

CONSUMPTION'S FUNCTIONAL EQUATION

According to Keynes, consumption depends upon income.

Consumption's standard equation is as: $C = C_0 + cY$

where C_0 is the autonomous consumption, cY represents induced consumption and c = marginal propensity to consume (M.P.C.)

EXAMPLE. $C - 0.6Y - 40 = 0$. While $Y = 0, 50, 100, 150$, we find different levels of consumption with the above equation.

Converting the above equation in standard form: $C = 40 + 0.6 Y$

Putting the values of Y

$$\text{When } Y = 0, \quad C = 40 + 0.6(0) = 40$$

$$\text{When } Y = 50, \quad C = 40 + 0.6(50) = 40 + 30 = 70$$

$$\text{When } Y = 100, \quad C = 40 + 0.6(100) = 40 + 60 = 100$$

$$\text{When } Y = 150, \quad C = 40 + 0.6(150) = 40 + 90 = 130$$

Thus we have following pairs of values of Y and C .

Y	0	50	100	150
C	40	70	100	130

With these values of Y and C we can construct the graph of consumption.

SAVING'S FUNCTIONAL EQUATION

According to Keynes, savings depend upon income of people.

Saving's standard equation is as: $S = -S_0 + sY$

where $-S_0$ = autonomous savings, sY = induced saving and s = marginal propensity to save (M.P.S.)

EXAMPLE. $S + 40 - 0.4Y = 0$.

While $Y = 0, 50, 100, 150$, we find different levels of savings with the above equation.

Converting the above equation in standard form: $S = -40 + 0.4 Y$

Putting the values of Y

$$\text{When } Y = 0, \quad S = -40 + 0.4(0) = -40$$

When $Y = 50$, $S = -40 + 0.4(50) = -40 + 20 = -20$
 When $Y = 100$, $S = -40 + 0.4(100) = -40 + 40 = 0$
 When $Y = 150$, $S = -40 + 0.4(150) = -40 + 60 = 20$
 Thus we have following pairs of values of Y and S .

Y	0	50	100	150
S	-40	-20	0	20

With these values of Y and S we can construct the graph of savings.

INVESTMENT'S FUNCTIONAL EQUATION

According to Clark, investment depends upon income.

Investment's standard equation is as: $I = I_0 + eY$

where I_0 = autonomous investment, eY = induced investment, i.e., investment increases along with increase in income and e = marginal propensity to investment (M.P.I.)

EXAMPLE. $I = 40 + 0.2Y$. While $Y = 0, 100, 200, 300$, we find different levels of investment with the above equation.

Putting the values of Y : $I = 40 + 0.2Y$

When $Y = 0$, $I = 40 + 0.2(0) = 40$

When $Y = 100$, $I = 40 + 0.2(100) = 40 + 20 = 60$

When $Y = 200$, $I = 40 + 0.2(200) = 40 + 40 = 80$

When $Y = 300$, $I = 40 + 0.2(300) = 40 + 60 = 100$

Thus we have following pairs of values of Y and I .

Y	0	100	200	300
I	40	60	80	100

With these values of Y and I we can construct the graph of investment function.

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DETERMINATION OF EQUILIBRIUM LEVEL OF N. I.

The theory of NI determination is attributed to Prof. Keynes. Accordingly, NI is determined where

1. Savings are equal to investment
2. Aggregate demand is equal to Aggregate supply

1. NI determination in a Two Sector Economy — Saving and Investment Method

First we present equilibrium in general form:

As $S = -S_0 + sY$, while $I = I_0$ (autonomous investment)

At equilibrium: $S = I$. Putting the values of S and I : $-S_0 + sY = I_0$

$$sY = I_0 + S_0 \Rightarrow \bar{Y} = \frac{I_0 + S_0}{s} \Rightarrow \bar{Y} = \frac{1}{s} (I_0 + S_0) \text{ where } \bar{Y} \text{ is given the}$$

name of equilibrium level of national income (NI). Here $s = \text{MPS}$.

Thus the last equation shows the equilibrium level of NI in two sector economy.

Q. Given the following investment function: $I = 130 + 0.25Y$, (i) find the investment level at income of Rs 300, (ii) find the income at which investment is Rs. 200 (iii) draw the graph of the function. (UOH:2007/S)

(i): $I = 130 + 0.25(300) = 205$
 (ii): $200 = 130 + 0.25Y \Rightarrow 0.25Y = 200 - 130, 0.25Y = 70 \Rightarrow Y = \frac{70}{0.25} = 280.$

EXAMPLE. If $S = -40 + 0.4Y$, $I = 80$, we find equilibrium level of NI and prove that at equilibrium level of NI savings are equal to investment. the value of multiple (K) is also found out.

At equilibrium : $S = I$

$$-40 + 0.4Y = 80 \Rightarrow 0.4Y = 80 + 40 \Rightarrow Y = \frac{120}{0.4} = 300$$

$$S = -40 + 0.4Y = -40 + 0.4(300) = -40 + 120 = 80$$

Here we find the value of investment multiplier (K_I) which is as: $\frac{1}{MPS} = \frac{1}{0.4} = 2.5$

(Note: The proof of this formula is available in my book Principles of Economics II)

By assuming different values of Y and putting them in S we get different values of S and then plotting we obtain Fig. 5.

When $Y = 0$, $S = -40 + 0.4(0) = -40$

When $Y = 100$, $S = -40 + 0.4(100) = -40 + 40 = 0$

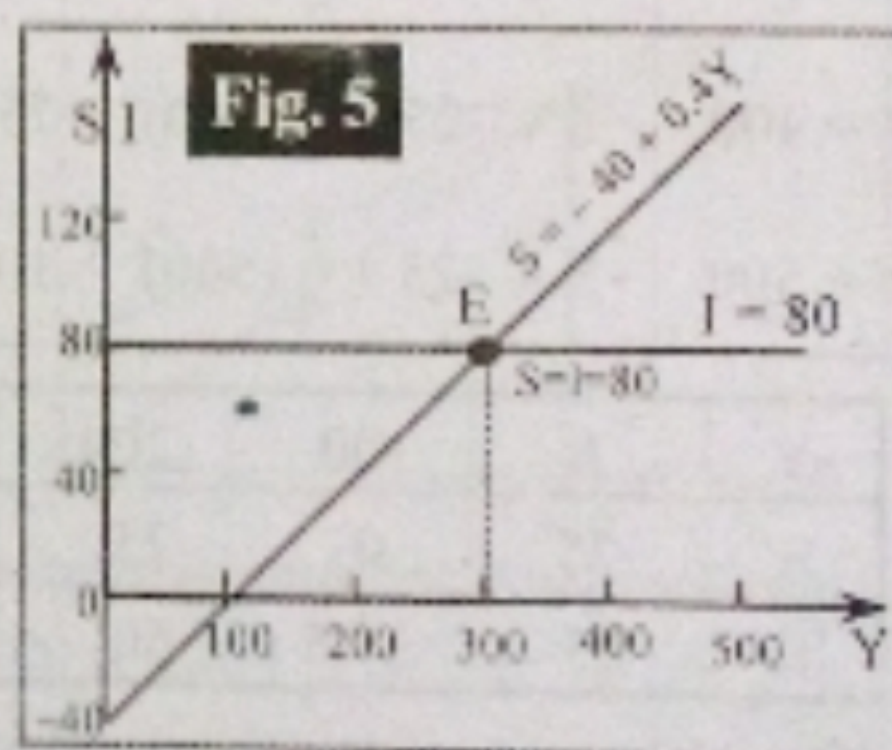
When $Y = 200$, $S = -40 + 0.4(200) = -40 + 80 = 40$

When $Y = 300$, $S = -40 + 0.4(300) = -40 + 120 = 80$

When $Y = 400$, $S = -40 + 0.4(400) = -40 + 160 = 120$

Y	0	100	200	300	400
S	-40	0	40	80	120
I	80	80	80	80	80

By plotting these pairs of values we get Fig. 5. The Fig. shows that at equilibrium level of NI (300) the savings are equal to investment (80= 80).



Now we include **induced investment** (eY). Thus investment function in its general form will be as: $I = I_0 + eY$, $0 < e < 1$. While $S = -S_0 + sY$

At equilibrium: $S = I$. Putting the values of I and S, we have

$$-S_0 + sY = I_0 + eY \Rightarrow sY - eY = S_0 + I_0$$

$$Y(s - e) = S_0 + I_0 \Rightarrow \bar{Y} = \frac{S_0 + I_0}{s - e} \Rightarrow \bar{Y} = \frac{1}{s - e} (S_0 + I_0). \text{ Here}$$

$s = MPS$ and $e = MPI$.

The last equation also shows equilibrium level of NI in a two sector economy in the presence of induced investment.

EXAMPLE. If $S = -25 + \frac{1}{4}Y$, $I = 25 + \frac{1}{8}Y$

We find equilibrium level of NI and show the equality between S and I. We also find the value of super multiplier $K_s = \frac{1}{MPS - MPI}$.

(Note: The proof of this formula is available in my book of Principles of Economics II)

At equilibrium: $S = I$. Putting the values of S and I and solving for Y , we have

$$-25 + \frac{1}{4} Y = 25 + \frac{1}{8} Y$$

$$-\frac{1}{8} Y + \frac{1}{4} Y = 25 + 25$$

$$\frac{1}{8} Y = 50 \Rightarrow Y = 50(8) = 400$$

Putting the value $Y = 400$ in S and I

$$-25 + \frac{1}{4} Y = 25 + \frac{1}{8} Y$$

$$-25 + \frac{1}{4}(400) = 25 + \frac{1}{8}(400)$$

$$-25 + 100 = 25 + 50 \Rightarrow 75 = 75$$

Assuming different values of Y and putting them in S and I equations, we get

$Y = 0$	$S = -25 + \frac{1}{4}(0) = -25$	$I = 25 + \frac{1}{8}(0) = 25$
$Y = 100$	$S = -25 + \frac{1}{4}(100) = 0$	$I = 25 + \frac{1}{8}(100) = 37.5$
$Y = 200$	$S = -25 + \frac{1}{4}(200) = 25$	$I = 25 + \frac{1}{8}(200) = 50$
$Y = 300$	$S = -25 + \frac{1}{4}(300) = 50$	$I = 25 + \frac{1}{8}(300) = 62.5$
$Y = 400$	$S = -25 + \frac{1}{4}(400) = 75$	$I = 25 + \frac{1}{8}(400) = 75$
$Y = 500$	$S = -25 + \frac{1}{4}(500) = 100$	$I = 25 + \frac{1}{8}(500) = 87.5$

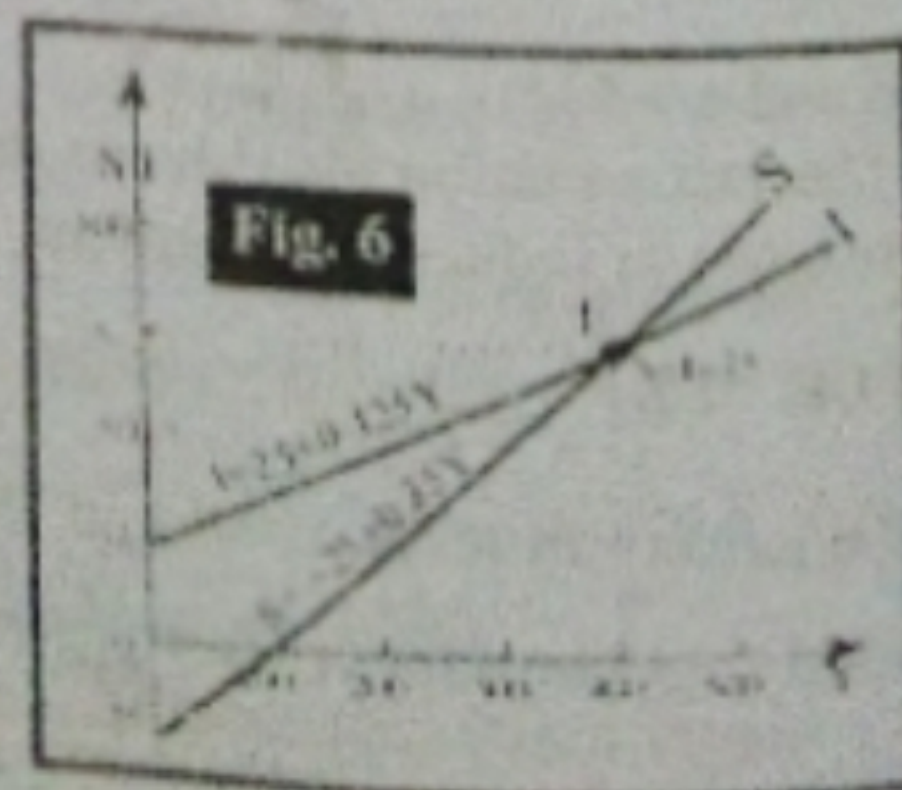
$-Y$	0	100	200	300	400	500
S	-25	0	25	50	75	100
I	25	37.5	50	62.5	75	87.5

The value of multiplier is also found here. But it is told that in such case the multiplier is not the simple one, rather it is super multiplier (K_s) whose formula is as:

$K_s = \frac{1}{s - e}$. Here $s = \text{MPS}$ which is available in saving function, i.e., $= \frac{1}{4} = 0.25$ and $e = \text{MPI}$ which is available in investment function, i.e., $= \frac{1}{8} = 0.125$. Thus

$$K_s = \frac{1}{s - e} = \frac{1}{0.25 - 0.125} = 8.$$

With the help of values of Y and S and Y and I we construct Fig. 6. This figure shows that at $Y = 400$, S and I are equal, i.e. when $Y = 400$, $S = 75 = I$.



2. NI DETERMINATION IN A TWO SECTOR ECONOMY

Aggregate Demand and Aggregate Supply Method OR Consumption and Investment Method

We suppose that investment is autonomous ($I = I_0$) while consumption equation is as:

$$C = C_0 + cY$$

Putting the values of C and I in NI equation, i.e., $Y = C + I$

$$Y = C + I \Rightarrow Y = C_0 + cY + I_0 \Rightarrow Y - cY = C_0 + I_0$$

$$Y(1 - c) = C_0 + I_0 \Rightarrow Y = \frac{C_0 + I_0}{1 - c} \Rightarrow Y = \frac{1}{1 - c} (C_0 + I_0)$$

The last equation shows equilibrium level of NI. Here $c = \text{MPC}$.

EXAMPLE. If $C = 40 + 0.6Y$, $I = 80$. We find the equilibrium level of NI and prove that $Y = C + I$. Also find the value of multiplier (K).

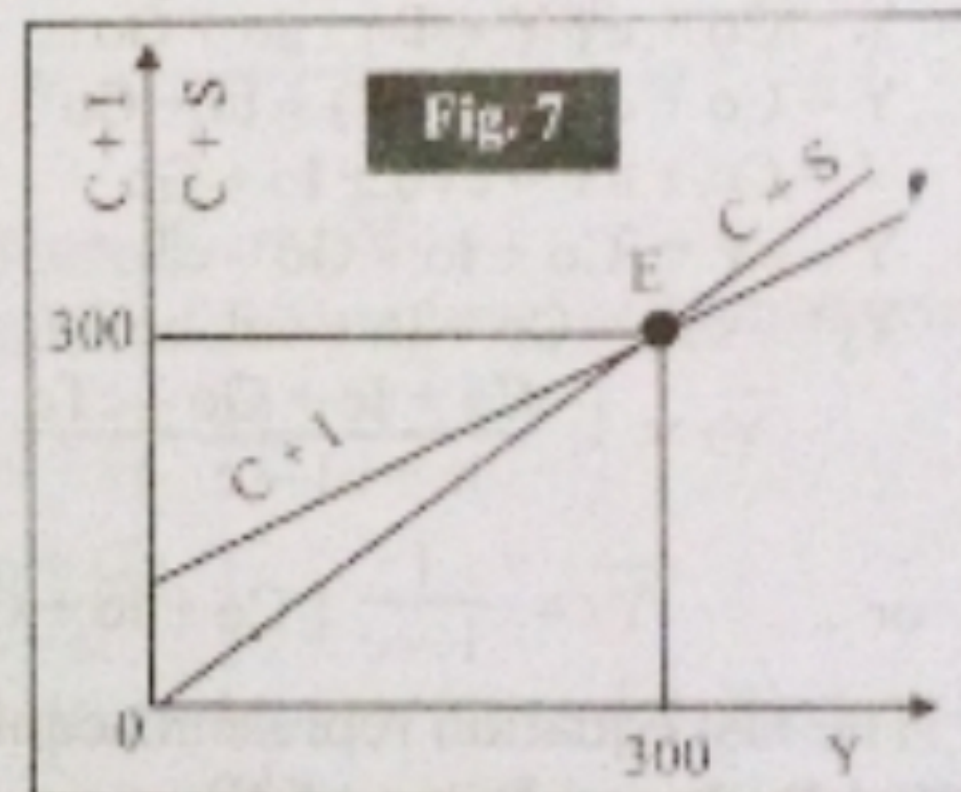
Putting the values of Y and C and solving for Y

$$Y = 40 + 0.6Y + 80$$

$$Y - 0.6Y = 40 + 80$$

$$Y(1 - 0.6) = 120$$

$$Y(0.4) = 120 \Rightarrow Y = \frac{120}{0.4} = 300$$



Putting $Y = 300$ in C,

$$C = 40 + 0.6(300) = 40 + 180 = 220$$

Putting the value of Y, C and I, we have

$$Y = C + I \Rightarrow 300 = 220 + 80 \Rightarrow 300 = 300$$

Here value of investment multiplier (K_I) is also calculated: (Its proof is in my book Principles of Economics II)

$$K_I = \frac{1}{1 - \text{MPC}} = \frac{1}{1 - 0.6} = \frac{1}{0.4} = 2.5$$

Now we include **induced investment** (eY). Thus the investment function in its standard form will be as: $I = I_0 + eY$ and $C = C_0 + cY$

Putting the values of C and I to get equilibrium level of NI in its general form.

$$Y = C_0 + cY + I_0 + eY \Rightarrow Y - cY - eY = C_0 + I_0 \Rightarrow Y(1 - c - e) = C_0 + I_0 \Rightarrow$$

$$Y = \frac{C_0 + I_0}{1 - c - e} \Rightarrow Y = \frac{1}{1 - c - e} (C_0 + I_0)$$

This equation shows equilibrium level of NI.

EXAMPLE. If $C = 40 + 0.6Y$, $I = 80 + 0.2Y$. We find the equilibrium level of NI and prove that $Y = C + I$. We also find the value of super multiplier (K_s).

Putting the values of Y and C and solving Putting the value of Y, C and I, we have for Y

$$Y = C + I$$

$$Y = 40 + 0.6Y + 80 + 0.2Y$$

$$Y - 0.6Y - 0.2Y = 40 + 80$$

$$Y(1 - 0.6 - 0.2) = 120$$

$$Y(0.2) = 120 \Rightarrow Y = \frac{120}{0.2} = 600$$

$$600 = [40 + 0.6(600)] + [80 + 0.2(600)]$$

$$600 = [40 + 360] + [80 + 120]$$

$$600 = 400 + 200 \Rightarrow 600 = 600$$

$$K_s = \frac{1}{1 - c - c} = \frac{1}{1 - 0.6 - 0.2} = 5$$

(Note: The proof of this formula is available in my book Principles of Economics II)
Moreover, at this level of NI (600), savings are also equal to investment (200),
i.e., $S = I$, i.e. $200 = 200$

$$i.e., Y = 600$$

3. NI DETERMINATION IN 3 SECTOR ECONOMY When Taxes are Autonomous ($T = T_o$).

As NI in 3 sector of economy is

$$Y = C + I + G$$

Putting the values in Y

$$Y = C_o + cY_d + I_o + G_o$$

$$Y = C_o + c(Y - T) + I_o + G_o$$

$$Y = C_o + c(Y - T_o) + I_o + G_o$$

$$Y = C_o + cY - cT_o + I_o + G_o$$

$$Y - cY = C_o + I_o + G_o - cT_o$$

$$Y(1 - c) = C_o + I_o + G_o - cT_o$$

$$Y = \frac{C_o + I_o + G_o - cT_o}{1 - c}$$

Whereas

$$C = C_o + cY_d$$

$$Y_d = Y - T$$

$$T = T_o$$

$$G = G_o$$

$$I = I_o$$

where $Y_d =$ disposable income

$$\text{or } \bar{Y} = \frac{1}{1 - c} [C_o + I_o + G_o - cT_o]$$

The last equation represents equilibrium level of NI in 3 sector of economy and it is called **Reduced Form of NI**.

Now we find equilibrium level of consumption.

$$C = C_o + cY_d \quad \bar{C} = c[\bar{Y} - T]$$

Putting the values of \bar{Y} and T

$$\bar{C} = C_o + c \left[\frac{C_o + I_o + G_o - cT_o}{1 - c} \right] - T_o$$

The last equation shows the equilibrium level of consumption.

EXAMPLE. If $G = G_o = 20$, $T_o = 50$, $I_o = 40$, $c = 0.6$, $C_o = 100$, we find equilibrium level of NI, and value of tax multiplier (K_T).

By putting the above values in \bar{Y} , we get

$$\bar{Y} = \frac{1}{1 - c} [C_o + I_o + G_o - cT_o] = \frac{1}{1 - 0.6} [100 + 40 + 20 - (0.6)(50)]$$

$$= \frac{1}{0.4} \times 130 = 325. \text{ This is equilibrium level of NI. } K_o = \frac{1}{1 - c} = \frac{1}{1 - 0.6} = 2.5$$

$$\text{Here } K_T = \frac{-c}{1 - c} = \frac{-0.6}{1 - 0.6} = -1.5. \quad Y_d = Y - T_o = 325 - 50 = 275$$

(The proof of K_T and K_o are available in my book Principles of Economics II)

Q. If $C = 20 + 0.8 Y_d$, $I_o = 40$
 $G_o = 30$ and $T_o = 30$ (i) find
equilibrium level of national
income and consumption (ii)
also prove $Y = C + I + G$
(OAU: 2015)

$$\bar{C} = 100 + 0.6 Y_d = 100 + 0.6(275) = 265, \quad \bar{S} = Y_d - C = 275 - 265 = 10$$

$$\bar{Y} = C + I + G \Rightarrow 325 = 265 + 40 + 20 = 325$$

$$S + T = I_0 + G_0 \Rightarrow 10 + 50 = 40 + 20 \Rightarrow 60 = 60$$

Thus we proved that at equilibrium level ($Y = 325$), $S + T = I + G$ and $Y = C + I + G$.

3 - A. NI DETERMINATION IN 3 SECTOR ECONOMY — When Induced Taxes are included.

NI in 3 sector economy gives

$$Y = C + I + G$$

$$Y = C_0 + cY_d + I_0 + G_0$$

$$Y = C_0 + c[Y - T] + I_0 + G_0$$

$$Y = C_0 + c[Y - tY] + I_0 + G_0$$

$$Y = C_0 + cY - ctY + I_0 + G_0$$

$$Y - cY + ctY = C_0 + I_0 + G_0$$

$$\text{as } I = I_0, G = G_0$$

$$C = C_0 + cY_d$$

$$Y_d = Y - T$$

$$T = tY$$

Here $t =$ marginal propensity to tax (MPT). Its value is as: $0 < t < 1$.

$$Y(1 - c + ct) = C_0 + I_0 + G_0 \Rightarrow \bar{Y} = \frac{1}{1 - c + ct} [C_0 + I_0 + G_0]$$

This is equilibrium level of NI. As $T = tY$ then $\bar{T} = t\bar{Y}$

This is equilibrium level of taxes. Now we find equilibrium level of consumption.

$$C = C_0 + c[\bar{Y} - \bar{T}] \Rightarrow \bar{C} = C_0 + c \left[\frac{C_0 + I_0 + G_0}{1 - c + ct} \right] - t\bar{Y}$$

$$\bar{C} = C_0 + c \left[\frac{C_0 + I_0 + G_0}{1 - c + ct} - t \left(\frac{C_0 + I_0 + G_0}{1 - c + ct} \right) \right]$$

EXAMPLE. If $C = 40 + 0.6Y_d$, $Y_d = Y - T$, $T = tY$, $I = 80$, $G = 20$, $t = 0.2$. Find equilibrium level of income (\bar{Y}) equilibrium level of consumption (\bar{C}) and value of multiplier. Then

putting these values in \bar{Y} : $\bar{Y} = \frac{1}{1 - c + ct} [C_0 + I_0 + G_0]$

$$\bar{Y} = \frac{1}{1 - 0.6 + 0.6(0.2)} [40 + 80 + 20] = \frac{1}{0.52} \times 140 = 269.23$$

This is equilibrium level of NI.

$$(i) \quad \bar{C} = C_0 + c[\bar{Y} - \bar{T}] = 40 + 0.6 [269.23 - 53.83] \\ = 40 + 0.6 [215.40] = 40 + 129.24 = 169.24$$

This is equilibrium level of C.

$$(ii) \quad \bar{T} = t\bar{Y} \Rightarrow \bar{T} = 0.2(269.23) = 53.83$$

This is equilibrium level of T.

$$(iii) \quad S = Y_d - C \Rightarrow S = Y - T - C = 269.23 - 53.83 - 169.24 = 46.16$$

This is equilibrium level of S. Thus

$S + T = I + G$ $46.16 + 53.83 = 80 + 20$ $100 = 100$	$\bar{Y} = \bar{C} + I_0 + G_0$ $269.23 = 169.24 + 80 + 20$ $269.23 = 269.24$
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3-B. NI DETERMINATION IN A 3 SECTOR ECONOMY

When Autonomous and Induced Taxes are included,

NI in 3 sector economy gives

$$Y = C + I + G$$

$$Y = C_0 + cY_d + I_0 + G_0$$

$$Y = C_0 + c[Y - T] + I_0 + G_0$$

$$Y = C_0 + c[Y - (T_0 + tY)] + I_0 + G_0$$

$$Y = C_0 + cY - cT_0 - ctY + I_0 + G_0$$

$$Y - cY + ctY = C_0 + I_0 + G_0 - cT_0$$

$$Y(1 - c + ct) = C_0 + I_0 + G_0 - cT_0$$

$$\bar{Y} = \frac{1}{1 - c + ct} [C_0 + I_0 + G_0 - cT_0]$$

This is reduced form of NI or this is equilibrium level of NI.

$$T = T_0 + tY, \quad \bar{T} = T_0 + t\bar{Y}$$

$$\bar{T} = T_0 + t \left[\frac{C_0 + I_0 + G_0 - cT_0}{1 - c + ct} \right]$$

This is equilibrium level of taxes.

Now we find equilibrium level of consumption, $C = C_0 + c[\bar{Y} - \bar{T}]$

$$\bar{C} = C_0 + c \left[\left\{ \frac{C_0 + I_0 + G_0 - cT_0}{1 - c + ct} \right\} - t \left\{ \frac{C_0 + I_0 + G_0 - cT_0}{1 - c + ct} \right\} + T_0 \right]$$

The last equation represents equilibrium level of consumption.

EXAMPLE 1: $Y = C + I_0 + G_0$ Solution to the Q given in the box of this page.

$$S = -40 + 0.4Y_d, I_0 = 40, G_0 = 50, T = 10 + 0.2Y, Y_d = Y - T$$

$$MPC = 1 - MPS = 1 - 0.4 = 0.6, -S_0 = C_0 = 40$$

$$Y = C + I_0 + G_0 = 40 + 0.6Y_d + 40 + 50 = 40 + 0.6(Y - T) + 90 \Rightarrow$$

$$Y = 130 - 0.6Y_d [Y - (10 + 0.2Y)] \Rightarrow Y = 130 + 0.6[Y - 10 - 0.2Y] \Rightarrow$$

$$Y = 130 + 0.6Y - 6 - 0.12Y \Rightarrow Y - 0.6Y + 0.12Y = 130 - 6 \Rightarrow$$

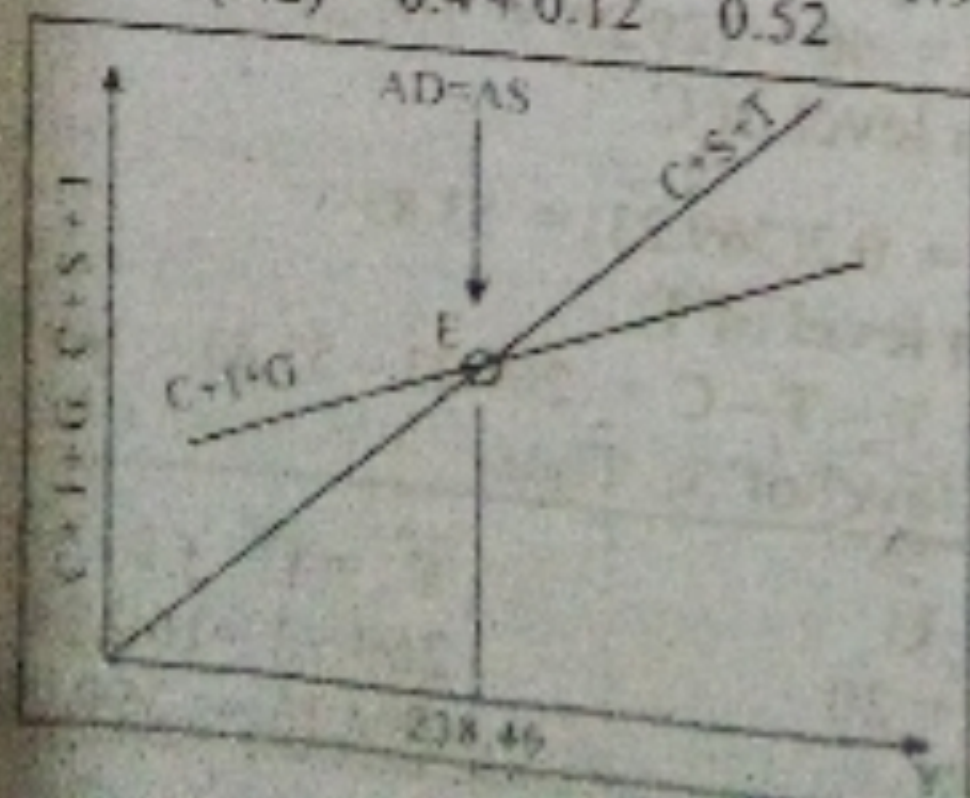
$$Y(1 - 0.6 + 0.12) = 124 \Rightarrow Y(0.52) = 124 \Rightarrow \bar{Y} = 124/0.52 = 238.46$$

$$\bar{T} = 10 + 0.2(238.46) = 57.69$$

$$C = 40 + 0.6(Y - T) = 40 + 0.6(238.46 - 57.69) = 148.46$$

$$\bar{Y} = \bar{C} + I_0 + G_0 \Rightarrow 238.46 = 148.46 + 40 + 50 \Rightarrow 238.46 = 238.46$$

$$K = \frac{1}{1 - c + ct} = \frac{1}{1 - 0.6 + 0.6(0.2)} = \frac{1}{0.4 + 0.12} = \frac{1}{0.52} = 1.923$$



Q. With the help of national income determination model, $Y = C + I + G$ while $S = -40 + 0.4Y_d, I_0 = 40, G_0 = 50, T = 10 + 0.2Y, Y_d = Y - T$, find (i) equilibrium level of income and consumption (ii) value of multiplier (K) and (iii) show equilibrium level of income graphically. (UOS-2015/11)

2. GENERAL EQUILIBRIUM IN MONEY MARKET

According to Hicks, LM curve shows general equilibrium in money market. Therefore, "LM curve shows the combinations of rate of interest and level of NI where $M_d = M_s$, or money market is in equilibrium". According to Hicks, M_d has two parts: (1) The transactive demand for money (M_{td}) which depends upon level of income (Y). Its general and standard form are as: $M_{td} = f(Y)$ and $M_{td} = kY$ where k = proportion of cash balance.

(2) Speculative demand for money (M_{sd}) which depends upon the rate of interest. There exists an inverse relationship between M_{sd} and rate of interest. Its general and standard forms are as: $M_{sd} = f(i)$ and $M_{sd} = -mi$

where m is the slope of M_{sd} curve and negative sign shows inverse relationship between interest rate and M_{sd} . $M_d = M_{td} + M_{sd}$

M_s is the supply of money which is fixed in short-run. It is as: $M_s = M_o$

At equilibrium $M_d = M_{td} + M_{sd}$ Summing two demands $M_d = kY - mi$

$$M_s = M_o \quad , \quad M_d = M_o$$

$$kY - mi = M_o \quad \Rightarrow \quad kY = M_o + mi$$

$$Y = \frac{M_o + mi}{k} \quad \text{or} \quad Y = \frac{M_o}{k} + \frac{m}{k} i \quad : \text{LM equation}$$

Numerical Example 1. If the following information are given regarding money market, the level of income is found at some rate of interest where $M_d = M_s$.

$$M_{td} = 0.25 Y, \quad M_{sd} = -200i, \quad M_s = M_o = 300$$

Solution.

$$M_d = M_{td} + M_{sd}$$

$$M_d = 0.25 Y + (-200i)$$

$$M_d = 0.25 Y - 200i$$

$$M_s = M_o = 300$$

$$M_d = M_s$$

$$0.25 Y - 200i = 300$$

$$0.25 Y = 300 + 200i$$

$$Y = \frac{300 + 200i}{0.25}$$

By supposing 10% as interest rate we get the following level of NI

$$Y = 1200 + 800(10/100)$$

$$Y = 1200 + 80 = 1280$$

Putting the values of Y and i

$$M_d = 0.25 Y - 200i$$

$$M_d = 0.25(1280) - 200(10/100)$$

$$M_d = 320 - 20 = 300$$

$Y = 1200 + 800i$ The LM equation

Thus, $i = 10\%$, $Y = 1280$, where $M_d = M_s = 300$.

If $i = 15\%$, then $Y = 1200 + 800(15/100) = 1320$

$$M_{td} = 0.25Y = 0.25(1320) = 330$$

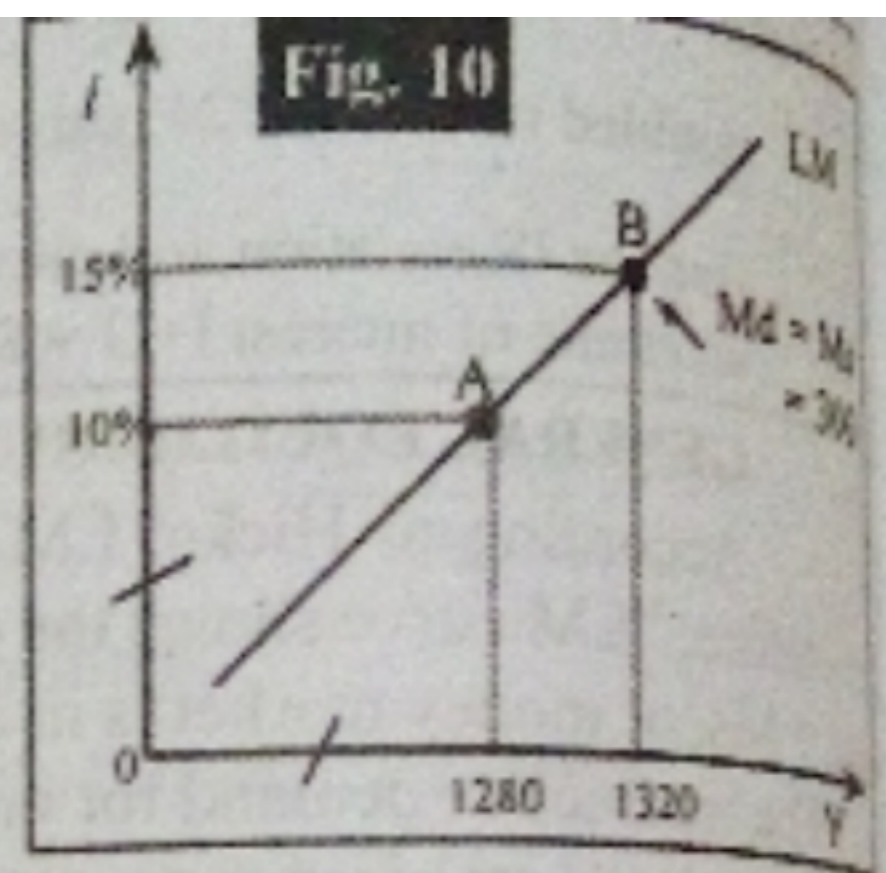
$$M_{sd} = -200(15/100) = -30,$$

$$M_d = M_{td} + M_{sd} = 330 - 30 = 300$$

Thus, $i = 10\%$, $Y = 1280$, where $M_d = M_s = 300$.

Thus, $i = 15\%$, $Y = 1320$, where $M_d = M_s = 300$.

All this is shown in Fig. 10.



Example 2. If following are the information regarding money market, find the level of NI at some rate of interest where $M_d = M_s$.

$$M_{td} = L_1 = 0.25Y, \quad M_{sd} = L_2 = -62.5i, \quad M_s = 500 = M_o.$$

It is told that some people represent M_d by L , M_{td} by L_1 and M_{sd} by L_2 .

Solution.

$$L = L_1 + L_2 = 0.25Y + (-62.5i)$$

$$= 0.25Y - 62.5i$$

$$M_s = 500, \quad M_s = M_d$$

$$M_o = M_d$$

$$500 = 0.25Y - 62.5i$$

$$500 + 62.5i = 0.25Y$$

$$0.25Y = 500 + 62.5i$$

$$Y = \frac{500 + 62.5i}{0.25}$$

$$Y = 2000 + 250i$$

Thus, when $i = 10\%$ and $Y = 2025$, where $M_d = M_s = 500$. This represents general equilibrium in money market.

If $i = 10\%$ then

$$Y = 2000 + 250(10/100)$$

$$Y = 2000 + 25$$

$$Y = 2025$$

$$L_1 = 0.25Y = 0.25(2025) = 506.25$$

$$L_2 = -62.5i = -62.5(10/100) = -6.25$$

$$L = L_1 + L_2$$

$$= (506.25) + (-6.25) = 500$$

$$M_s = M_o = 500$$

EXERCISE-8

No.	Questions	Answers								
(1)	$M_d = 0.25Y - 30i$ $M_s = M_o = 350$	$Y = 1400 + 120i$ <table border="1"> <thead> <tr> <th>i</th> <th>Y</th> <th>Md</th> <th>Ms</th> </tr> </thead> <tbody> <tr> <td>10%</td> <td>1412</td> <td>350</td> <td>350</td> </tr> </tbody> </table> <p>[$i = 10\%$ is supposed here.]</p>	i	Y	Md	Ms	10%	1412	350	350
i	Y	Md	Ms							
10%	1412	350	350							

(2).	$M_{td} = 0.25Y$ $M_{sd} = -1500r$ $M_s = 375$	$Y = 1500 + 6000r$ <table border="1"> <tr> <td>r</td> <td>Y</td> <td>Md</td> <td>Ms</td> </tr> <tr> <td>5%</td> <td>1800</td> <td>375</td> <td>375</td> </tr> </table> <p>where $r = 5\%$ is rate of interest which is supposed here.</p>	r	Y	Md	Ms	5%	1800	375	375
r	Y	Md	Ms							
5%	1800	375	375							
(3).	$M_{sd} = 135 - 600i$ $M_{td} = 0.25Y$ $M_s = 200$	$Y = 260 + 2400i$ <table border="1"> <tr> <td>i</td> <td>Y</td> <td>Md</td> <td>Ms</td> </tr> <tr> <td>5%</td> <td>380</td> <td>200</td> <td>200</td> </tr> </table> <p>[N.B. $i = 5\%$ is supposed]</p>	i	Y	Md	Ms	5%	380	200	200
i	Y	Md	Ms							
5%	380	200	200							

3. SIMULTANEOUS EQUILIBRIUM IN GOODS AND MONEY MARKETS

We have already told that IS represents general equilibrium in goods markets while LM shows general equilibrium in money market. Therefore, according to Hicks the simultaneous equilibrium in goods and money markets takes place where IS intersects LM curve. Now we explain them with examples.

Numerical Example 1:

If the following informations are given regarding goods and money markets, find equilibrium rate of interest and level of NI where goods and money markets are in equilibrium. If

$$\begin{aligned}
 Y &= C + I \\
 Y &= 102 + 0.7Y + 150 - 100i \\
 Y - 0.7Y &= 252 - 100i \\
 Y(1 - 0.7) &= 252 - 100i \\
 Y(0.3) &= 252 - 100i \\
 Y &= \frac{252 - 100i}{0.3}
 \end{aligned}$$

$$\begin{aligned}
 C &= 102 + 0.7Y \quad (\text{UAIK:2012}) \\
 I &= 150 - 100i \\
 L_T &= M_{td} = 0.25Y \\
 L_s &= M_{sd} = 124 - 200i \\
 M_s &= M_s = M_0 = 300
 \end{aligned}$$

Note please: $M_{sd} = L_s =$ speculative demand for money, $M_{td} = L_T =$ transactive demand for money $Y = 840 - 333.33i \dots$ (1) The IS Equation.

$$\begin{aligned}
 M_d &= M_{td} + M_{sd} \\
 M_d &= M_s \Rightarrow L = M \\
 &= 0.25Y + 124 - 200i \Rightarrow L = L_T + L_s
 \end{aligned}$$

$$0.25Y + 124 - 200i = 300$$

$$0.25Y = 300 - 124 + 200i$$

$$0.25Y = 176 + 200i$$

$$Y = \frac{176 + 200i}{0.25}$$

$$Y = 704 + 800i \dots (2)$$

The LM equation

Subtracting (2) from (1)

$$Y = 840 - 333.33i$$

$$\pm Y = \pm 704 \pm 800i$$

$$0 = 136 - 1133.33i$$

$$1133.33i = 136$$

$$i = \frac{136}{1133.33} = 0.12 = 12\%$$

Putting the value of i in (1) and (2)

$$Y = 840 - 333.33(0.12) = 800$$

$$Y = 704 + 800(0.12) = 800$$

$M_{td} = 0.25Y$. Putting the value of Y in M_{td}

$$M_{td} = 0.25(800) = 200$$

$$M_{sd} = 124 - 200i$$

Putting the value of i in M_{sd}

$$M_{sd} = 124 - 200(0.12) = 100$$

$$M_d = M_{td} + M_{sd}$$

$$M_d = 200 + 100 = 300$$

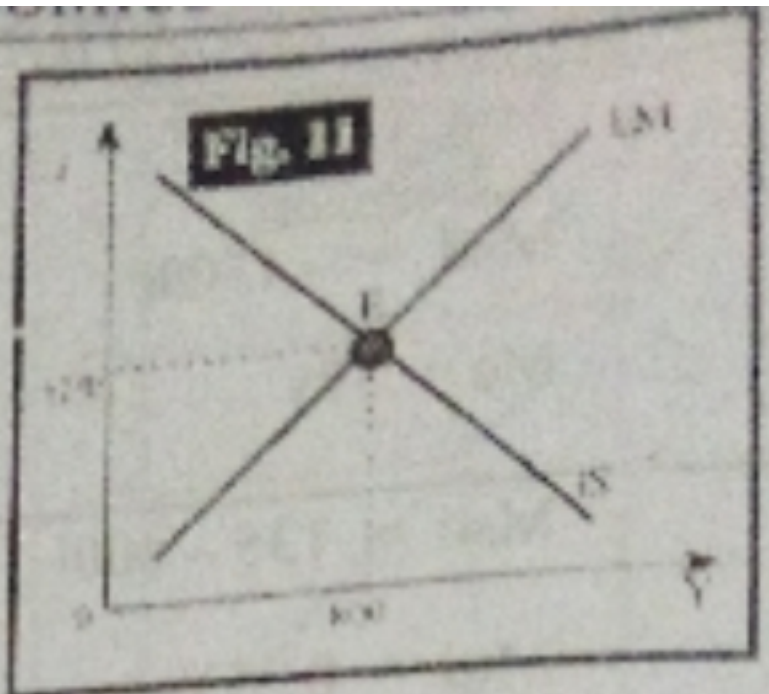
$$C = 102 + 0.7Y$$

$$C = 102 + 0.7(800) = 662$$

$$S = Y - C = 800 - 662 = 138$$

$$I = 150 - 100i = 150 - 100(0.12) = 138$$

i	Y	S	I	$Md = M_{td} + M_{sd}$	M_s
12%	800	138	138	$300 = 200 + 100$	300



All this is shown in Fig. 11. Thus equilibrium rate of interest (i) = 12% and equilibrium level of NI = 800 where $S = I$ and $Md = Ms$. If $M_s = 317$, then we also find values of i and Y .

$$L_T + L_s = M \Rightarrow \quad \text{(UAIJK:2012/A)}$$

$$0.25Y + 124 - 200i = 317 \Rightarrow 0.25Y = 317 - 124 + 200i \Rightarrow Y = 772 + 800i,$$

$$\text{New LM: } Y = 772 + 800i, \text{ old IS: } Y = 840 - 333.33i$$

$$Y = 840 - 333.33i$$

$$-Y = -772 \pm 800i$$

$$0 = 68 - 1133.33i \Rightarrow 1133.33i = 68 \Rightarrow$$

$$i = 0.06, Y = 840 - 333.33(0.06) = 820, Y = 772 + 800(0.06) = 820$$

Thus new $Y = 820$ and $i = 0.06 = 6\%$

Example 2. With the following equations, find simultaneous equilibrium in goods and money markets:

Goods Market Equilibrium

$$Y = C + I + G$$

$$Y = 10 + 0.8Y_d + 100 - 200i + 15$$

$$Y = 10 + 0.8[Y - T] + 100 - 200i + 15$$

$$Y = 10 + 0.8[Y - 30] + 100 - 200i + 15$$

$$Y = 10 + 0.8Y - 24 + 100 - 200i + 15$$

$$Y - 0.8Y = 101 - 200i$$

$$Y(1 - 0.8) = 101 - 200i$$

$$Y(0.2) = 101 - 200i$$

$$Y = \frac{101 - 200i}{0.2} \quad \text{OR } Y = 505 - 1000i \quad \dots (1)$$

Money Market Equilibrium

$$Md = Ms$$

$$0.25Y - 40i = 100$$

$$0.25Y = 100 + 40i$$

$$Y = \frac{100 + 40i}{0.25}$$

$$Y = 400 + 160i \quad \dots (2) \quad i = \frac{105}{1160} = 0.0905172$$

Putting the value of i in (1) and (2)

$$Y = 505 - 1000i = 505 - 1000(0.0905172)$$

$$C = 10 + 0.8Y_d$$

$$Y_d = Y - T, T = 30$$

$$G = G_0 = 15$$

$$I = 100 - 200i$$

$$Md = 0.25Y - 40i, Mo = 100$$

$$Q: \text{ If } C = 48 + 0.8Y$$

$$I = 98 - 75i$$

$$M = 250$$

$$L = L_T + L_s \Rightarrow$$

$$0.3Y + 52 - 150i$$

(i) find IS and LM equations

(ii) find equilibrium level of income and rate of interest.

(UAIJK: 2013)

Goods, market equilibrium

$$Y = C + I$$

$$Y = 48 + 0.8Y + 98 - 75i$$

Money market equilibrium

$$L = M \Rightarrow L_T + L_s = M$$

$$0.3Y + 52 - 150i = 250$$

For further solution, see Example 1

$$Y = 505 - 90.5172 = 414.48275$$

$$Y = 400 + 160i = 400 + 160(0.0905172) = 414.48275$$

$$M_d = 0.25Y - 40i = 0.25(414.48275) - 40(0.0905172) = 100$$

$$C = 10 + 0.8Y = 10 + 0.8[Y - T]$$

$$C = 10 + 0.8[414.48275 - 30] = 317.58624$$

$$Y_d = Y - T = 414.4828 - 30 = 384.4828$$

$$S = Y_d - C = 384.4828 - 317.58624 = 66.89656$$

$$I = 100 - 200i = 100 - 200(0.0905172)$$

$$I = 100 - 18.10344 = 81.89$$

Thus at 9% rate of interest the equilibrium level of NI is 414.48, where

$$M_d = M_s = 100 \text{ and } I + G = S + T = 96.90$$

i	Y	M_d	M_s	$S + T$	$I + G$
9%	414.48	100	100	$66.89 + 30 = 96.90$	$81.89 + 15 = 96.90$

Numerical Example 3. With the following equations, find equilibrium level of interest and equilibrium level of NI.

$$C = 130 + 0.5 Y_d, Y_d = Y - T, T = 20 + 0.2Y, G = G_0 = 112,$$

$$I = 200 - 600i, M_s = 300, M_d = 0.5Y, M_{sd} = 50 - 600i$$