ECONOMIC APPLICATION OF DERIVATIVES

INTRODUCTION

In economics, particularly in micro-economics, the "*Marginal Analysis*" commands a great importance. Marginal means *any addition or change which occurs in one variable due to change in other variable.* It means that the concept of marginal depicts the rate of change or slope. While to show the rate of change in mathematics, the concept of derivative is used. Now we see the cases where the concept of derivative is applicable in economics.

Demand Function is: $Q = f(P)$

Slope of demand function = marginal demand function = derivative = $\frac{dQ}{dP}$

Utility Function is: $U = f(Q)$.

Derivative of this function is called *Marginal Utility* (MU) = $\frac{dU}{dQ}$.

Production function is: $Q = f(L)$

Derivative of production function is marginal product (MP) = $\frac{dQ}{dL}$.

Supply Function is: $Q = f(P)$

Slope of supply function = marginal supply function = derivative = $\frac{dQ}{dP}$.

Cost Function is: $C = f(Q)$.

Derivative of cost function is marginal cost (MC) = $\frac{dC}{dQ}$.

Revenue Function is: $R = f(Q)$.

Derivative of revenue function is marginal revenue (MR) = $\frac{dR}{dQ}$.

Consumption Function is: $C = f(Y)$.

Derivative of consumption function is called *marginal propensity to consume*

$$(MPC) = \frac{dC}{dY}.$$

Saving Function is: $S = f(Y)$.

Derivative of saving function is called *marginal propensity to save* (MPS) = $\frac{dS}{dY}$.

1. DEMAND FUNCTION AND ELASTICITY OF DEMAND

As we told earlier that demand function is Q = f(P). Thus its slope = $\frac{dQ}{dP}$. While average demand function = $\frac{Q}{P}$. If we divide the derivative by average demand function we get elasticity of demand.

EXAMPLE–1. If Q = 100 – 2P, while P = 25, we find elasticity of demand.

Solution. Now $Q = 100 - 2P \Rightarrow \frac{dQ}{dP} = 0 - 2 = -2$

$$E = \frac{dQ}{dP} \div \frac{Q}{P} = (-2) \div \frac{Q}{P} = 2 \div \frac{25}{5} = -\frac{50}{50} = -1$$

[As When P = 25 , Q = 100 – 2(25) = 50]

Thus ignoring –ve sign, E = 1

EXAMPLE–2. If $Q = 250 - P - 0.05P^2$, and P = 10, we find elasticity of demand.

Solution. Now $Q = 250 - P - 0.05\,P^2$

When P = 10, $Q = 250 - 10 - 0.05(10)^2 = 250 - 10 - 5 = 235$

Now $Q = 250 - P - 0.05\,P^2$, $\frac{dQ}{dP} = 0 - 1 - 0.05\,(2)P^{2-1} = -1 - 0.1\,P$

So When P = 10, $\frac{dQ}{dP} = -1 - 0.1(10) = -1 - 1 = -2$

$$E = \frac{dQ}{dP} \div \frac{Q}{P} = (-2) \div \frac{235}{10} = -2 \times \frac{10}{235} = -\frac{20}{235} = -\frac{4}{47}$$

Thus ignoring –ve sign, $E = \frac{4}{47}$ which is less than 1.

2. SUPPLY FUNCTION AND ELASTICITY OF SUPPLY

A supply function is Q = f (P), its slope or derivative = $\frac{dQ}{dP}$. While average supply function = $\frac{Q}{P}$. The elasticity of supply is obtained by dividing the derivative by average supply function. It is as :

$$E = \frac{dQ}{dP} \div \frac{Q}{P}$$

EXAMPLE–1. If $Q = 200 + 4P + 0.1\,P^2$, and P = 20, we find elasticity of supply.

Solution. Now $Q = 200 + 4P + 0.1\,P^2$

When P = 20, $Q = 200 + 4(20) + 0.1(20)^2 = 200 + 80 + 0.1(400)$
$= 200 + 80 + 40 = 320$

Now $Q = 200 + 4P + 0.1\,P^2$, $\frac{dQ}{dP} = 0 + 4 + 0.1\,(2)P^{2-1} = 4 + 0.2P$

So When $P = 20$, $\frac{dQ}{dP} = 4 + 0.2(20) = 4 + 4 = 8$

$$E = \frac{dQ}{dP} \div \frac{Q}{P} = (8) \div \frac{320}{20} = 8 \times \frac{20}{320} = \frac{160}{320} = \frac{1}{2}$$

Thus elasticity of supply $= \frac{1}{2}$ which is less than 1.

(Note, the same question is solved below by supposing $P = 10$, above it was solved with $P = 20$)

(b) Find the Elasticity of Supply at Price Rs.10/- and Rs.20/- per Unit, when supply equation is given as $q_s = 200 + 4P + 0.1P^2$ (UOP: 2014)

$q_s = 200 + 4(10) + 0.1(10)^2 = 250$

$q_s = 200 + 4P + 0.1P^2$, $dq_s / dP = 4 + 0.2P = 4 + 0.2(10) = 6$

$E_s = dq_s / dP \div q_s / P = 6 \div 250 / 10 = 6 \times 10 / 250 = 6 / 25 = 0.24$

EXAMPLE–2. If $13P - Qs = 27$, and $P = 3$, we find elasticity of supply.

Solution. Now $Q = 13P - 27$

When $P = 3$, $Q = 13(3) - 27 = 39 - 27 = 12$

Now $Q = 13P - 27 \Rightarrow \frac{dQ}{dP} = 13 - 0 = 13$

$$E = \frac{dQ}{dP} \div \frac{Q}{P} = (13) \div \frac{12}{3} = (13) \div 4 = \frac{13}{4} = 3.25$$

Thus elasticity of supply $= 3.25$ which is greater than 1.

SOLVED EXAMPLES OF ELASTICITY OF DEMAND AND ELASTICITY OF SUPPLY

EXAMPLE–1. If $Q = 300 - 3P - 0.2\,P^2$, and $P = 30$, we find elasticity of demand.

Solution. Now $Q = 300 - 3P - 0.2\,P^2$

When $P = 30$, $Q = 300 - 3(30) - 0.2(30)^2 = 300 - 90 - 0.2(900)$

$= 300 - 90 - 180 = 30$

Now $Q = 300 - 3P - 0.2\,P^2$

$\frac{dQ}{dP} = 0 - 3 - 0.2\,(2)P^{2-1} = -3 - 0.4P$

So When $P = 30$, $\frac{dQ}{dP} = -3 - 0.4(30) = -3 - 12 = -15$

$$E = \frac{dQ}{dP} \div \frac{Q}{P} = (-15) \div \frac{30}{30} = (-15) \div 1 = -15$$

Thus elasticity of demand = 15, ignoring –ve sign.

EXAMPLE–2. If $Q = 150 - 2P - 0.5\,P^2$, and $P = 10$, we find elasticity of demand.

Solution. Now $Q = 150 - 2P - 0.5\,P^2 = 150 - 20 - 0.5(100)$

When $P = 10$, $Q = 150 - 2(10) - 0.5(10)^2 = 150 - 20 - 50 = 80$

Now $Q = 150 - 2P - 0.5\,P^2$, $\frac{dQ}{dP} = 0 - 2 - 0.5\,(2)P^{2-1} = -2 - P$

C: Find Equilibrium Price ($\overline{P}$), Equilibrium … (Ed) and Elasticity of Supply (Es) in each of the followings :

14.	$Qs + 32 - 7P = 0$ $Qd - 128 + 9P = 0$	$\left[\begin{array}{l}\text{Ans: } \overline{P} = 10; \ \overline{Q} = 38 \\ Es = 1.84; \ Ed = 2.36\end{array}\right]$
15.	$P = 5 - \frac{X}{2}$ $6P = 6 + X$	$\left[\begin{array}{l}\text{Ans: } \overline{P} = 2; \ \overline{X} = 6 \\ Ed = 0.66; \ Es = 2\end{array}\right]$
16.	$Dx = 40 - P$ $Sx = -10 + \frac{3}{2}P$	$\left[\begin{array}{l}\text{Ans: } \overline{P} = 20; \ \overline{X} = 20 \\ Ed = 1; \ Es = 1.5\end{array}\right]$
17.	$Qd = 5 - P^2$ $Qs = 2P - 3$	$\left[\begin{array}{l}\text{Ans: } \overline{P} = 2; \ \overline{Q} = 1 \\ Ed = 8; \ Es = 4\end{array}\right]$
18.	$Qd = 10 - 3P^2$ $Qs = 4 + P$	$\left[\begin{array}{l}\text{Ans: } \overline{P} = 1.25; \ \overline{Q} = 5.3 \\ Ed = 1.76; \ Es = 0.23\end{array}\right]$
19.	$Qd = 10 - 3P$ $Qs = 2P^2 + 4P + 6$	$\left[\begin{array}{l}\text{Ans: } \overline{P} = 0.5; \ \overline{Q} = 8.5 \\ Ed = 0.17; \ Es = 0.35\end{array}\right]$
20.	$Q = 32 - 4P - P^2$ $P = \frac{Q}{20} + 1$ (UAJK:2015)	$\left[\begin{array}{l}\text{Ans: } \overline{P} = 2; \ \overline{Q} = 20 \\ Ed = 0.8; \ Es = 2\end{array}\right]$
21.	$Q = 10P + P^2$ $Q = 96 - 8P - 2P^2$	$\left[\begin{array}{l}\text{Ans: } \overline{P} = 3.4; \ \overline{Q} = 45.6 \\ Ed = 1.61; \ Es = 1.25\end{array}\right]$
22.	$Qd = -P^2 + 4$ (UOG:2010) $Qs = 4P - 1$	$\left[\begin{array}{l}\text{Ans: } \overline{P} = 1; \ \overline{Q} = 3 \\ Ed = 0.66; \ Es = 1.33\end{array}\right]$
23.	$Qd = -2P + 15$ $Qs = P + 3$	$\left[\begin{array}{l}\text{Ans: } \overline{P} = 4; \ \overline{Q} = 7 \\ Ed = 1\frac{1}{7}; \ Es = \frac{4}{7}\end{array}\right]$

3. UTILITY FUNCTION AND DERIVATIVE

As the utility function is : $U = f(Q)$

Its derivative will be marginal utility: $MU = \frac{dU}{dQ}$.

EXAMPLE-1. $U = 4Q - Q^2$, we find MU and construct schedule.

Solution. $MU = \frac{dU}{dQ} = 4 - 2Q$

To construct schedule we suppose the values of Q and put them in MU.

When Q = 1, $MU = 16 - 2Q = 4 - 2(1) = 4 - 2 = 2$
When Q = 2, $MU = 16 - 2Q = 4 - 2(2) = 4 - 4 = 0$
When Q = 3, $MU = 16 - 2Q = 4 - 2(3) = 4 - 6 = -2$

Q	MU
1	2
2	0
3	-2

EXAMPLE–2. $U = 16Q - 4Q^2$, we find MU and construct schedule.

Solution. $MU = \frac{dU}{dQ} = 16 - 8Q$

To construct schedule we suppose the values of Q and put them in MU.

When Q = 1, $MU = 16 - 8Q = 4 - 8(1) = 16 - 8 = 8$
When Q = 2, $MU = 16 - 8Q = 4 - 8(2) = 16 - 16 = 0$
When Q = 3, $MU = 16 - 8Q = 4 - 8(3) = 16 - 24 = -8$

Q	MU
1	8
2	0
3	-8

4. PRODUCTION FUNCTION AND DERIVATIVE

As the production function is: $Q = f(L)$

Its derivative will be marginal production of labor : $MP_L = \frac{dQ}{dL}$

while the average product of labor is: $AP_L = \frac{Q}{L}$

If the production function is: $Q = f(K)$

Its derivative will be marginal production of capital : $MP_K = \frac{dQ}{dK}$

while the average product of capital is : $AP_K = \frac{Q}{K}$.

EXAMPLE–1. $Q = 10L - 15L^2 + 3L^3$, we find AP_L, MP_L.

Solution. $AP_L = \frac{Q}{L} = \frac{10L - 15L^2 + 3L^3}{L} = \frac{10L}{L} - \frac{15L^2}{L} + \frac{3L^3}{L} = 10 - 15L + 3L^2$

$$Q = 10L - 15L^2 + 3L^3$$

$$MP_L = \frac{dQ}{dL} = 10(1)L^{1-1} - 15(2)L^{2-1} + 3(3)L^{3-1} = 10 - 30L + 9L^2$$

EXAMPLE–2. $Q = 20K - 30K^2 + 6K^3$, we find AP_K, MP_K.

Solution. $AP_K = \frac{Q}{K} = \frac{20K - 30K^2 + 6K^3}{K} = \frac{20K}{K} - \frac{30K^2}{K} + \frac{6K^3}{K} = 20 - 30K + 6K^2$

$$Q = 20K - 30K^2 + 6K^3$$

$$MP_K = \frac{dQ}{dK} = 10(1)K^{1-1} - 30(2)K^{2-1} + 6(3)K^{3-1} = 10 - 60K + 18K^2$$

5. COST FUNCTION AND DERIVATIVE

As cost function in its general form is: $C = f(X)$

i.e. $C = f(Q)$ or $TC = f(Q)$ where $Q = X$.

Its derivative will be marginal cost: $MC = \frac{dC}{dQ}$

The slope of MC is the second derivative of cost function. It is as :

$$\text{Slope of MC} = \frac{d^2C}{dQ^2} = \frac{d(MC)}{dQ}$$

The average cost function is called average cost (AC) = $AC = \frac{C}{Q}$

while the slope of AC is the derivative of $AC = \frac{d(AC)}{dQ}$

As we know that C = TFC + TVC

Thus in cost equation the term without Q represents TFC (Total Fixed Costs) while the terms with Q represent TVC (Total Variable Costs).

The $AVC = \frac{TVC}{Q}$, while AFC (Average Fixed Costs) is as: $AFC = \frac{TFC}{Q}$.

EXAMPLE. $C = Q^3 - 12Q^2 + 60\,Q$ **, we find** (IUBWR:2014/I)

(1) Average Cost (AC) and Marginal Cost (MC) functions,

(2) Slopes of AC and MC functions,

(3) At what value of Q, the MC = AC.

Solution. (1) $AC = \frac{C}{Q} = \frac{Q^3 - 12Q^2 + 60Q}{Q} = Q^2 - 12Q + 60$

$$C = Q^3 - 12\,Q^2 + 60\,Q,\ MC = \frac{dC}{dQ} = 3Q^2 - 24Q + 60$$

(2) $AC = Q^2 - 12\,Q + 60$

Slope of AC = Derivative of $AC = \frac{d(AC)}{dQ} = 2Q - 12$

$$MC = 3Q^2 - 24Q + 60$$

> Q. Find Marginal cost function when (a):
>
> $AC = 97/Q + 14 - 7Q + 3Q^2$
>
> $C = AC\,(Q)$
>
> $C = \left(\frac{97}{Q} + 14 - 7Q + 3Q^2\right)Q$
>
> $C = 97 + 14Q - 7Q^2 + 3Q^3$
>
> $MC = \frac{dC}{dQ} = 14 - 14Q + 9Q^2$
>
> (b): **(UOPR:2012,2016)**
>
> $AC - 13/Q - 0.75 - 0.85Q = 0$
>
> $AC = 13/Q - 0.75 - 0.85Q$
>
> $C = AC(Q) = \left(\frac{13}{Q} + 0.75 + 0.85Q\right)Q$
>
> $C = 13 + 0.75Q^2 + 0.85Q^2$
>
> $MC = \frac{dC}{dQ} = 1.5Q + 1.70Q.$
>
> (c): $AC = \frac{1}{2}Q^2 + Q + 7$
>
> $C = \frac{1}{2}Q^3 + Q^2 + 7Q$
>
> $MC = \frac{3}{2}Q^2 + 2Q + 7$

Slope of MC = Derivative of $MC = \frac{d(MC)}{dQ} = 6Q - 24$, (3) AC = MC gives

$$Q^2 - 12Q + 60 = 3Q^2 - 24Q + 60$$

$$3Q^2 - Q^2 - 24Q + 12Q + 60 - 60 = 0$$

$$2Q^2 - 12Q = 0 \quad \Rightarrow \quad 2Q(Q - 6) = 0$$

$$2Q = 0 \quad \Rightarrow Q = 0 \text{ and } Q - 6 = 0 \Rightarrow Q = 6$$

Thus at Q = 0, Q = 6 both AC and MC are equal.

EXAMPLE-1. If $C = Q^3 - 60Q^2 + 1800\,Q$, we find

(1) AC and MC functions, (UAJK:2012/A)(QAU:2016)

(2) Slopes of AC and MC functions,

(3) Moreover, without making diagram we show that MC curve crosses AVC at its minimum $\Rightarrow$ (MC = AVC).

Solution. (1) $AC = \dfrac{C}{Q} = \dfrac{Q^3 - 60Q^2 + 1800Q}{Q} = Q^2 - 60Q + 1800$

$C = Q^3 - 60Q^2 + 1800Q$

$MC = \dfrac{dC}{dQ} = 3Q^{3-1} - 60(2)Q^{2-1} + 1800(1)Q^{1-1} = 3Q^2 - 120Q + 1800Q$

(2) $AC = Q^2 - 60Q + 1800$

Slope of AC = derivative of AC = $\dfrac{d(AC)}{dQ} = 2Q - 60$

$MC = 3Q^2 - 120Q + 1800Q$

Slope of MC = derivative of MC = $\dfrac{d(AC)}{dQ} = 6Q - 120$

> Here is Example 1.
> $TC = TVC = Q^3 - 60Q^2 + 1800Q$
> $AVC = \dfrac{TVC}{Q} = \dfrac{Q^3 - 60Q^2 + 1800Q}{Q}$
> $AVC = Q^2 - 60Q + 1800$
> As $MC = 3Q^2 - 120Q + 1800$
> $MC = AVC \Rightarrow$
> $3Q^2 - 120Q + 1800 = Q^2 - 60Q + 1800 \Rightarrow$
> $3Q^2 - Q^2 - 120Q + 60Q + 1800 - 1800 = 0$
> $2Q^2 - 60Q + 0 = 0 \Rightarrow$
> $2Q(Q - 30) = 0 \Rightarrow 2Q = 0 \Rightarrow$
> $Q = 0, Q - 30 = 0 \Rightarrow Q = 30$
> Thus at Q = 0 and Q = 30 MC = AVC or MC crosses AVC at it minimum.

EXAMPLE–2. If $C = 0.001x^3 - 0.09x^2 + 20x$, we find

(1) AC and slope of AC function,

(2) MC and slope of MC function.

Solution. (1) $AC = \dfrac{C}{x} = \dfrac{0.001x^3 - 0.09x^2 + 20x}{x}$

$= 0.001x^2 - 0.09x + 20$

Slope of AC = $\dfrac{d(AC)}{dx} = 0.002x - 0.09$

(2) $C = 0.001x^3 - 0.09x^2 + 20x$

$MC = \dfrac{dC}{dx} = 0.001(3)x^{3-1} - 0.09(2)x^{2-1} + 20(1)x^{1-1}$

$MC = 0.003x^2 - 0.18x + 20$

Slope of MC = derivative of MC = $\dfrac{d(MC)}{dx} = 0.006x - 0.18$

EXAMPLE–3. If $C = 1000 + 25Q - 5Q^2 + Q^3$, we find

(1) AC and MC functions, (2) Slopes of AC and MC functions,

(3) AVC, (4) At what Q, AVC = MC. (UOH:2007/S)

Solution. (1) $C = 1000 + 25Q - 5Q^2 + Q^3$

$AC = \dfrac{C}{Q} = \dfrac{1000 + 25Q - 5Q^2 + Q^3}{Q} = \dfrac{1000}{Q} + 25 - 5Q + Q^2$

$C = 1000 + 25Q - 5Q^2 + Q^3$

$MC = \dfrac{dC}{dQ} = 0 + 25 - 10Q + 3Q^2 = 25 - 10Q + 3Q^2$

(2) $AC = \dfrac{1000}{Q} + 25 - 5Q + Q^2 = 1000Q^{-1} + 25 - 5Q + Q^2$

Slope of AC = derivative of AC = $\frac{d(AC)}{dQ}$ = $2Q - 5 - 1000\,Q^{-2} + 0$

$$= 2Q - 5 - \frac{1000}{Q^2}$$

MC = $25 - 10Q + 3Q^2$ Slope of MC = derivative of MC = $\frac{d(MC)}{dQ}$ = $6Q - 10$

(3) $C = 1000 + 25Q - 5Q^2 + Q^3$

The terms with Q represent TVC, while the term without Q represents TFC, accordingly TVC = $25Q - 5Q^2 + Q^3$

$$AVC = \frac{TVC}{Q} = \frac{25Q - 5Q^2 + Q^3}{Q} = 25 - 5Q + Q^2$$

> Q: If
> $TC = 200Q - 24Q^2 + Q^3$
> (i) Find MC (ii) AC
> (IUBWR.2015/3rd year)

(4) Equating AVC = MC

$$25 - 5Q + Q^2 = 25 - 10Q + 3Q^2$$

$$3Q^2 - Q^2 - 10Q + 5Q + 25 - 25 = 0 \Rightarrow 2Q^2 - 5Q = 0 \Rightarrow Q(2Q - 5) = 0$$

$$Q = 0,\ 2Q - 5 = 0 \Rightarrow 2Q = 5 \Rightarrow \quad Q = 2.5$$

Thus at Q = 0, Q = 2.5 both AVC and MC are equal which can be checked by putting these values of Q in AVC and MC.

(b): $C = AC \times Q\,(1.5Q + 4 + \frac{46}{Q})\,Q = 1.5Q^2 + 4Q + 46$, $MC = \frac{dC}{dQ} = 3Q + 4$.

6. REVENUE FUNCTION AND DERIVATIVE

As revenue function is: $R = f(Q)$

Average revenue function will be: $AR = \frac{R}{Q}$

Marginal revenue function will be : $MR = \frac{dR}{dQ}$

The slope of AR = derivative of AR $= \frac{d(AR)}{dQ}$

Slope of MR = derivative of MR $= \frac{d(MR)}{dQ}$

which is also second derivative of revenue function.

Q: If $AR = 60 - 2Q$, (i) find TR function (ii) MR function (iii) slope of AR function, (iv) slope of MR function (UOG:2011)

Q: If $P = 30\text{-}2Q$, find (a) TR function (b) MR and AR function (c) show MR and AR function graphically. (UOH:2011)

Q. If $AR = 3 - 2.5Q$ Calculate MR: (UOPR:2016)

$R = AR(Q) = 3Q - 2.5Q^2$

$MR = \frac{dR}{dQ} = 3 - 5Q$

EXAMPLE–1. $R = 250Q - 3Q^2$, we find

(1) AR and MR functions

(2) Slopes of AR and MR functions and their relationships

(3) At what value of Q, the MR and P will be zero.

Solution. (1) $R = 250Q - 3Q^2$

$$AR = \frac{R}{Q} = \frac{250Q - 3Q^2}{Q} = 250 - 3Q$$

Now $R = 250Q - 3Q^2 \Rightarrow MR = \frac{dR}{dQ} = 250 - 6Q$

(2) $AR = \frac{R}{Q} = \frac{250Q - 3Q^2}{Q} = 250 - 3Q$

Slope of AR = Derivative of AR

$$= \frac{d(AR)}{dQ} = -3$$

Slope of MR = Derivative of MR

$$= \frac{d(MR)}{dQ} = -6$$

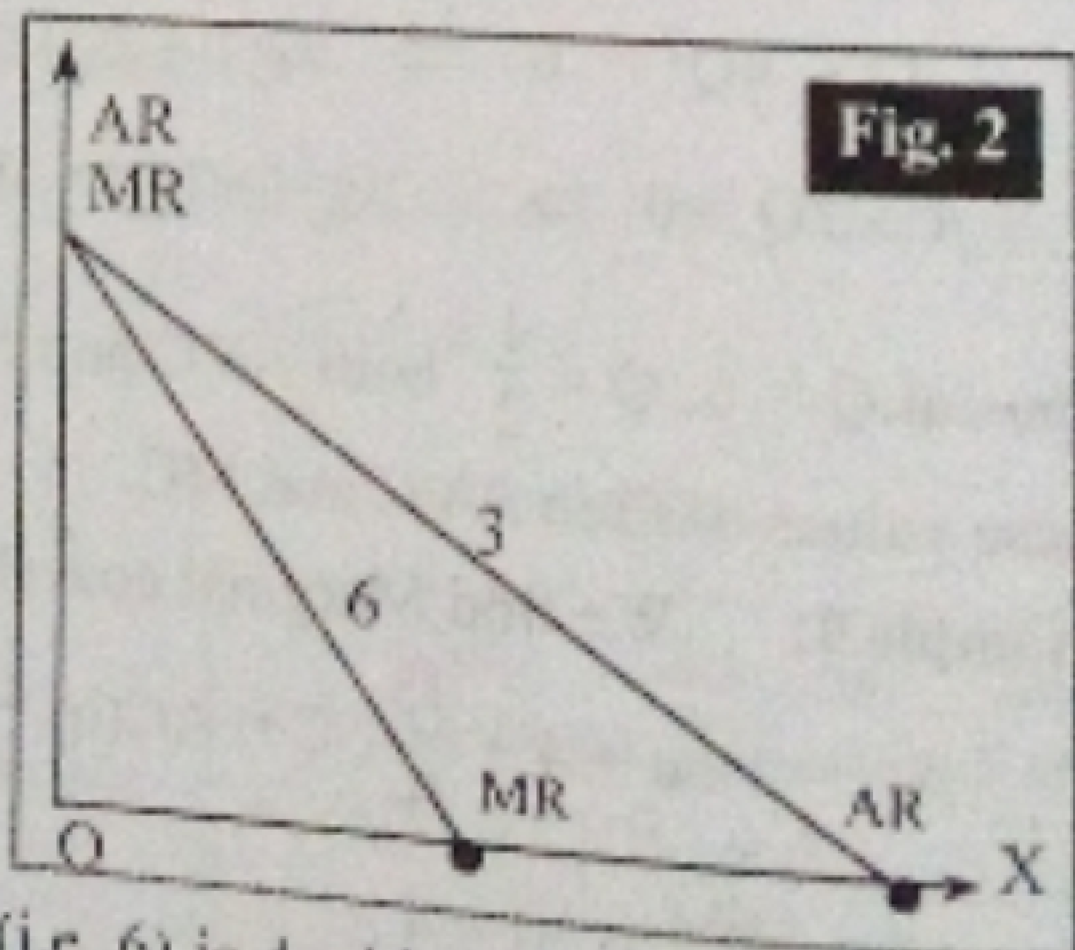

Fig. 2

Ignoring the –ve sign we see that slope of MR(i.e. 6) is double to that of slope of AR (i.e. 3). Moreover, we find that MR falls double to that of AR as shown by Fig. 2.

(3) Now we find Q where MR = 0.

$MR = 250 - 6Q = 0 \Rightarrow 6Q = 250 \Rightarrow Q = \frac{250}{6} = 41.67$

Thus at $Q = 41.67$, $MR = 0$.

Again, it is reminded that $P = AR$

$P = 250 - 3Q = 0$ and $3Q = 250 \Rightarrow Q = \frac{250}{3} = 83.33$

Thus at $Q = 83.33$, P or AR is zero.

EXAMPLE–2. If $P = 50 - 2.5\,Q$, find **(1) MR, AR, R functions**

(2) Slopes of AR and MR functions (3) At what Q, P and MR are zero.

Solution. (1) $R = PQ = (50 - 2.5Q)Q = 50Q - 2.5\,Q^2$

$AR = \frac{R}{Q} = \frac{50Q - 2.5Q^2}{Q} = 50 - 2.5\,Q \Rightarrow$ Slope of $AR = \frac{d(AR)}{dQ} = -2.5$

$MR = \frac{dR}{dQ} = 50 - 5Q \Rightarrow$ Slope of $MR = \frac{d(MR)}{dQ} = 0 - 5 = -5$

Ignoring the negative signs or taking the absolute values we find that the slope of MR(i.e. 5) is double the slope of AR (i.e. 2.5).

(3) *Now we find Q where MR = 0.*

$MR = 50 - 5Q = 0$

$5Q = 50 \Rightarrow Q = \frac{50}{5} = 10$

Thus at $Q = 10$, $MR = 0$.

Now we find Q where P = 0

Again, it is reminded that $P = AR$

$P = 50 - 2.5Q = 0$

$2.5Q = 50 \Rightarrow Q = \frac{50}{2.5} = 20$

Thus at $Q = 20$, P or AR is zero.

EXAMPLE–3. If $Qd = 100 - 2P$, find **(1) R, AR, MR functions**

(2) Slopes of AR and MR functions (3) Q where, P and MR are zero.

Solution. (1) Ignoring d and solving for P

$$Q = 100 - 2P \Rightarrow 2P = 100 - Q \Rightarrow P = \frac{100 - Q}{2} \Rightarrow P = 50 - \frac{1}{2}\,Q$$

This must be remembered that in revenue function Q is the independent variable while R is the dependent variable. Therefore, we write

$R = PQ = \left(50 - \frac{1}{2}\,Q\right)Q = 50Q - \frac{1}{2}\,Q^2$

$AR = \frac{R}{Q} = \frac{50Q - \frac{1}{2}Q^2}{Q} = 50 - \frac{1}{2}\,Q$

Slope of $AR = \frac{d(AR)}{dQ} = -\frac{1}{2}$

$R = 50Q - \frac{1}{2}\,Q^2$

$MR = \frac{dR}{dQ} = 50 - Q$

Slope of $MR = \frac{d(MR)}{dQ} = 0 - 1 = -1$

Ignoring the negative signs or taking the absolute values we find that the slope of MR(i.e. 1) is double the slope of AR (i.e. 0.5).

(3) *Now we find Q where MR = 0*

$MR = 50 - Q = 0$

$\Rightarrow Q = 50$

Thus at $Q = 50$, $MR = 0$.

Now we find Q where P = 0

Again, it is reminded that $P = AR = 50 - \frac{1}{2}\,Q = 0$

$P = 50 - \frac{1}{2}\,Q = 0 \Rightarrow \frac{1}{2}\,Q = 50 \Rightarrow Q = 100$

EXERCISE – 13

A: With the following cost functions, find (1) AC and MC functions (2) Slopes of AC and MC functions (3) At what values of Q, MC = AVC ? :

1.	$TC = Q^3 - 2Q^2 + 9Q + 650$	[*Ans.* MC = AVC; Q = 1; 0]
2.	$TC = \frac{1}{3}Q^3 - 4.5Q^2 + 14Q + 22$	[*Ans.* MC = AVC; Q = 6.75; 0]
3.	$C = 1000 + 100x - 10x^2 + x^3$	[*Ans.* MC = AVC; $x = 5; 0$]
4.	$TC = 500 + 3x - 2x^2 + x^3$	[*Ans.* MC = AVC; $x = 1; 0$]
5.	$TC = 3Q^3 - 7Q^2 + 12Q + 12$	[*Ans.* MC = AVC; Q = 7/6; 0]

B: With the following Revenue Functions find (1) AR and MR functions (2) Slopes of AR and MR functions (3) At what Q, P = 0 and MR = 0 ?

6.	$4P = 200 - Q$	[Ans. (MR = 0; Q = 100); (AR = P = 0; At Q = 200)]

7.	$5P = 75 - Q$	$\left[\text{Ans.}\begin{pmatrix} MR = 0 \\ Q = 37.5 \end{pmatrix}; \begin{pmatrix} AR = P = 0 \\ At\ Q = 75 \end{pmatrix}\right]$
8.	$TR = 75Q - 4Q^2$	$\left[\text{Ans.}\begin{pmatrix} MR = 0 \\ Q = 9.37 \end{pmatrix}; \begin{pmatrix} AR = P = 0 \\ At\ Q = 18.74 \end{pmatrix}\right]$
9.	$P = 12 - Q$	$\left[\text{Ans.}\begin{pmatrix} MR = 0 \\ Q = 6 \end{pmatrix}; \begin{pmatrix} AR = P = 0 \\ At\ Q = 12 \end{pmatrix}\right]$
10.	$2Q = 30 - P$	$\left[\text{Ans.}\begin{pmatrix} MR = 0 \\ Q = 7.5 \end{pmatrix}; \begin{pmatrix} AR = P = 0 \\ At\ Q = 15 \end{pmatrix}\right]$
11.	$5Q = 30 - P$	$\left[\text{Ans.}\begin{pmatrix} MR = 0 \\ Q = 3 \end{pmatrix}; \begin{pmatrix} AR = P = 0 \\ At\ Q = 6 \end{pmatrix}\right]$
12.	$Q = 50 - \frac{1}{2}P$	$\left[\text{Ans.}\begin{pmatrix} MR = 0 \\ Q = 25 \end{pmatrix}; \begin{pmatrix} AR = P = 0 \\ At\ Q = 50 \end{pmatrix}\right]$
13.	$Q = 20 - 2P$	$\left[\text{Ans.}\begin{pmatrix} MR = 0 \\ Q = 10 \end{pmatrix}; \begin{pmatrix} AR = P = 0 \\ At\ Q = 20 \end{pmatrix}\right]$
14.	$Qd = 60 - 3P$	$\left[\text{Ans.}\begin{pmatrix} MR = 0 \\ Q = 30 \end{pmatrix}; \begin{pmatrix} AR = P = 0 \\ At\ Q = 60 \end{pmatrix}\right]$

7. CONSUMPTION FUNCTION AND DERIVATIVE

The consumption expenditures depend upon income of the people. It is as : $C = f(Y)$. The average consumption function is called $APC = \frac{C}{Y}$, while the derivative or marginal consumption function is called $MPC = \frac{dC}{dY}$. The rate of change of APC is called slope of $APC = \frac{d(APC)}{dY}$, while the slope of $MPC = \frac{d(MPC)}{dY}$.

8. SAVING FUNCTION AND DERIVATIVE

The savings depend upon income. It is as : $S = f(Y)$. The average saving function is called $APS = \frac{S}{Y}$, while the derivative or marginal saving function is called $MPS = \frac{dS}{dY}$. The rate of change of APS is called slope of $APS = \frac{d(APS)}{dY}$, while the slope of $MPS = \frac{d(MPS)}{dY}$.

EXAMPLES OF CONSUMPTION AND SAVING FUNCTIONS

EXAMPLE–1. If $C = 40 + 0.6Y$, find

(1) APC function, slope of APC function

(2) MPC function and slope of MPC function

(3) **Value of multiplier (K).**

Solution. $C = 40 + 0.6Y$

> Q: If $C = 100 + 0.6Y_d$, $Y_d = Y - T$
> $T = 50$ then find (a): APC function and rate of change of APC function
> (b): MPC function and rate of change of MPC function. (UOH: 2006)