Business Statistics: A Decision-Making Approach

6th Edition

Chapter 10

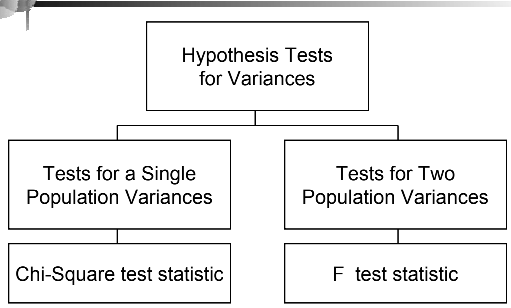
Hypothesis Tests for One and Two Population Variances

Chapter Goals

After completing this chapter, you should be able to:

- Formulate and complete hypothesis tests for a single population variance
- Find critical chi-square distribution values from the chi-square table
- Formulate and complete hypothesis tests for the difference between two population variances
- Use the F table to find critical F values

Hypothesis Tests for Variances



Single Population

Hypothesis Tests for Variances

Tests for a Single Population Variances *

Chi-Square test statistic

H_0: $\sigma^2 = \sigma_0^2$ H_A: $\sigma^2 \neq \sigma_0^2$	Two tailed test
H_0: $\sigma^2 \geq \sigma_0^2$ H_A: $\sigma^2 < \sigma_0^2$	Lower tail test
H_0: $\sigma^2 \leq \sigma_0^2$ H_A: $\sigma^2 > \sigma_0^2$	Upper tail test

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Chi-Square Test Statistic

Hypothesis Tests for Variances

Tests for a Single Population Variances

Chi-Square test statistic *

The chi-squared test statistic for a Single Population Variance is:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

where

χ^2 = standardized chi-square variable

n = sample size

s^2 = sample variance

σ^2 = hypothesized variance

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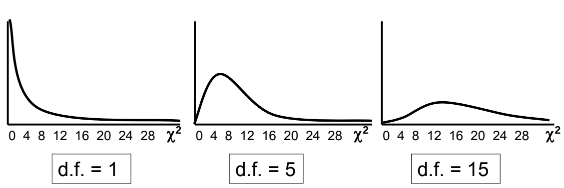
The Chi-square Distribution

- The chi-square distribution is a family of distributions, depending on degrees of freedom:
- d.f. = n - 1



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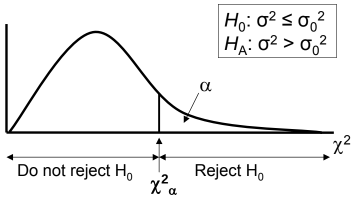
Finding the Critical Value

- The critical value, χ^2_α , is found from the chi-square table

Upper tail test:

H_0: $\sigma^2 \le \sigma_0^2$
H_A: $\sigma^2 > \sigma_0^2$



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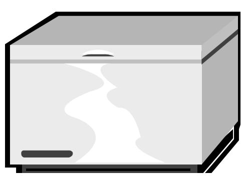
Example

- A commercial freezer must hold the selected temperature with little variation. Specifications call for a standard deviation of no more than 4 degrees (or variance of 16 degrees^2). A sample of 16 freezers is tested and yields a sample variance of $s^2 = 24$. Test to see whether the standard deviation specification is exceeded. Use $\alpha = .05$



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Finding the Critical Value

- The the chi-square table to find the critical value:

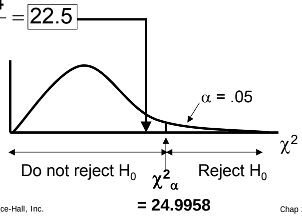
$\chi^2_\alpha = 24.9958$ ($\alpha = .05$ and $16 - 1 = 15$ d.f.)

The test statistic is:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(16-1)24}{16} = \boxed{22.5}$$

Since 22.5 < 24.9958, do not reject H_0

There is not significant evidence at the $\alpha = .05$ level that the standard deviation specification is exceeded



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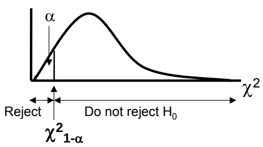
Lower Tail or Two Tailed Chi-square Tests

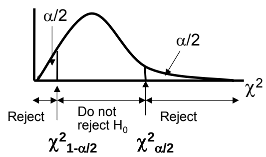
Lower tail test:

H_0: $\sigma^2 \geq \sigma_0^2$
H_A: $\sigma^2 < \sigma_0^2$



Two tail test:

H_0: $\sigma^2 = \sigma_0^2$
H_A: $\sigma^2 \neq \sigma_0^2$



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F Test for Difference in Two Population Variances

Hypothesis Tests for Variances → * Tests for Two Population Variances → F test statistic

H_0: $\sigma_1^2 - \sigma_2^2 = 0$
H_A: $\sigma_1^2 - \sigma_2^2 \neq 0$
Two tailed test

H_0: $\sigma_1^2 - \sigma_2^2 \geq 0$
H_A: $\sigma_1^2 - \sigma_2^2 < 0$
Lower tail test

H_0: $\sigma_1^2 - \sigma_2^2 \leq 0$
H_A: $\sigma_1^2 - \sigma_2^2 > 0$
Upper tail test

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F Test for Difference in Two Population Variances

Hypothesis Tests for Variances → Tests for Two Population Variances → * F test statistic

The F test statistic is:

(Place the larger sample variance in the numerator)

$$F = \frac{s_1^2}{s_2^2}$$

s_1^2 = Variance of Sample 1
$n_1 - 1$ = numerator degrees of freedom

s_2^2 = Variance of Sample 2
$n_2 - 1$ = denominator degrees of freedom

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The F Distribution

- The F critical value is found from the F table
- The are two appropriate degrees of freedom: numerator and denominator

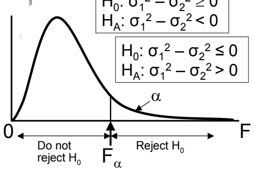
$$F = \frac{s_1^2}{s_2^2} \quad \text{where } df_1 = n_1 - 1 \text{ ; } \; df_2 = n_2 - 1$$

- In the F table,
  - numerator degrees of freedom determine the row
  - denominator degrees of freedom determine the column

Finding the Critical Value

$H_0: \sigma_1^2 - \sigma_2^2 \geq 0$
$H_A: \sigma_1^2 - \sigma_2^2 < 0$

$H_0: \sigma_1^2 - \sigma_2^2 \leq 0$
$H_A: \sigma_1^2 - \sigma_2^2 > 0$

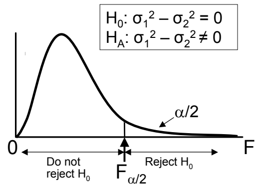


- rejection region for a one-tail test is

$$F = \frac{s_1^2}{s_2^2} > F_\alpha$$

(when the larger sample variance in the numerator)

$H_0: \sigma_1^2 - \sigma_2^2 = 0$
$H_A: \sigma_1^2 - \sigma_2^2 \neq 0$



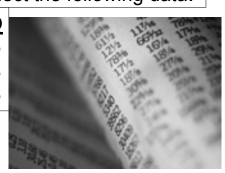
- rejection region for a two-tailed test is

$$F = \frac{s_1^2}{s_2^2} > F_{\alpha/2}$$

F Test: An Example

You are a financial analyst for a brokerage firm. You want to compare dividend yields between stocks listed on the NYSE & NASDAQ. You collect the following data:

	NYSE	NASDAQ
Number	**21**	**25**
Mean	**3.27**	**2.53**
Std dev	**1.30**	**1.16**



Is there a difference in the variances between the NYSE & NASDAQ at the α = 0.05 level?

F Test: Example Solution

- Form the hypothesis test:

$H_0: \sigma^2_1 - \sigma^2_2 = 0$	(there is no difference between variances)
$H_A: \sigma^2_1 - \sigma^2_2 \neq 0$	(there is a difference between variances)

- Find the F critical value for $\alpha = .05$:
  - Numerator:
    - $df_1 = n_1 - 1 = 21 - 1 = 20$
  - Denominator:
    - $df_2 = n_2 - 1 = 25 - 1 = 24$

$$F_{.05/2,\ 20,\ 24} = 2.327$$

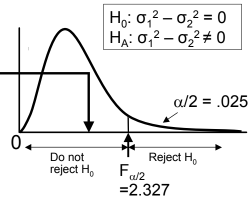
F Test: Example Solution

(continued)

- The test statistic is:

$$F = \frac{s_1^2}{s_2^2} = \frac{1.30^2}{1.16^2} = 1.256$$

$H_0: \sigma_1^2 - \sigma_2^2 = 0$
$H_A: \sigma_1^2 - \sigma_2^2 \neq 0$



- F = 1.256 is not greater than the critical F value of 2.327, so we do not reject H_0
- Conclusion: There is no evidence of a difference in variances at $\alpha = .05$

Using EXCEL and PHStat

EXCEL

- F test for two variances:
  - Tools | data analysis | F-test: two sample for variances

PHStat

- Chi-square test for the variance:
  - PHStat | one-sample tests | chi-square test for the variance
- F test for two variances:
  - PHStat | two-sample tests | F test for differences in two variances

Chapter Summary

- Performed chi-square tests for the variance
- Used the chi-square table to find chi-square critical values
- Performed F tests for the difference between two population variances
- Used the F table to find F critical values

