Electrodynamics-II

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Lecture Contents

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Assignment

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Problem 4.5

Problem 4.5 In Fig. 4.6, $\mathbf{p}_1$ and $\mathbf{p}_2$ are (perfect) dipoles a distance r apart. What is the torque on $\mathbf{p}_1$ due to $\mathbf{p}_2$? What is the torque on $\mathbf{p}_2$ due to $\mathbf{p}_1$? [In each case I want the torque on the dipole *about its own center*. If it bothers you that the answers are not equal and opposite, see Prob. 4.29.]

Formulas Used

$$\mathbf{E}_{\text{dip}}(r,\theta) = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}).$$

$$\mathbf{N} = \mathbf{p} \times \mathbf{E}.$$

$\mathbf{p}_1$

Solution:

What is the torque on $\mathbf{p}_1$ due to $\mathbf{p}_2$?

$\mathbf{p}_2$

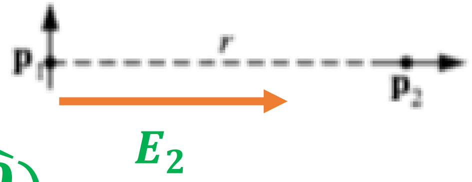
($\theta = \pi$ in Eq. 3.103)

At the first we calculate the electric field of $\mathbf{p}_2$ at the position of $\mathbf{p}_1$

In this case ($\theta = \pi$ in Eq. 3.103)

Problem 4.5

$$\mathbf{E}_{\text{dip}}(r,\theta) = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}).$$



$$\longrightarrow \quad E_2(r,\theta) = \frac{p_2}{4\pi\epsilon_0 r^3}(2\cos(\pi)\hat{r} + \sin(\pi)\hat{\theta})$$

$$\longrightarrow \quad \mathbf{E}_2 = \frac{p_2}{4\pi\epsilon_0 r^3}(-2\,\hat{\mathbf{r}}) \text{ (points to the } \textit{right}\text{).}$$

Torque on $\mathbf{p}_1$: $\mathbf{N}_1 = \mathbf{p}_1 \times \mathbf{E}_2 = \boxed{\dfrac{2p_1p_2}{4\pi\epsilon_0 r^3}}$ (points *into* the page).

b) **What is the torque on $\mathbf{p}_2$ due to $\mathbf{p}_1$?**

($\theta = \pi/2$ in Eq. 3.103)

($\theta = \pi/2$ in Eq. 3.103)

In this case

$$E_1(r,\theta) = \frac{p_1}{4\pi\epsilon_0 r^3}\left(2\cos\left(\frac{\pi}{2}\right)\hat{r} + \sin\left(\frac{\pi}{2}\right)\hat{\theta}\right)$$

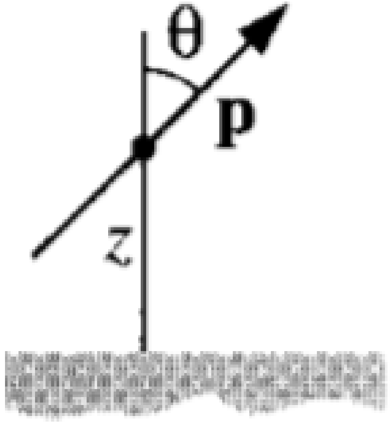
Problem 4.5

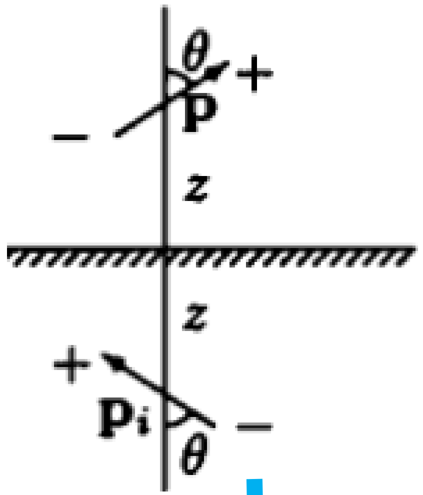
$$\mathbf{E}_1 = \frac{p_1}{4\pi\epsilon_0 r^3}\hat{\boldsymbol{\theta}} \text{ (points } \textit{down}\text{).}$$

Torque on $\mathbf{p}_2$: $\mathbf{N}_2 = \mathbf{p}_2 \times \mathbf{E}_1 = p_2 E_1 \sin 90^\circ = p_2 E_1$

$$= \boxed{\frac{p_1 p_2}{4\pi\epsilon_0 r^3}} \text{ (points } \textit{into} \text{ the page).}$$

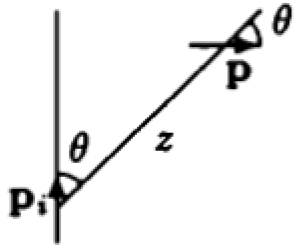
Problem 4.6 A (perfect) dipole **p** is situated a distance z above an infinite grounded conducting plane (Fig. 4.7). The dipole makes an angle θ with the perpendicular to the plane. Find the torque on **p**. If the dipole is free to rotate, in what orientation will it come to rest?





- Using Image Charge $\boldsymbol{p_i}$, Redraw, placing the image charge at origin
  - Electric field due to image charge at the place of original dipole is

$$\mathbf{E}_i = \frac{p}{4\pi\epsilon_0(2z)^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}})$$



Pi is at origin

Problem:4.6

$$\mathbf{E}_i = \frac{p}{4\pi\epsilon_0(2z)^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}})$$

Components of original dipole is

$$\mathbf{p} = p\cos\theta\,\hat{\mathbf{r}} + p\sin\theta\,\hat{\boldsymbol{\theta}}.$$

Torque acting on the dipole **p** due to infinite grounded plane is

$$\begin{aligned}
\mathbf{N} &= \mathbf{p}\times\mathbf{E}_i = \frac{p^2}{4\pi\epsilon_0(2z)^3}\left[(\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}) \times (2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}})\right] \\
&= \frac{p^2}{4\pi\epsilon_0(2z)^3}\left[\cos\theta\sin\theta\,\hat{\boldsymbol{\phi}} + 2\sin\theta\cos\theta(-\hat{\boldsymbol{\phi}})\right] \\
&= \frac{p^2\sin\theta\cos\theta}{4\pi\epsilon_0(2z)^3}(-\hat{\boldsymbol{\phi}}) \quad \text{(out of the page).}
\end{aligned}$$