

Electrodynamics-II

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5.1.3 Currents

The current in a wire is the charge per unit time passing a given point

By definition, the negative charges moving to left count the same as the positive charge to right.

$$\text{Formula: } I = \frac{Q}{t}$$

$$\text{Unit: } 1\text{A} = 1\text{C/s}$$

Effect of Current

- Currents produce heat
- Currents produce magnetic fields

If charge is moving with velocity v , it covers distance

$$\Delta l = v\Delta t$$

in time Δt .

How much charge moves in time Δt through point P?

It will be charge equal to charge in segment of $\Delta l = v\Delta t$

Linear charge density is $\lambda = \frac{\text{charge}}{\text{length}}$

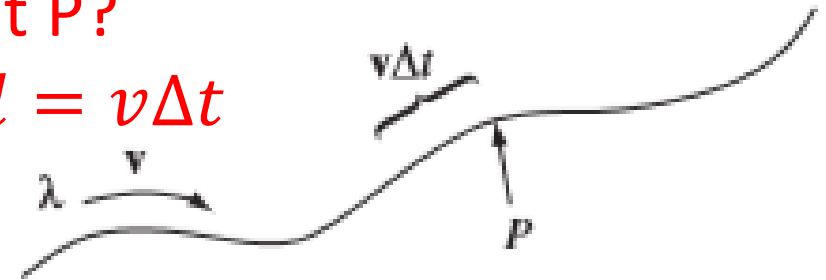


FIGURE 5.9

So $\lambda \Delta l = \lambda v\Delta t$ is the amount of charge moving through point P in time Δt

And rate of charge passing through point P = $\frac{Q}{t} = \frac{\lambda v\Delta t}{\Delta t} = \lambda v$

And therefore current in the wire is

$$I = \lambda v$$

In vector form

$$\vec{I} = \lambda \vec{v}$$

Current in the conductor is due to flow of electrons but in the electrolyte it is due to both positive and negative charges.

The magnetic force on the segment of current carrying wire having charge dq is

$$\vec{F} = \int \vec{v} \times \vec{B} dq = \int \vec{v} \times \vec{B} \lambda dl \quad \text{because } \lambda = \frac{dq}{dl}$$

$$\begin{aligned}\vec{F} &= \int \lambda \vec{v} \times \vec{B} dl = \vec{F} = \int \vec{I} \times \vec{B} dl = \int I \hat{I} \times \vec{B} dl \\ &= \int I (dl \hat{I} \times \vec{B})\end{aligned}$$

Direction of length is along the direction of current

$$\hat{l} = \hat{I}$$

$$\vec{F} = \int I (dl \hat{l} \times \vec{B}) = \int I (\vec{dl} \times \vec{B})$$

$$\vec{F} = \int I (\vec{dl} \times \vec{B})$$

Which is formula for force acting on line current

Surface Current Density

When charge flows over a *surface*, we describe it by the **surface current density**, \mathbf{K} , defined as follows: Consider a “ribbon” of infinitesimal width dl_{\perp} , running parallel to the flow (Fig. 5.13). If the current in this ribbon is $d\mathbf{I}$, the surface current density is

$$\mathbf{K} \equiv \frac{d\mathbf{I}}{dl_{\perp}}. \quad (5.22)$$

In words, K is the *current per unit width*. In particular, if the (mobile) surface charge density is σ and its velocity is \mathbf{v} , then

$$\mathbf{K} = \sigma \mathbf{v}. \quad (5.23)$$

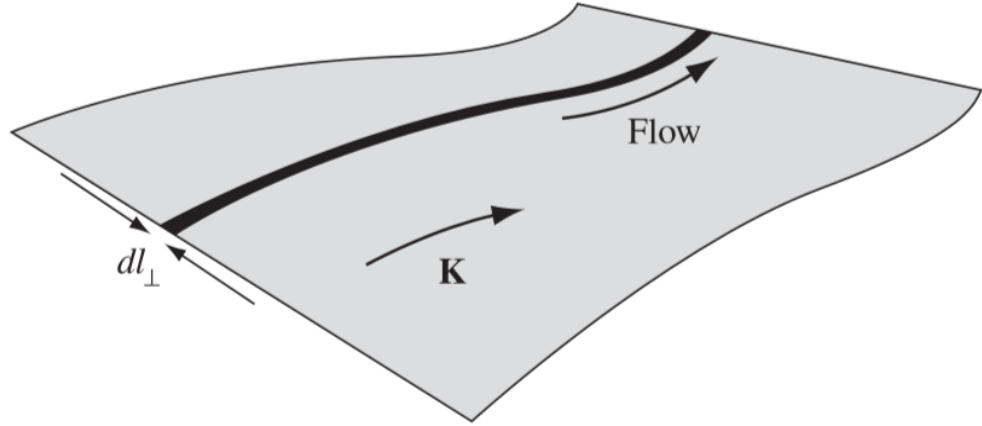


FIGURE 5.13

Force on surface current

In general, \mathbf{K} will vary from point to point over the surface, reflecting variations in σ and/or \mathbf{v} . The magnetic force on the surface current is

$$\vec{F} = \int \vec{v} \times \vec{B} dq = \int \vec{v} \times \vec{B} \sigma da$$

$$\vec{F} = \int \sigma \vec{v} \times \vec{B} da = \int \vec{K} \times \vec{B} da$$

$$\vec{F} = \int \vec{K} \times \vec{B} da$$

Volume current density \vec{J}

When the flow of charge is distributed throughout a three-dimensional region, we describe it by the **volume current density**, \mathbf{J} , defined as follows: Consider a “tube” of infinitesimal cross section da_{\perp} , running parallel to the flow (Fig. 5.14). If the current in this tube is $d\mathbf{I}$, the volume current density is

$$\mathbf{J} \equiv \frac{d\mathbf{I}}{da_{\perp}}. \quad (5.25)$$

In words, J is the *current per unit area*. If the (mobile) volume charge density is ρ and the velocity is \mathbf{v} , then

$$\mathbf{J} = \rho\mathbf{v}. \quad (5.26)$$

Fig. 5.14 Volume current moving through the body of the wire

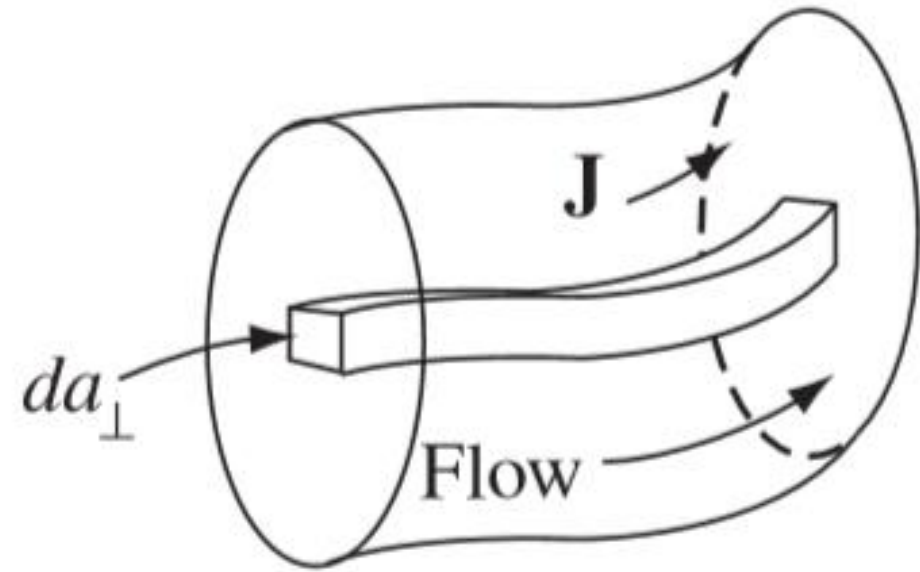


FIGURE 5.14

Magnetic Force on volume currents

The magnetic force on a volume current is therefore

$$\mathbf{F}_{\text{mag}} = \int (\mathbf{v} \times \mathbf{B}) \rho d\tau = \int (\mathbf{J} \times \mathbf{B}) d\tau. \quad (5.27)$$