Electrodynamics-II

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Lecture Contents

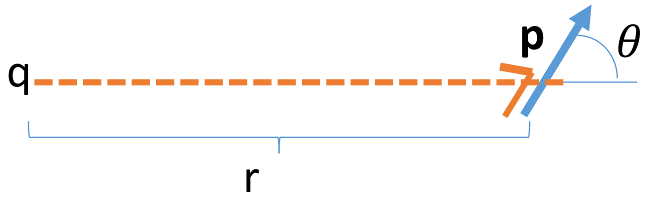
Problem 4.9

Problem 4.9 A dipole **p** is a distance r from a point charge q, and oriented so that **p** makes an angle θ with the vector **r** from q to **p**.

(a) What is the force on **p**?

(b) What is the force on q?

Solution:



Formulas used

(a) $\mathbf{F} = (\mathbf{p} \cdot \nabla)\mathbf{E}$ (Eq. 4.5);

(b) $$\mathbf{E}_{\text{dip}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}].$$

$$\mathbf{F} = q\mathbf{E}$$

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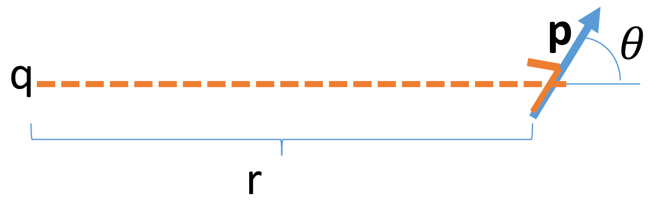


$$\mathbf{p}\cdot\nabla = \left(p_x\frac{\partial}{\partial x} + p_y\frac{\partial}{\partial y} + p_z\frac{\partial}{\partial z}\right)$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} \quad , \quad \hat{\mathbf{r}} = \frac{x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}}{(x^2+y^2+z^2)^{3/2}}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\frac{x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}}{(x^2+y^2+z^2)^{3/2}}$$

$$\mathbf{F} = (\mathbf{p} \cdot \nabla)\mathbf{E} = \left(p_x \frac{\partial}{\partial x} + p_y \frac{\partial}{\partial y} + p_z \frac{\partial}{\partial z}\right) \frac{q}{4\pi\epsilon_0} \frac{x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}}{(x^2+y^2+z^2)^{3/2}}$$

$$= \frac{q}{4\pi\epsilon_0}\left(p_x \frac{\partial}{\partial x} \frac{x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}}{(x^2+y^2+z^2)^{3/2}} + p_y \frac{\partial}{\partial y} \frac{x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}}{(x^2+y^2+z^2)^{3/2}}\right.$$

$$\left. + \; p_z \frac{\partial}{\partial z} \frac{x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}}{(x^2+y^2+z^2)^{3/}}\right)$$

At first, we take only x component

$$F_x = \left(p_x \frac{\partial}{\partial x} + p_y \frac{\partial}{\partial y} + p_z \frac{\partial}{\partial z}\right) \frac{q}{4\pi\epsilon_0} \frac{x}{(x^2+y^2+z^2)^{3/2}}$$

$$= \frac{q}{4\pi\epsilon_0}\left(p_x \frac{\partial}{\partial x} \frac{x}{(x^2+y^2+z^2)^{3/2}} + p_y \frac{\partial}{\partial y} \frac{x}{(x^2+y^2+z^2)^{3/2}}\right.$$

$$\left. + \; p_z \frac{\partial}{\partial z} \frac{x}{(x^2+y^2+z^2)^{3/2}}\right)$$

At first, we take only x component

$$= \frac{q}{4\pi\epsilon_0}\left(p_x \frac{\partial}{\partial x}\frac{x}{(x^2+y^2+z^2)^{3/2}} + p_y \frac{\partial}{\partial y}\frac{x}{(x^2+y^2+z^2)^{3/2}} + p_z \frac{\partial}{\partial z}\frac{x}{(x^2+y^2+z^2)^{3/2}}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\left\{p_x\left[\frac{1}{(x^2+y^2+z^2)^{3/2}} - \frac{3}{2}x\frac{2x}{(x^2+y^2+z^2)^{5/2}}\right] + p_y\left[-\frac{3}{2}x\frac{2y}{(x^2+y^2+z^2)^{5/2}}\right]\right.$$

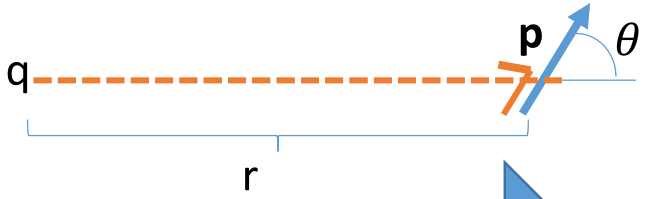
$$\left. + p_z\left[-\frac{3}{2}x\frac{2z}{(x^2+y^2+z^2)^{5/2}}\right]\right\} = \frac{q}{4\pi\epsilon_0}\left[\frac{p_x}{r^3} - \frac{3x}{r^5}(p_x x + p_y y + p_z z)\right] = \frac{q}{4\pi\epsilon_0}\left[\frac{\mathbf{p}}{r^3} - \frac{3\mathbf{r}(\mathbf{p}\cdot\mathbf{r})}{r^5}\right]_x$$

$$\mathbf{F} = \frac{q}{4\pi\epsilon_0}\left[\frac{\mathbf{p}}{r^3} - \frac{3r\hat{r}(\mathbf{p}.r\hat{r})}{r^5}\right] \quad \rightarrow \quad \mathbf{F} = \boxed{\frac{1}{4\pi\epsilon_0}\frac{q}{r^3}\left[\mathbf{p} - 3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}\right].}$$

(b) What is the force on q?

$$\mathbf{E}_{\text{dip}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}].$$

3.104 Equation



r is from q to p as given in question

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\{3[\mathbf{p}\cdot(-\hat{\mathbf{r}})](-\hat{\mathbf{r}}) - \mathbf{p}\} = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}[3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}]$$

We have to find the electric field at the position of q in opposite direction therefore taken as -r

F=qE=