Electrodynamics II

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Problem 4.16 Suppose the field inside a large piece of dielectric is $\mathbf{E}_0$, so that the electric displacement is $\mathbf{D}_0 = \epsilon_0 \mathbf{E}_0 + \mathbf{P}$.

(a) Now a small spherical cavity (Fig. 4.19a) is hollowed out of the material. Find the field at the center of the cavity in terms of $\mathbf{E}_0$ and $\mathbf{P}$. Also find the displacement at the center of the cavity in terms of $\mathbf{D}_0$ and $\mathbf{P}$.

(b) Do the same for a long needle-shaped cavity running parallel to $\mathbf{P}$ (Fig. 4.19b).

(c) Do the same for a thin wafer-shaped cavity perpendicular to $\mathbf{P}$ (Fig. 4.19c).

[Assume the cavities are small enough that $\mathbf{P}$, $\mathbf{E}_0$, and $\mathbf{D}_0$ are essentially uniform. *Hint:* Carving out a cavity is the same as superimposing an object of the same shape but opposite polarization.]

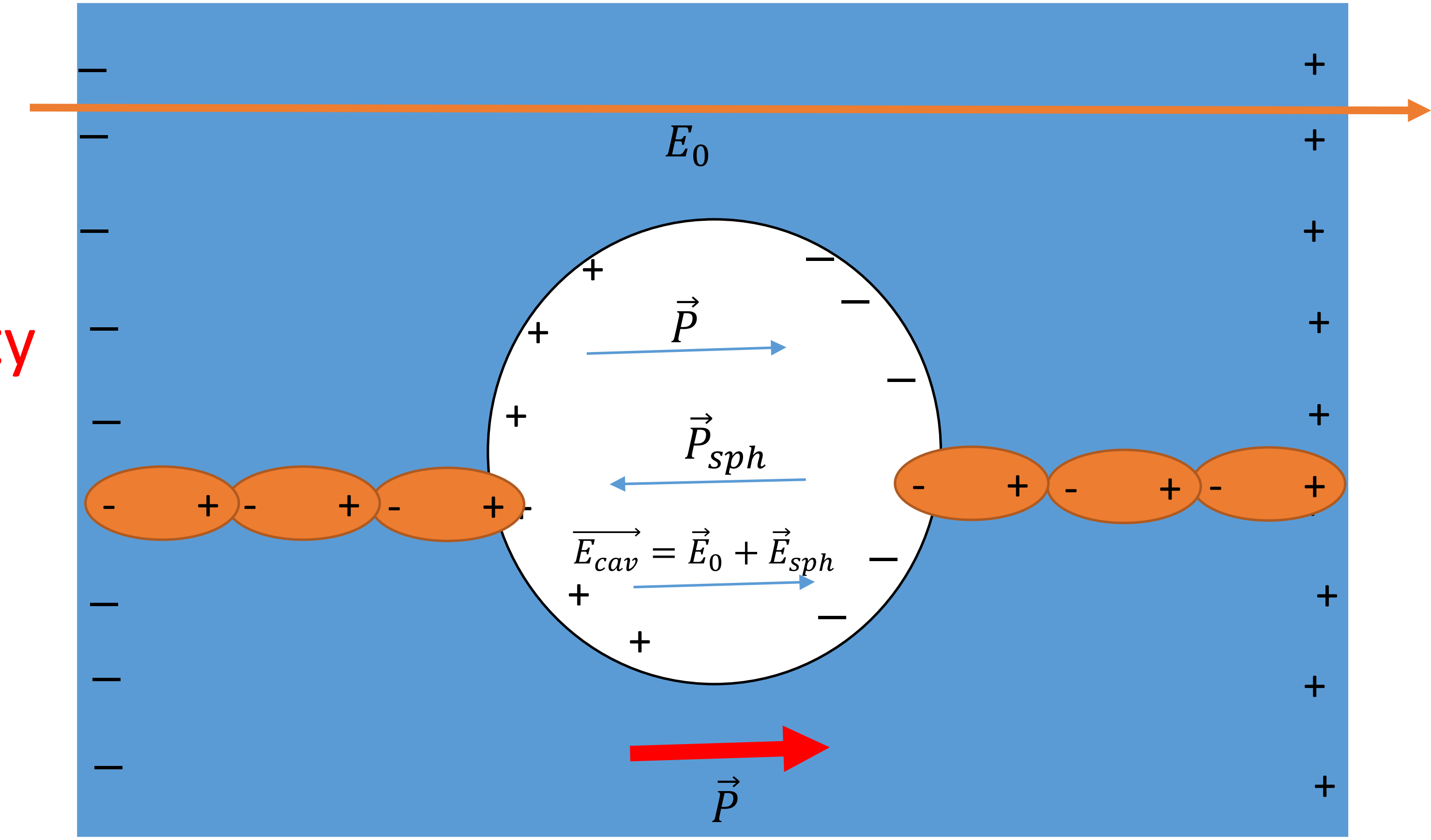
Problem 4.16 (a)

- The Electric field is from Left side to right side.
- External Polarization is from left side to right side.
- The electric field due to cavity Is from left side

The total Electric field is (Ex.4.2)

$$\vec{E}_{sph} = \frac{-\vec{P}_{sph}}{3\epsilon_0}$$

The polarization of hollow sphere $\vec{P}_{sph} = -\vec{P}$



Problem 4.16 (a)

$$\vec{E}_{cavity} = \vec{E}_0 + \vec{E}_{sph}$$

$$\vec{E}_{cavity} = \vec{E}_0 + \frac{-\vec{P}_{sph}}{3\epsilon_0} = \vec{E}_0 - \frac{\vec{P}_{sph}}{3\epsilon_0}$$

Because $\vec{P}_{sph} = -\vec{P}$

$$\vec{P}_{cavity} = \vec{P}_{sph} + \vec{P} = -\vec{P} + \vec{P} = 0$$

$$\boxed{\vec{E}_{cavity} = \vec{E}_0 + \frac{\vec{P}}{3\epsilon_0}}$$

$$\vec{D}_{cavity} = \epsilon_0 \vec{E}_{cavity} + \vec{P}_{cavity}$$

$$\vec{D}_{cavity} = \epsilon_0 \left(\vec{E}_0 + \frac{\vec{P}}{3\epsilon_0} \right) + 0$$

Problem 4.16 (a)

Given $\vec{D}_0 = \epsilon_0 \vec{E}_0 + \vec{P}$ ⟶ $\vec{E}_0 = \frac{\vec{D}_0}{\epsilon_0} - \frac{\vec{P}}{\epsilon_0}$

Put value of $\vec{E}_0$

$$\vec{D}_{cavity} = \epsilon_0\left(\vec{E}_0 + \frac{\vec{P}}{3\epsilon_0}\right) = \epsilon_0\left(\frac{\vec{D}_0}{\epsilon_0} - \frac{\vec{P}}{\epsilon_0} + \frac{\vec{P}}{3\epsilon_0}\right) = \vec{D}_0 - \frac{2\vec{P}}{3\epsilon_0}$$

$$\boxed{\vec{D}_{cavity} = \vec{D}_0 - \frac{2\vec{P}}{3\epsilon_0}}$$

Problem 4.16 (b)

(b) Do the same for a long needle-shaped cavity running parallel to **P** (Fig. 4.19b).

(b) Needle

Problem 4.16 (b)

$$\vec{E}_{cavity} = \vec{E}_0 + \vec{E}_{needle}$$

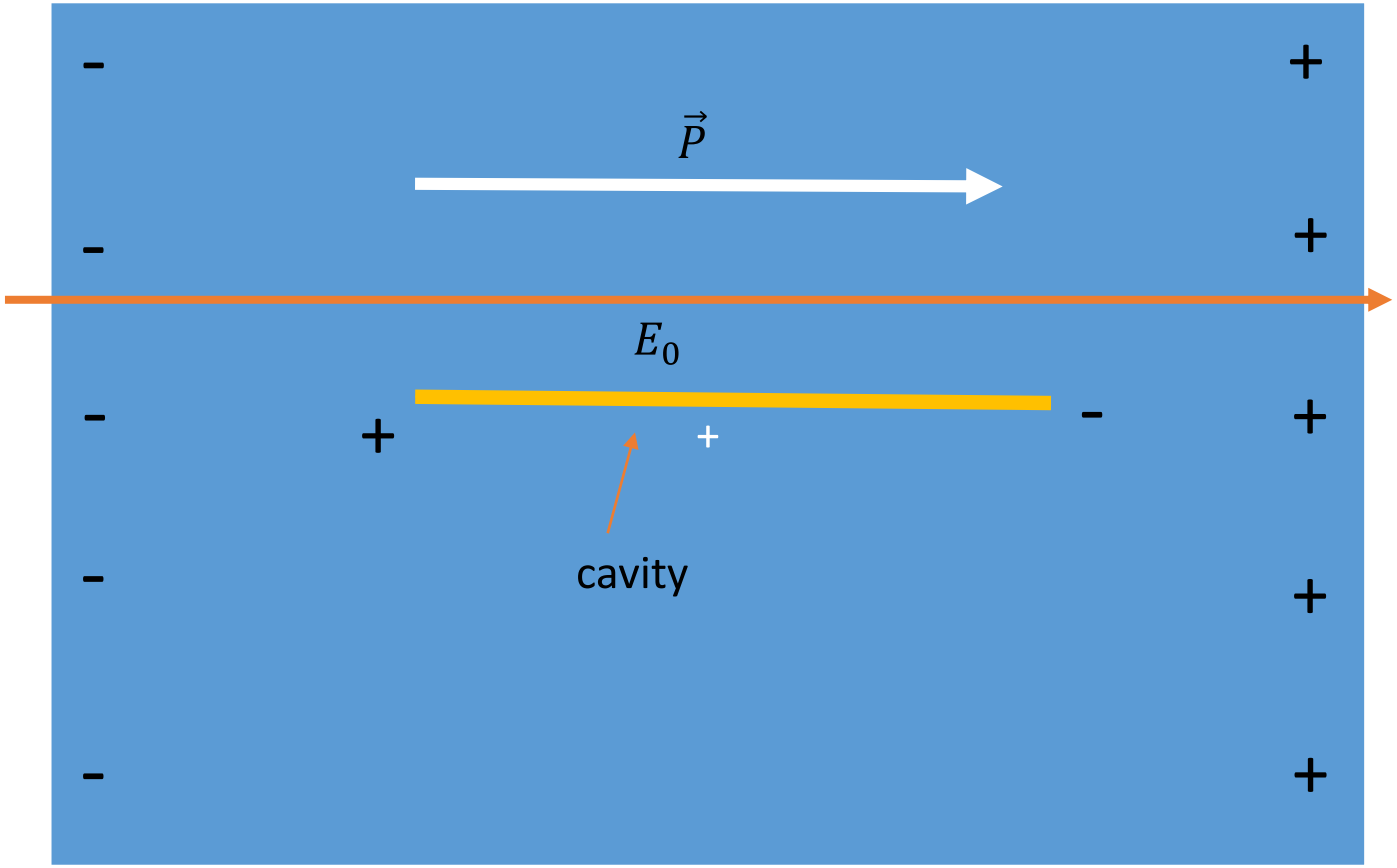
because $\vec{E}_{needle} \approx 0$

$\Rightarrow$ $\vec{E}_{cavity} \approx \vec{E}_0$

As

$$\vec{D}_{cavity} = \epsilon_0 \vec{E}_{cavity} + \vec{P}_{cavity}$$

$\Rightarrow$ $\vec{D}_{cavity} = \epsilon_0 \vec{E}_0 + \vec{P}_{cavity}$



Problem 4.16 (b)

Because $\vec{P}_{cavity} = 0$ → $\vec{D}_{cavity} = \epsilon_0 \vec{E}_0 + 0$

$$\vec{D}_{cavity} = \epsilon_0 \vec{E}_0$$

Given $\vec{E}_0 = \frac{\vec{D}_0}{\epsilon_0} - \frac{\vec{P}}{\epsilon_0}$ → $\vec{D}_{cavity} = \epsilon_0 \left(\frac{\vec{D}_0}{\epsilon_0} - \frac{\vec{P}}{\epsilon_0} \right)$

→ $$\boxed{\vec{D}_{cavity} = \vec{D}_0 - \vec{P}}$$

Problem 4.16 (c)

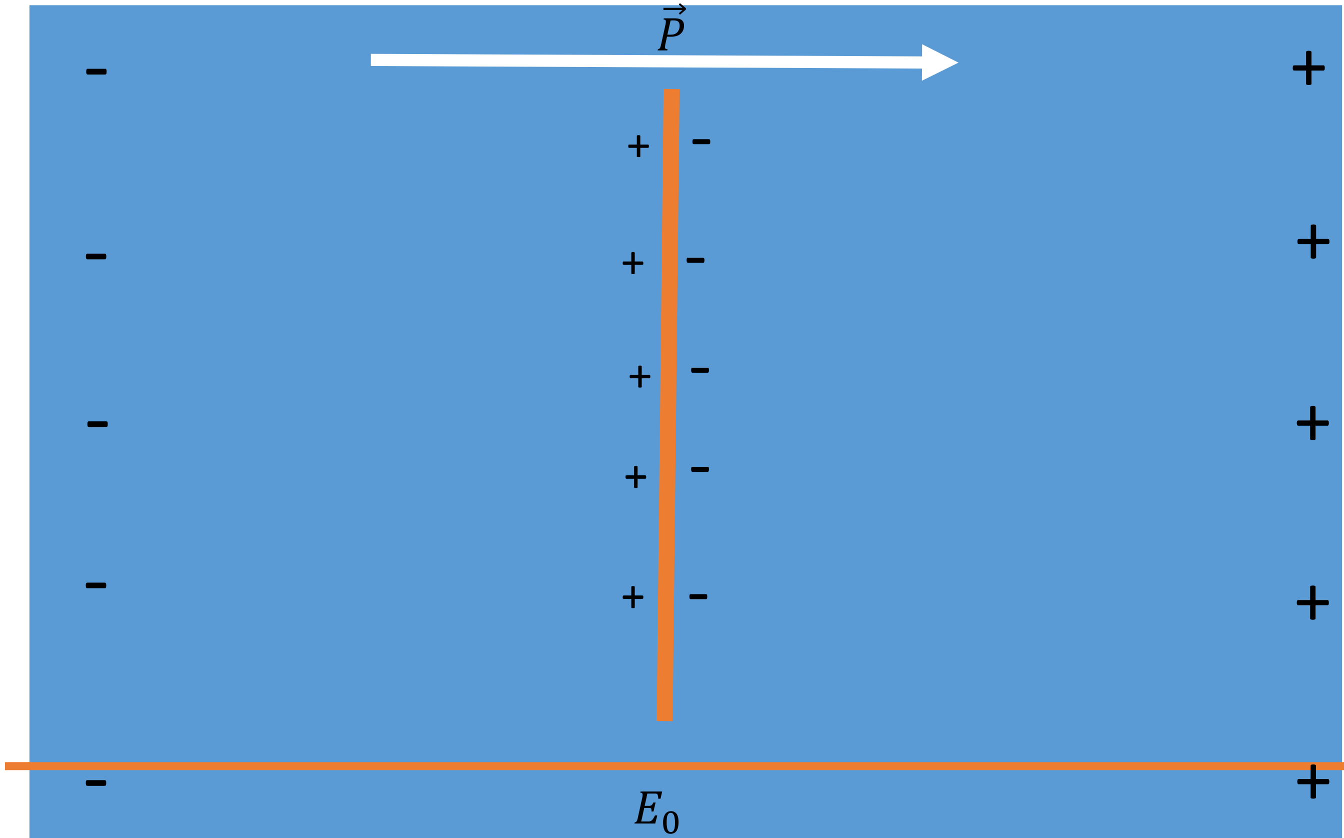
(c) Do the same for a thin wafer-shaped cavity perpendicular to **P** (Fig. 4.19c).

Electric field inside the capacitor

$$\vec{E}_{cavity} = \vec{E}_0 + \vec{E}_{cap}$$

Because $\vec{E}_{cap} = \frac{\sigma_b}{\epsilon_0}\hat{n}$

And $\sigma_b = \vec{P}.\hat{n} = P$



Problem 4.16 (c)

$$\vec{E}_{cavity} = \vec{E}_0 + \frac{\sigma_b}{\epsilon_0}\hat{n} = \vec{E}_0 + \frac{\vec{P}}{\epsilon_0}$$

$$\vec{D}_{cavity} = \epsilon_0 \vec{E}_{cavity} + \vec{P}_{cavity}$$

Given $\vec{E}_0 = \frac{\vec{D}_0}{\epsilon_0} - \frac{\vec{P}}{\epsilon_0}$ $\Rightarrow$ $\vec{D}_{cavity} = \epsilon_0\left(\vec{E}_0 + \frac{\vec{P}}{\epsilon_0}\right) + 0 = \vec{D}_0$

$$\boxed{\vec{D}_{cavity} = \vec{D}_0}$$