Subject: Electrodynamics II

Lecture Delivered By Muhammad Amer Mustafa

University of Sargodha,Sub Campus Bhakkar

Lecture Content

- Gauss's law of Dielectric
- Example 4.4

Assignment

- Problem 4.15

4.3 The Electric Displacement

4.3.1 The Guass's Law in the presence of Dielectrics

As we Know

$$Surface\ Volume\ Bound\ charge\ Density = \sigma_b \equiv \vec{P}.\hat{n}$$

$$Volume\ Bound\ Charge\ Density = \rho_b \equiv -\vec{\nabla}.\vec{P}$$

The Effect of polarization is to produce accumulations of bound charges , ρ_b within the volume and σ_b on the surface.

The field due to polarization is just the field of this bound charge. We are now ready to put it all together: the field attributable to bound charge plus the field due to every thing else (Free Charge)

What is Free Charge in Dielectric?
Any charge which is not a result of Polarization
for e.g. ions embedded in the material

The Guass's Law in the presence of Dielectrics

Within the Dielectric total charge density can be written as

$$\rho = \rho_b + \rho_f$$

And then Gauss's law is

$$\epsilon_0 \vec{\nabla}.\vec{E} = \rho = \rho_b + \rho_f$$

$$\epsilon_0 \vec{\nabla}.\vec{E} = -\vec{\nabla}.\vec{P} + \rho_f$$

Where ***E*** is now total field

$$\epsilon_0 \vec{\nabla}.\vec{E} + \vec{\nabla}.\vec{P} = \rho_f$$

The Guass's Law in the presence of Dielectrics

$$\epsilon_0 \vec{\nabla}.\vec{E} + \vec{\nabla}.\vec{P} = \rho_f$$

$$\vec{\nabla}.(\epsilon_0 \vec{E} + \vec{P}) = \rho_f$$

The expression in parenthesis, designated by the letter $\vec{D}$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \qquad (4.21)$$

$\vec{D}$ is known as electric displacement

The Guass's Law in the presence of Dielectrics

In terms of $\vec{D}$ Gauss's law is

$$\vec{\nabla}.\vec{D} = \rho_f$$

Which is also called the Gauss's law of Dielectrics

$$\int \vec{\nabla}.\vec{D} d\tau = \int \rho_f \, d\tau$$

$where$

$$\int \rho_f \, d\tau = Q_f = Total\ free\ charge$$

Apply the Divergence Theorem on left side

$$\oint \vec{D}.\overrightarrow{da} = Q_f$$

Which is integral form of Gauss's law of Dielectrics

Example 4.4

A long straight wire, carrying uniform line charge λ, is surrounded by rubber insulation out to a radius a (Fig. 4.17). Find the electric displacement.

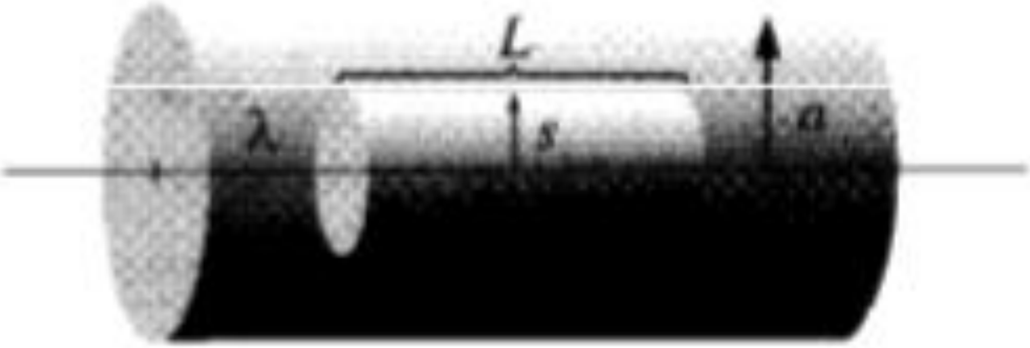
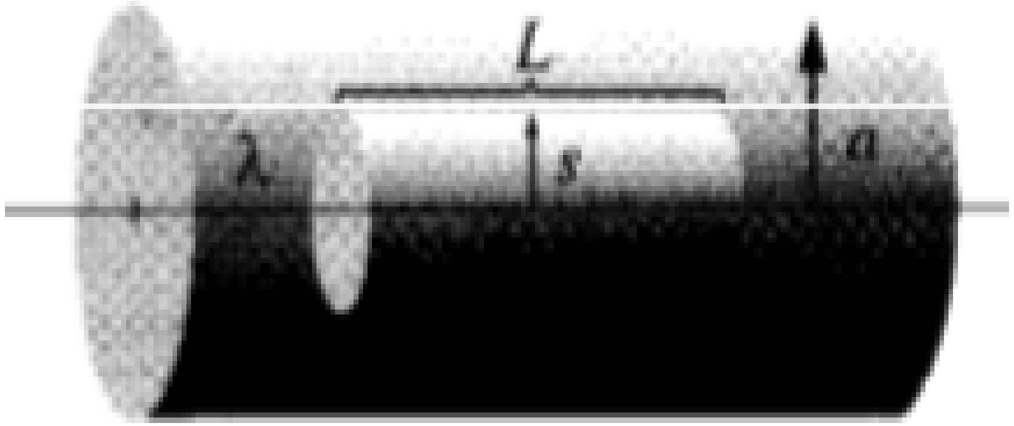


Figure 4.17

$$\oint \vec{D}.\overrightarrow{da} = Q_f$$



$$D \oint da \ = Q_f$$

and $$Q_f = \lambda L$$

$$D 2\pi s L = \lambda L$$

$$\vec{D} = \frac{\lambda}{2\pi s}\hat{s}$$

Example 4.4

Notice that this Formula holds both within the insulation and outside it.

Outside the insulation, **P**=0

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 \vec{E}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{s}$$

Inside the rubber the electric field cannot be determined since we do not know **P**.