Electrodynamics II

Lecture Delivered By Muhammad Amer Mustafa

University of Sargodha,Sub Campus Bhakkar

Student Assignment

Problem 4.11

4.1.4 Polarization

What happens to piece of dielectric when it is placed in electric field?

If a substance consist of neutral atom is placed in an external electric field, the field will induce tiny dipole moment in each atom pointing in the same direction as the field.

What is Polarized material?

The material is called the polarized, if lot of dipoles pointing in the direction of Electric field and such phenomenon is called the polarization.

4.1.4 Polarization

Definition of Polarization$(\vec{P})$

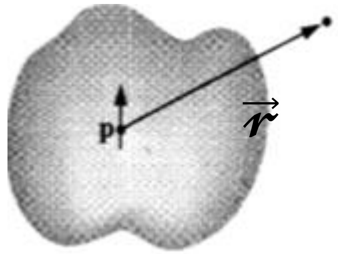
Polarization is defined as total dipole moment per unit volume of the material

What is freez in Polarization?

The polarization of a material which persists after the field is removed.

4.2 The Field of a Polarized Object

$\vec{\boldsymbol{r}}$ =Separation vector



- 4.2.1 Bound Charges
- Suppose we have piece of some polarized material.
- We want to find out potential outside the object due to this polarized object. Let we take a point P outside the object where we want to find the potential.

This polarized material has many dipoles. The potential at **r** due to single infinitesimal dipole can be written as

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\hat{\boldsymbol{r}}.\vec{p}}{\boldsymbol{r}^2}$$

Where $\vec{\boldsymbol{r}}$ is a vector from dipole to point at which we are evaluating the symbol.

4.2 The Field of a Polarized Object

We divide the whole material into small volume elements $d\tau'$

In present context, polarization of material is $\vec{P}$ so dipole moment $\vec{p}$ of single volume element can be written as

$$\vec{p} = \vec{P} d\tau'$$

And potential $\mathrm{d}V(\vec{r})$ due to single polarized volume element is

$$\mathrm{d}V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\hat{\boldsymbol{\mathscr{r}}}.\vec{P}}{\boldsymbol{\mathscr{r}}^2} d\tau'$$

Potential due to whole object can be found by integrating the above equation

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\hat{\boldsymbol{\mathscr{r}}}.\vec{P}}{\boldsymbol{\mathscr{r}}^2} d\tau'$$

4.2 The Field of a Polarized Object

- After some little effort we can conclude

$$\nabla'\left(\frac{1}{\mathscr{r}}\right) = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\hat{\boldsymbol{\mathscr{r}}}.\vec{P}}{\boldsymbol{\mathscr{r}}^2} d\tau' = \frac{1}{4\pi\varepsilon_0}\int \vec{P}.\left(\frac{\hat{\boldsymbol{\mathscr{r}}}}{\boldsymbol{\mathscr{r}}^2}\right) d\tau'$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \vec{P}.\nabla'\left(\frac{1}{\mathscr{r}}\right) d\tau'$$

4.2 The Field of a Polarized Object

$$\overrightarrow{\nabla'}.\left(\frac{\vec{P}}{\mathscr{r}}\right)=\frac{1}{\mathscr{r}}(\overrightarrow{\nabla'}.\vec{P})+\vec{P}.\nabla'\left(\frac{1}{\mathscr{r}}\right)$$

$$\nabla\cdot(f\mathbf{A})=f(\nabla\cdot\mathbf{A})+\mathbf{A}\cdot(\nabla f),$$

$$\vec{P}.\overrightarrow{\nabla'}\left(\frac{1}{\mathscr{r}}\right)=\overrightarrow{\nabla'}.\left(\frac{\vec{P}}{\mathscr{r}}\right)-\frac{1}{\mathscr{r}}(\overrightarrow{\nabla'}.\vec{P})$$

Put this in above equation

$$V(\vec{r})=\frac{1}{4\pi\varepsilon_0}\int\vec{P}.\nabla'\left(\frac{1}{\mathscr{r}}\right)d\tau'=\frac{1}{4\pi\varepsilon_0}\int\overrightarrow{\nabla'}.\left(\frac{\vec{P}}{\mathscr{r}}\right)d\tau'-\frac{1}{4\pi\varepsilon_0}\int\frac{1}{\mathscr{r}}(\overrightarrow{\nabla'}.\vec{P})d\tau'$$

4.2 The Field of a Polarized Object

$$\frac{1}{4\pi\varepsilon_0}\int \overrightarrow{\nabla'}.\left(\frac{\vec{P}}{\mathscr{r}}\right) d\tau' = \frac{1}{4\pi\varepsilon_0}\oint \frac{1}{\mathscr{r}}\vec{P}.\overrightarrow{da'}$$ **Using Divergence Theorem**

so

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \vec{P}.\nabla'\left(\frac{1}{\mathscr{r}}\right) d\tau' = \frac{1}{4\pi\varepsilon_0}\oint \frac{1}{\mathscr{r}}\vec{P}.\overrightarrow{da}' - \frac{1}{4\pi\varepsilon_0}\int \frac{1}{\mathscr{r}}(\overrightarrow{\nabla'}.\vec{P})d\tau'$$

4.2 The Field of a Polarized Object

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\oint \frac{1}{r}\vec{P}.\overrightarrow{da}' - \frac{1}{4\pi\varepsilon_0}\int \frac{1}{r}(\overrightarrow{\nabla'}.\vec{P})d\tau'$$

$$\boxed{V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}')}{r} d\tau'.}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma(\mathbf{r}')}{r} da'.$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\oint \frac{1}{r}\vec{P}.\hat{n}\, da' - \frac{1}{4\pi\varepsilon_0}\int \frac{1}{r}(\overrightarrow{\nabla'}.\vec{P})d\tau'$$

4.2 The Field of a Polarized Object

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\oint \frac{1}{r}\vec{P}.\hat{n}\,da' - \frac{1}{4\pi\varepsilon_0}\int \frac{1}{r}(\vec{\nabla'}.\vec{P})d\tau'$$

First term looks like a Potential of a surface charge and while 2nd term looks like a potential of volume charge

$$Volume\ Bound\ Charge\ Density = \rho_b \equiv -\vec{\nabla'}.\vec{P}$$

$$Surface\ Volume\ Bound\ charge\ Density = \sigma_b \equiv \vec{P}.\hat{n}$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\oint \frac{\sigma_b}{r}\,da' + \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_b}{r}d\tau'$$