# ⇒ Utility Maximization

## ⇒ Definition:-

Utility maximization refers to concepts that individual and firms seek to get the highest satisfaction from their economics dicision.

## ⇒ Utility maximizing Rule:-

The rule of utility maximizing is that

Marginal utility per dollar of each goals should be equal to each other.

$$\Rightarrow \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

## ⇒ Finding value of utility:-

There are following steps to find the value of utility:-

⇒ Utility function is given ⇒ (1)

⇒ Necessary Condition:-

- Find its first order derivative

with respect of quantity Q.

- Put it equal to zero and then find the value of Q.

⇒ **Sufficient Condition:-**

Find the 2nd order derivative of utility function.

- If it is less than zero then utility is maximum.
- If it is greater than zero then utility is minimum.
- Put value of Q in utility function eq ① to get value of utility at that value of Q.

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## ⇒ Question:-

$R = -3Q^2 + 250Q$

Find TR and AR, Aslo find Slope of MR and AR:

## ⇒ Solution:-

1): $TR = R = -3Q^2 + 250Q$

2). $AR = \frac{R}{Q} = \frac{-3Q^2}{Q} + \frac{250Q}{Q}$

$\Rightarrow AR = -3Q + 250$

3). $MR = \frac{dR}{dQ} = \frac{d}{dQ}(-3Q^2 + 250Q)$

$\Rightarrow MR = -6Q + 250$

4). Slope of $AR = \frac{dAR}{dQ}$

$= \frac{d}{dQ}(-3Q + 250)$

$\Rightarrow$ Slope of $AR = -3$

5). Slope of $MR = \frac{dMR}{dQ}$

$= \frac{d}{dQ}(-6Q + 250)$

$\Rightarrow$ Slope of $MR = -6$

$\Rightarrow$ As $P=0$, So, $AR=0$

$$250-3Q=0$$
$$3Q=+250$$
$$\boxed{Q=83.33}$$

$\Rightarrow$ As $P=0$, So, $MR=0$

$$-6Q+250=0$$
$$6Q=250$$
$$\boxed{Q=41.67}$$

## $\Rightarrow$ Question:-

If $P=50-2Q$

$$TC=Q^3-10Q^2+55Q+15$$

a). find $\pi$ and P. $\Rightarrow$ **Solution:-**

We know that

$$\pi=TR-TC$$

Also,

$$TR=P\times Q$$
$$TR=50Q-2Q^2$$
$$\pi=TR-TC$$
$$\pi=50Q-2Q^2-Q^3+10Q^2-55Q-15$$
$$\pi=-Q^3+8Q^2-5Q-15$$

NC/

$$\frac{d\pi}{dQ}=-3Q^2+16Q-5$$

Put $\frac{d\pi}{dQ}=0$

So,

$$-3Q^2-16Q-5(1)=0$$

$$-(3Q^2+16Q+5(1))=0$$

$$3Q^2-16Q+5=0$$

$$3Q^2-15Q-Q+5=0$$

$$3Q(Q-5)-1(Q-5)=0$$

$$(Q-5)(3Q-1)=0$$

$Q-5=0$ $\qquad\qquad$ $3Q-1=0$

$\boxed{Q=5}$ $\qquad\qquad$ $3Q=1$

$Q=1/3$

Now, $\qquad\qquad$ $\boxed{Q=0.334}$

SC/

$$\frac{d\pi^2}{dQ^2}=-6Q+16-0$$

$$=-6Q+16$$

Put $Q=5$ and $Q=0.334$ in above equation:-

$=-6(5)+16$ $\qquad\qquad$ $-6(0.334)+16$

$=-30+16$ $\qquad\qquad$ $14>0$

$\boxed{-14<0}$

At $Q=5$, the profit is at maximum.

Put value of Q in Price and profit equation

$$P=50-2(5)$$

$$P=50-10$$

$$\boxed{P=40}$$

Now, put the value of Q in $\pi$.

$$\pi = -(5)^3 + 8(5)^2 - 5(5) - 15$$

$$\pi = -125 + 200 - 25 - 15$$

$$\boxed{\pi = 35}$$

b). When government provides subsidy of Rs 3 per unit, then find new value of P and Q.

Now, if govt. gives subsidy then,

$$\pi = TR - TC + S$$

$$\pi = 50Q - 2Q^2 - Q^3 + 10Q^2 - 55Q - 15 + 3Q$$

$$\pi = 50Q - 2Q^2 - Q^3 + 10Q^2 - 52Q - 15$$

$$\pi = -Q^3 + 8Q^2 - 2Q - 15$$

NC/

$$\frac{d\pi}{dQ} = \frac{d}{dQ}(-Q^3 + 8Q^2 - 2Q - 15)$$

$$\frac{d\pi}{dQ} = -3Q^2 + 16Q - 2 - 0$$

put $\frac{d\pi}{dQ} = 0$

$$-3Q^2 + 16Q - 2 = 0$$

$$-3Q^2 + 16Q - 2 = 0$$

$$3Q^2 - 16Q + 2 = 0$$

$$Q = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(3)(2)}}{2(3)}$$

$$Q = \frac{16 \pm \sqrt{256-24}}{6}$$

$$Q = \frac{16 \pm \sqrt{232}}{6}$$

$$Q = \frac{16 \pm 15.23}{6}$$

$$Q = \frac{16 + 15.23}{6} \qquad Q = \frac{16 - 15.23}{6}$$

$$\boxed{Q = 5.2} \qquad \boxed{Q = 0.1283}$$

SC|

$$\frac{d\pi^2}{dQ} = -6Q + 16$$

Put $Q = 5.2$ and $Q = 0.1283$ in above equation

$$= -6(5.2) + 16 \qquad = -6(0.12) + 16$$

$$= -31.2 + 16 \qquad = 15.4 + 16$$

$$-15.2 < 0 \qquad = 31.96 > 0$$

Hence, $\pi$ is maximum at

$Q = 5.2$

$P = 50 - 2(5.2)$ ; $\boxed{P = 39.6}$

$\pi = -(5.2)^3 + 8(5.2)^2 - 2(5.2) - 15$

$\pi = -140.608 + 216.32 - 10.4 - 15$

$\boxed{\pi = 50.312}$

| Before Subsidy | | | After Subsidy | | |
|---|---|---|---|---|---|
| P | Q | $\pi$ | P | Q | $\pi$ |
| 40 | 5 | 25 | 39.6 | 5.2 | 50.31 |