline{A}$.
  - $\overline{A}$ is the event that A does not occur.
  - Note that $\Pr(\overline{A}) = 1 - \Pr(A)$.

Mutually Exclusive and Independent Events

- A and B are mutually exclusive if both cannot occur at the same time.
- A and B are independent events if and only if

$$\Pr(A \cap B) = \Pr(A) \Pr(B).$$

Probability Laws

- Multiplication Law: If $A_1, \cdots, A_k$ are independent events, then

$$\Pr(A_1 \cap A_2 \cap \cdots \cap A_k) = \Pr(A_1)\Pr(A_2)\cdots\Pr(A_k).$$

- Addition Law: If A and B are any events, then

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

Note: This law can be extended to more than 2 events.

Conditional Probability

- The conditional probability of B given A

$$\Pr(B|A) = \frac{\Pr(A \cap B)}{\Pr(A)}$$

- A and B are independent events if and only if

$$\Pr(B|A) = \Pr(B) = \Pr(B|\overline{A})$$

Random Variable

- A random variable is a variable whose value is a numerical outcome of a random phenomenon
- Usually denoted by X, Y or Z.
- Can be
  - Discrete - a random variable that has finite or countable infinite possible values
    - Example: the number of days that it rains yearly
  - Continuous - a random variable that has an (continuous) interval for its set of possible values
    - Example: amount of preparation time for the SAT

Probability Distribution

- The probability distribution for a random variable X gives
  - the possible values for X, and
  - the probabilities associated with each possible value (i.e., the likelihood that the values will occur)
- The methods used to specify discrete prob. distributions are similar to (but slightly different from) those used to specify continuous prob. distributions.