Subject: Electrodynamics II

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5.3 Divergence and Curl of B

5.3.1 Straight Line Currents

At first, we calculate the curl of **B** due to straight line current. As we know magnetic field due to straight line current is like a circular shape around the wire as shown in figure below. For instance, current is coming out of page shown in figure 2.

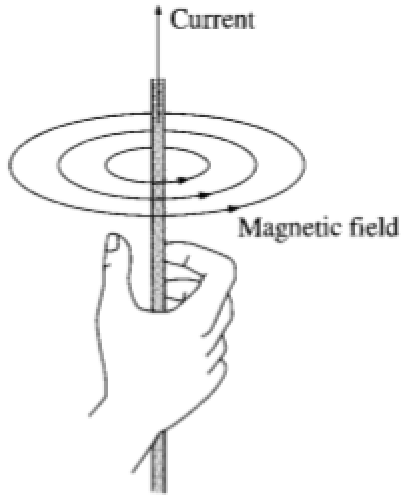


Fig.1

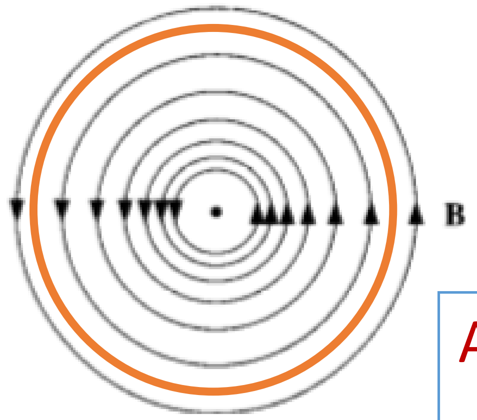


Fig.2

Additional Information:

'.' shows the current flowing out of page

'×' shows current is flowing into the page.

Straight Line Currents

- Wire is perpendicular to page.
- A distance ‘s’ from the wire at any position of page, magnetic field is

$$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\emptyset}$$

Where direction of $\hat{\emptyset}$ is anticlock wise around the wire.

Lets calculate the closed Line Integral of B along the magnetic field in closed circular path as shown in fig.2 by yellow line.

$$\oint \vec{B}.\overrightarrow{dl} = \oint \frac{\mu_0 I}{2\pi s}\hat{\emptyset}\,.\overrightarrow{dl}$$

Straight Line Currents

Wire has cylindrical shape. So using the cylindrical coordinates value of infinitesimal displacement $\overrightarrow{dl}$

$\overrightarrow{dl} = ds\hat{s} + sd\emptyset\widehat{\emptyset} + dz\hat{z}$ put this value in above equation

$$\oint \vec{B}.\overrightarrow{dl} = \oint \frac{\mu_0 I}{2\pi s}\widehat{\emptyset}\,.\,(ds\hat{s} + sd\emptyset\widehat{\emptyset} + dz\hat{z})$$

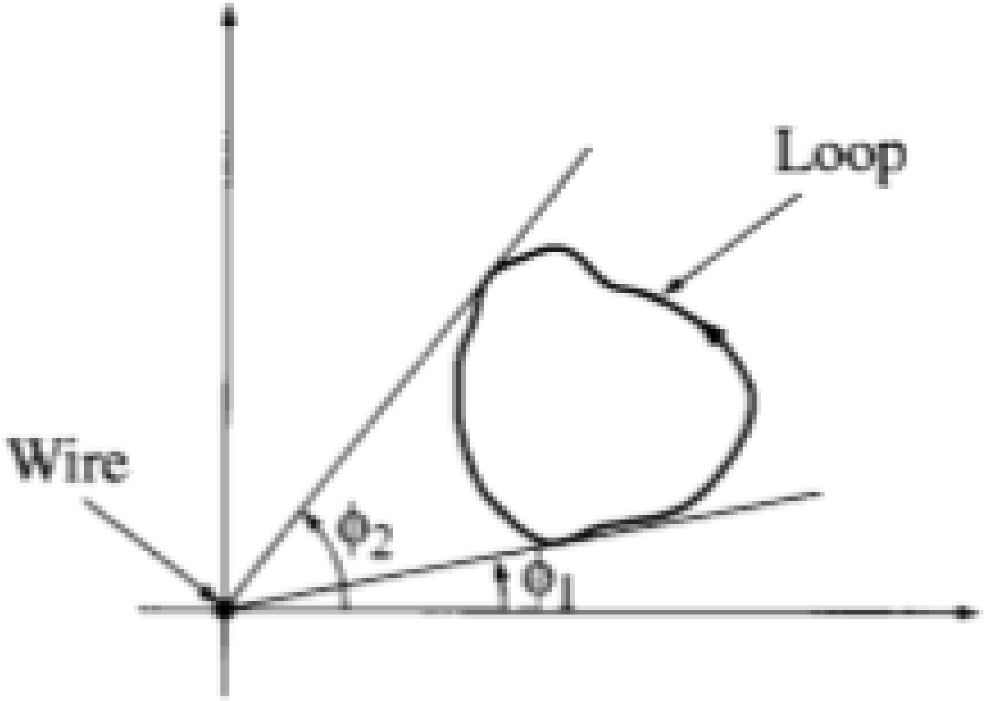
$$\oint \vec{B}.\overrightarrow{dl} = \frac{\mu_0 I}{2\pi}\oint \frac{1}{s}\widehat{\emptyset}\,.\,(ds\hat{s} + sd\emptyset\widehat{\emptyset} + dz\hat{z}) = \frac{\mu_0 I}{2\pi}\oint \frac{1}{s}\,sd\emptyset = \frac{\mu_0 I}{2\pi}\oint_0^{2\pi} d\emptyset = \mu_0 I$$

Straight Line Currents

$$\oint \vec{B}.\vec{dl}=\mu_0 I \qquad (5.42)$$

Where I is the current enclosed by yellow loop.

If current enclosed by the loop is zero,
then closed integral will be zero.
For example consider the following shape

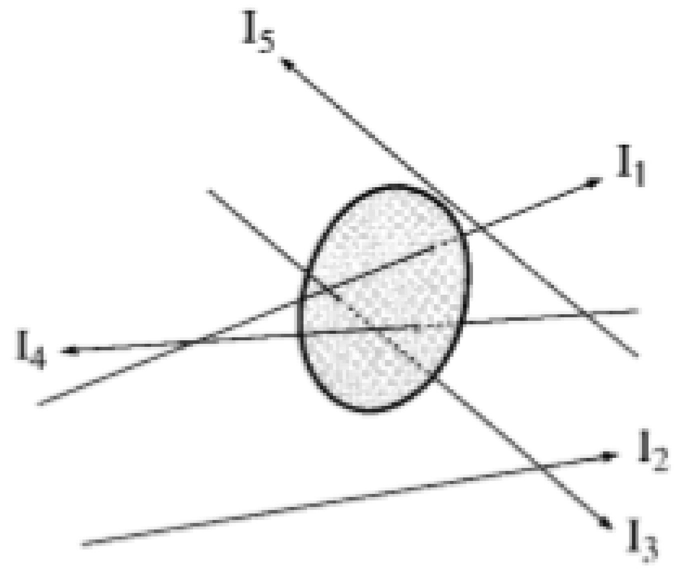


In this case closed integral is zero due to zero enclosed current.

Straight Line Currents

In this case only $I_1 . I_4, I_3$ are enclosed current because they are enclosed by integration path.
So above equation can be written as

$$\oint \vec{B} . \overrightarrow{dl} = \mu_0 I_{enc}$$



If the flow of charge is represented by volume current density $\vec{J}$, the enclosed current is

$$I_{enc} = \int \vec{J} . \overrightarrow{da}$$

Then above equation is $$\oint \vec{B} . \overrightarrow{dl} = \mu_0 \int \vec{J} . \overrightarrow{da}$$

Straight Line Currents

- Apply Stokes Theorem

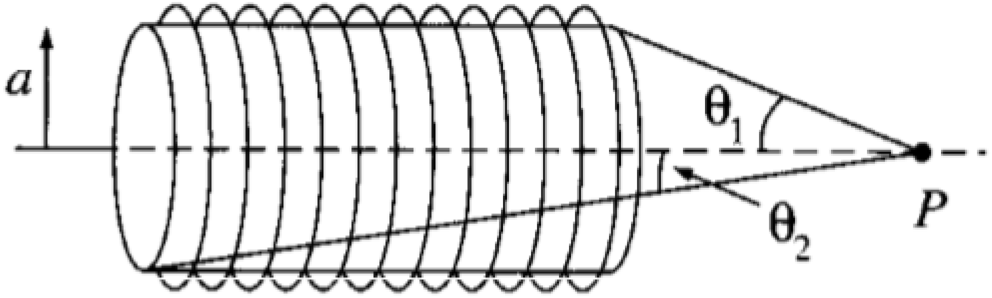
$$\int (\vec{\nabla} \times \vec{B}).\overrightarrow{da} = \mu_0 \int \vec{J}\,.\overrightarrow{da}$$

$$\boldsymbol{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}} \qquad (5.44)$$

With minimal labor we actually obtained the general formula of curl of **B** by using the straight line currents.

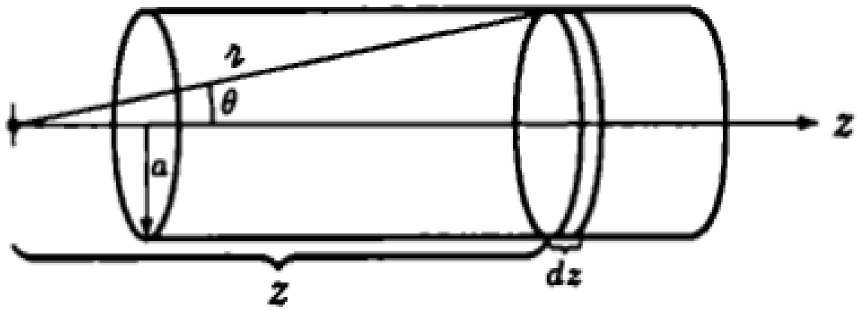
Problem 5.11

Problem 5.11 Find the magnetic field at point P on the axis of a tightly wound solenoid (helical coil) consisting of n turns per unit length wrapped around a cylindrical tube of radius a and carrying current I (Fig. 5.25). Express your answer in terms of θ_1 and θ_2 (it's easiest that way). Consider the turns to be essentially circular, and use the result of Ex. 5.6. What is the field on the axis of an *infinite* solenoid (infinite in both directions)?



Problem 5.11

Using equation 5.38 in the previous example 5.6



$$B(z) = \frac{\mu_0 I}{4\pi}\left(\frac{\cos\theta}{r^2}\right)2\pi R = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}.$$

Above magnetic field is due to circular ring at distance z from the center of ring.

Use above equation for a ring of width dz, with $I \longrightarrow nIdz$

Where n=turns per unit length wrapped around cylindrical tube of radius a

I ⟶ current of one turn

ndz ⟶ number of turns in the ring width dz

$nIdz$ ⟶ current in number of turns in the ring width dz

Problem 5.11

Put $\mathrm{R} = a$ and value of I

$$B(z) = \frac{\mu_0 I}{4\pi}\left(\frac{\cos\theta}{r^2}\right)2\pi R = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}.$$

$$dB = \frac{\mu_0 n I}{2}\frac{a^2}{(a^2+z^2)^{3/2}}dz$$

It is magnetic field due small ring of width dz

and integrate to calculate total magnetic field due to solenoid

$$B = \frac{\mu_0 n I}{2}\int\frac{a^2}{(a^2+z^2)^{3/2}}dz$$

Problem 5.11

But $z = acot\theta$

so $dz = -acosec^2\theta$

$$dz = -\frac{a}{sin^2\theta}d\theta$$ and $$\frac{1}{(a^2+z^2)^{3/2}} = -\frac{sin^3\theta}{a^3}d\theta$$

$$\Rightarrow \quad B = \frac{\mu_0 nI}{2}\int \frac{a^2}{(a^2+z^2)^{3/2}}dz = \frac{\mu_0 nI}{2}\int \frac{a^2 sin^3\theta}{a^3 sin^2\theta}(-ad\theta)$$

Problem 5.11

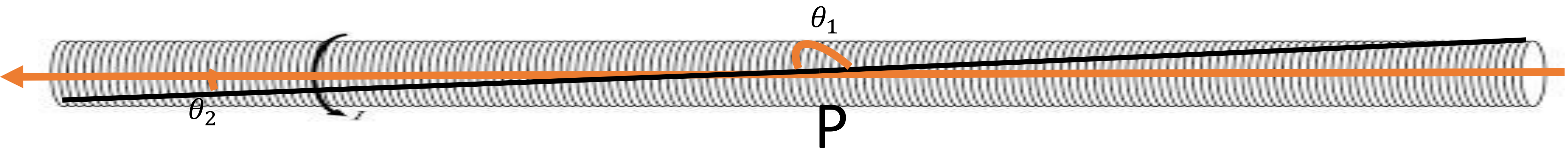
$$B = \frac{-\mu_0 nI}{2}\int sin\theta d\theta = \frac{-\mu_0 nI}{2}\int_{\theta_1}^{\theta_2} sin\theta d\theta$$

$$B = \frac{-\mu_0 nI}{2}\int_{\theta_1}^{\theta_2} sin\theta d\theta = \frac{-\mu_0 nI}{2}(-cos\theta)\Big|_{\theta_1}^{\theta_2}$$

$$= \frac{\mu_0 nI}{2}(cos\theta_2 - cos\theta_1)$$

For an infinite solenoid, θ_2=0, $\theta_1 = \pi$

$$B = \frac{\mu_0 nI}{2}(cos\theta_2 - cos\theta_1) = \frac{\mu_0 nI}{2}(1-(-1)) = \mu_0 nI$$



θ_1
θ_2
P