Business Statistics: A Decision-Making Approach

6th Edition

Chapter 8

Introduction to Hypothesis Testing

Chapter Goals

After completing this chapter, you should be able to:

- Formulate null and alternative hypotheses for applications involving a single population mean or proportion
- Formulate a decision rule for testing a hypothesis
- Know how to use the test statistic, critical value, and p-value approaches to test the null hypothesis
- Know what Type I and Type II errors are
- Compute the probability of a Type II error

What is a Hypothesis?

- A hypothesis is a claim (assumption) about a population parameter:

  - population mean

    Example: The mean monthly cell phone bill of this city is μ = $42

  - population proportion

    Example: The proportion of adults in this city with cell phones is p = .68

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The Null Hypothesis, H_0

- States the assumption (numerical) to be tested

> Example: The average number of TV sets in U.S. Homes is at least three ($H_0 : \mu \geq 3$)

- Is always about a population parameter, not about a sample statistic

$H_0 : \mu \geq 3$ $H_0 : \bar{x} \geq 3$

The Null Hypothesis, H_0

(continued)

- Begin with the assumption that the null hypothesis is true
  - Similar to the notion of innocent until proven guilty

- Refers to the status quo
- Always contains "=" , "≤" or "≥" sign
- May or may not be rejected

The Alternative Hypothesis, H_A

- Is the opposite of the null hypothesis
  - e.g.: The average number of TV sets in U.S. homes is less than 3 ($H_A: \mu < 3$)
- Challenges the status quo
- Never contains the "=" , "≤" or "≥" sign
- May or may not be accepted
- Is generally the hypothesis that is believed (or needs to be supported) by the researcher

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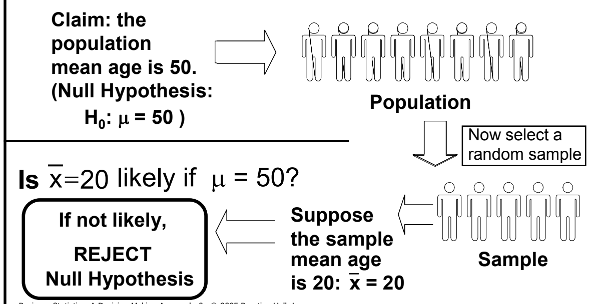
Hypothesis Testing Process



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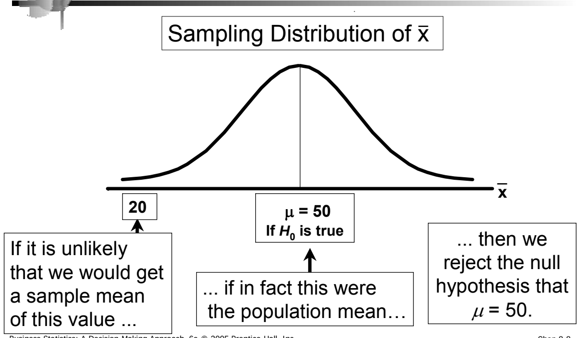
Reason for Rejecting H_0



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Level of Significance, α

- **Defines unlikely values of sample statistic if null hypothesis is true**
  - Defines rejection region of the sampling distribution
- Is designated by α , (level of significance)
  - Typical values are .01, .05, or .10
- Is selected by the researcher at the beginning
- Provides the critical value(s) of the test

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Level of Significance and the Rejection Region

Level of significance = α

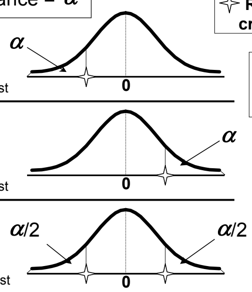
Represents critical value

Rejection region is shaded

H_0: μ ≥ 3
H_A: μ < 3
Lower tail test

H_0: μ ≤ 3
H_A: μ > 3
Upper tail test

H_0: μ = 3
H_A: μ ≠ 3
Two tailed test



Errors in Making Decisions

- **Type I Error**
  - Reject a true null hypothesis
  - Considered a serious type of error

The probability of Type I Error is α

- Called level of significance of the test
- Set by researcher in advance

Errors in Making Decisions

(continued)

- **Type II Error**
  - Fail to reject a false null hypothesis

The probability of Type II Error is β

Outcomes and Probabilities

Possible Hypothesis Test Outcomes		
	State of Nature	
Decision	H_0 True	H_0 False
Do Not Reject **H_0**	**No error** **(1 - α)**	**Type II Error** **(β)**
Reject **H_0**	**Type I Error** **(α)**	**No Error** **(1 - β)**

Key: Outcome (Probability)

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Type I & II Error Relationship

- Type I and Type II errors can not happen at the same time
  - Type I error can only occur if H_0 is true
  - Type II error can only occur if H_0 is false

If Type I error probability (α) ⇧, then
Type II error probability (β) ⇩

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Factors Affecting Type II Error

- All else equal,
  - β ⇧ when the difference between hypothesized parameter and its true value ⇩
  - β ⇧ when α ⇩
  - β ⇧ when σ ⇧
  - β ⇧ when *n* ⇩

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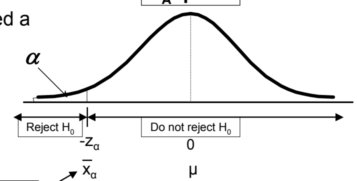
Critical Value Approach to Testing

- Convert sample statistic (e.g.: $\bar{x}$) to test statistic (Z or t statistic)
- Determine the critical value(s) for a specified level of significance α from a table or computer
- If the test statistic falls in the rejection region, reject H_0 ; otherwise do not reject H_0

Lower Tail Tests

- The cutoff value, $-z_\alpha$ or $\bar{x}_\alpha$, is called a critical value

H_0: μ ≥ 3
H_A: μ < 3

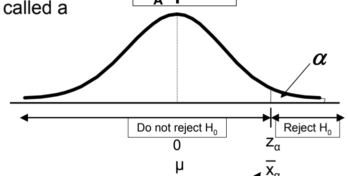


$$\bar{x}_\alpha = \mu - z_\alpha \frac{\sigma}{\sqrt{n}}$$

Upper Tail Tests

- The cutoff value, z_α or $\bar{x}_\alpha$, is called a critical value

H_0: μ ≤ 3
H_A: μ > 3



$$\bar{x}_\alpha = \mu + z_\alpha \frac{\sigma}{\sqrt{n}}$$

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Two Tailed Tests

- There are two cutoff values (critical values):

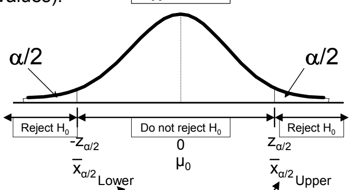
$\pm z_{\alpha/2}$

or

$\bar{x}_{\alpha/2\ Lower}$

$\bar{x}_{\alpha/2\ Upper}$

H_0: μ = 3
H_A: μ ≠ 3



$$\bar{x}_{\alpha/2} = \mu \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

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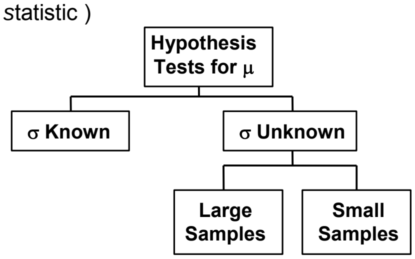
Critical Value Approach to Testing

- Convert sample statistic ($\bar{x}$) to a test statistic (*Z or t* statistic)



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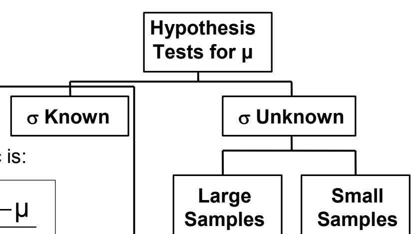
Calculating the Test Statistic



The test statistic is:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

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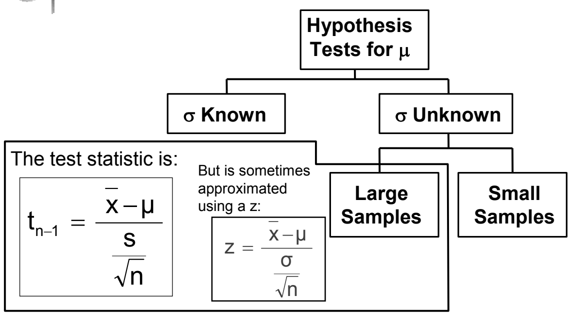
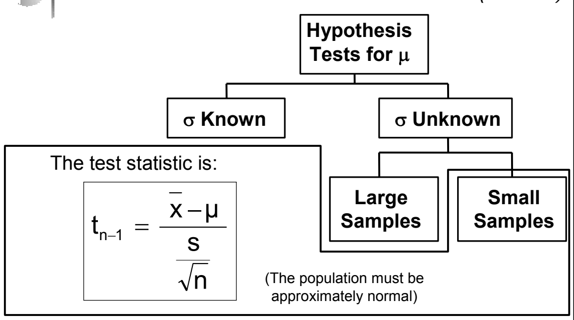
Calculating the Test Statistic

(continued)



Calculating the Test Statistic

(continued)



Review: Steps in Hypothesis Testing

- 1. Specify the population value of interest
- 2. Formulate the appropriate null and alternative hypotheses
- 3. Specify the desired level of significance
- 4. Determine the rejection region
- 5. Obtain sample evidence and compute the test statistic
- 6. Reach a decision and interpret the result

Hypothesis Testing Example

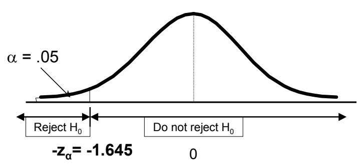
Test the claim that the true mean # of TV sets in US homes is at least 3. (Assume σ = 0.8)

- 1. Specify the population value of interest
  - The mean number of TVs in US homes
- 2. Formulate the appropriate null and alternative hypotheses
  - H_0: $\mu \ge 3$ H_A: $\mu < 3$ (This is a lower tail test)
- 3. Specify the desired level of significance
  - Suppose that $\alpha = .05$ is chosen for this test

Hypothesis Testing Example

(continued)

- 4. Determine the rejection region



This is a one-tailed test with $\alpha = .05$.
Since **σ is known**, the cutoff value is a z value:

Reject H_0 if $z < z_\alpha = -1.645$; otherwise do not reject H_0

Hypothesis Testing Example

- 5. Obtain sample evidence and compute the test statistic

Suppose a sample is taken with the following results: $n = 100$, $\bar{x} = 2.84$ ($\sigma = 0.8$ is assumed known)

- Then the test statistic is:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{.08} = \boxed{-2.0}$$

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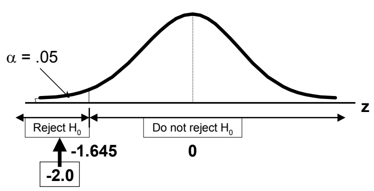
Hypothesis Testing Example

(continued)

- 6. Reach a decision and interpret the result



Since z = -2.0 < -1.645, we reject the null hypothesis that the mean number of TVs in US homes is at least 3

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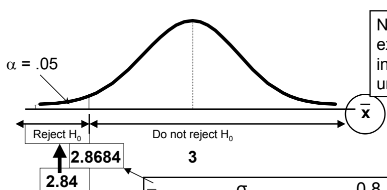
Hypothesis Testing Example

(continued)

- An alternate way of constructing rejection region:



$$\bar{x}_\alpha = \mu - z_\alpha \frac{\sigma}{\sqrt{n}} = 3 - 1.645 \frac{0.8}{\sqrt{100}} = 2.8684$$

Since $\bar{x}$ = 2.84 < 2.8684, we reject the null hypothesis

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p-Value Approach to Testing

- Convert Sample Statistic (e.g. $\bar{x}$) to Test Statistic (*Z or t* statistic)
- Obtain the p-value from a table or computer
- Compare the p-value with α
  - If p-value < α , reject H_0
  - If p-value ≥ α , do not reject H_0

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p-Value Approach to Testing

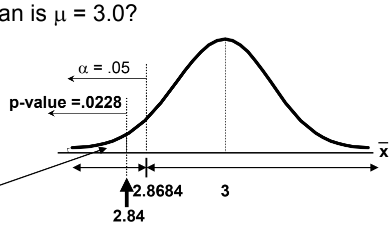
(continued)

- p-value: Probability of obtaining a test statistic more extreme (≤ or ≥) than the observed sample value given H_0 is true
  - Also called observed level of significance
  - Smallest value of α for which H_0 can be rejected

p-value example

- Example: How likely is it to see a sample mean of 2.84 (or something further below the mean) if the true mean is μ = 3.0?

$$P(\bar{x} < 2.84 \mid \mu = 3.0)$$
$$= P\left(z < \frac{2.84 - 3.0}{0.8/\sqrt{100}}\right)$$
$$= P(z < -2.0) = .0228$$

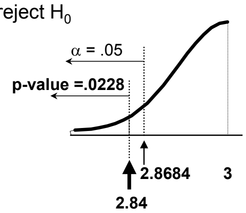


p-value example

(continued)

- Compare the p-value with α
  - If p-value < α, reject H_0
  - If p-value ≥ α, do not reject H_0

Here: p-value = .0228
α = .05
Since .0228 < .05, we reject the null hypothesis



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Example: Upper Tail z Test for Mean (σ Known)

A phone industry manager thinks that customer monthly cell phone bill have increased, and now average over $52 per month. The company wishes to test this claim. (Assume σ = 10 is known)

Form hypothesis test:

H_0: μ ≤ 52 the average is not over $52 per month

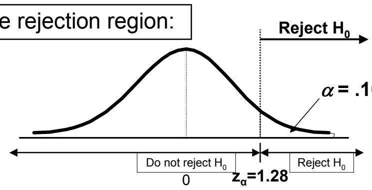
H_A: μ > 52 the average **is** greater than $52 per month (i.e., sufficient evidence exists to support the manager's claim)

Example: Find Rejection Region

(continued)

- Suppose that α = .10 is chosen for this test

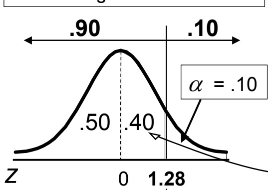
Find the rejection region:



Reject H_0 if z > 1.28

Review: Finding Critical Value - One Tail

What is z given α = 0.10?



Critical Value = 1.28

Standard Normal Distribution Table (Portion)

Z	.07	.08	.09
1.1	.3790	.3810	.3830
1.2	.3980	.3997	.4015
1.3	.4147	.4162	.4177

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Example: Test Statistic

(continued)

Obtain sample evidence and compute the test statistic

Suppose a sample is taken with the following results: n = 64, $\bar{x}$ = 53.1 (σ=10 was assumed known)

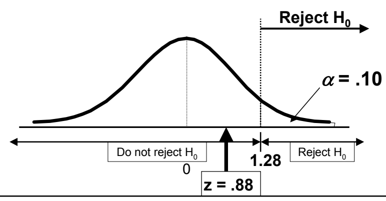
- Then the test statistic is:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{53.1 - 52}{\frac{10}{\sqrt{64}}} = \boxed{0.88}$$

Example: Decision

(continued)

Reach a decision and interpret the result:



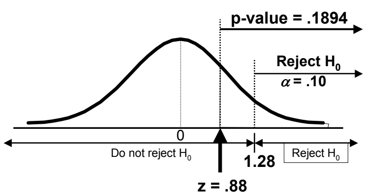
Do not reject H_0 since z = 0.88 ≤ 1.28

i.e.: there is not sufficient evidence that the mean bill is over $52

p -Value Solution

(continued)

Calculate the p-value and compare to α



$$P(\bar{x} \geq 53.1 \mid \mu = 52.0)$$

$$= P\left(z < \frac{53.1 - 52.0}{10/\sqrt{64}}\right)$$

$$= P(z \geq 0.88) = .5 - .3106$$

$$= \boxed{.1894}$$

Do not reject H_0 since p-value = .1894 > α = .10

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Example: Two-Tail Test (σ Unknown)

The average cost of a hotel room in New York is said to be \$168 per night. A random sample of 25 hotels resulted in $\bar{x}$ = \$172.50 and s = \$15.40. Test at the α = 0.05 level.

(Assume the population distribution is normal)

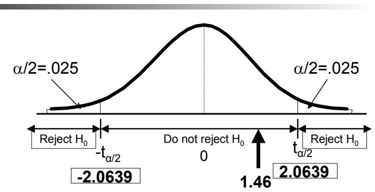
H_0: μ = 168
H_A: μ ≠ 168

Example Solution: Two-Tail Test

H_0: μ = 168
H_A: μ ≠ 168

- **α = 0.05**
- **n = 25**
- **σ is unknown, so use a t statistic**
- **Critical Value:**

$t_{24} = \pm 2.0639$



$$t_{n-1} = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

Do not reject H_0: not sufficient evidence that true mean cost is different than \$168

Hypothesis Tests for Proportions

- Involves categorical values
- Two possible outcomes
  - "Success" (possesses a certain characteristic)
  - "Failure" (does not possesses that characteristic)
- Fraction or proportion of population in the "success" category is denoted by p

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Proportions

(continued)

- Sample proportion in the success category is denoted by $\bar{p}$
  - $\bar{p} = \frac{x}{n} = \frac{\text{number of successes in sample}}{\text{sample size}}$
- When both np and n(1-p) are at least 5, $\bar{p}$ can be approximated by a normal distribution with mean and standard deviation
  - $\mu_{\bar{p}} = p$ $\quad \sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}}$

Hypothesis Tests for Proportions

- The sampling distribution of $\bar{p}$ is normal, so the test statistic is a z value:

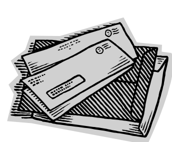
$$z = \frac{\bar{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

Hypothesis Tests for p

- **np ≥ 5 and n(1-p) ≥ 5**
- **np < 5 or n(1-p) < 5** → Not discussed in this chapter

Example: z Test for Proportion

A marketing company claims that it receives 8% responses from its mailing. To test this claim, a random sample of 500 were surveyed with 25 responses. Test at the α = .05 significance level.



Check:

n p = (500)(.08) = 40

n(1-p) = (500)(.92) = 460 ✓

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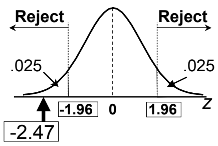
Z Test for Proportion: Solution

H_0: p = .08
H_A: p ≠ .08

α = .05
n = 500, $\bar{p}$ = .05

Critical Values: ± 1.96



Test Statistic:

$$z = \frac{\bar{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{.05 - .08}{\sqrt{\frac{.08(1-.08)}{500}}} = -2.47$$

Decision:

Reject H_0 at α = .05

Conclusion:

There is sufficient evidence to reject the company's claim of 8% response rate.

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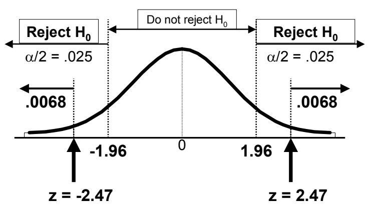
p -Value Solution

(continued)

Calculate the p-value and compare to α
(For a two sided test the p-value is always two sided)



p-value = .0136:

$$P(z \le -2.47) + P(x \ge 2.47)$$
$$= 2(.5 - .4932)$$
$$= 2(.0068) = 0.0136$$

Reject H_0 since p-value = .0136 < α = .05

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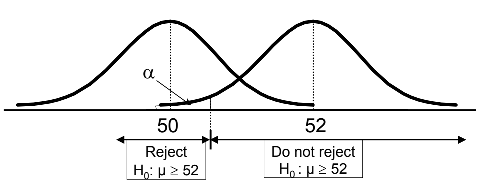
Type II Error

- **Type II error is the probability of failing to reject a false H_0**

Suppose we fail to reject **H_0: μ ≥ 52** when in fact the true mean is **μ = 50**



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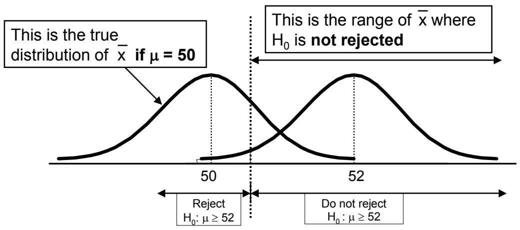
Type II Error

(continued)

- Suppose we do not reject H_0: $\mu \geq 52$ when in fact the true mean is $\mu = 50$



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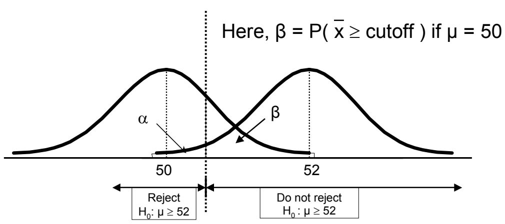
Type II Error

(continued)

- Suppose we do not reject H_0: $\mu \geq 52$ when in fact the true mean is $\mu = 50$



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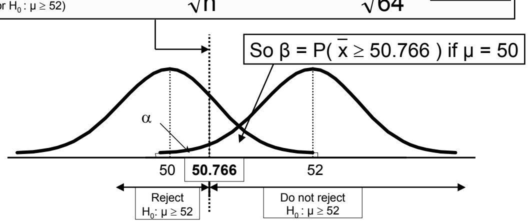
Calculating β

- Suppose n = 64 , σ = 6 , and α = .05

$$\text{cutoff} = \bar{x}_\alpha = \mu - z_\alpha \frac{\sigma}{\sqrt{n}} = 52 - 1.645 \frac{6}{\sqrt{64}} = \boxed{50.766}$$

(for H_0: $\mu \geq 52$)



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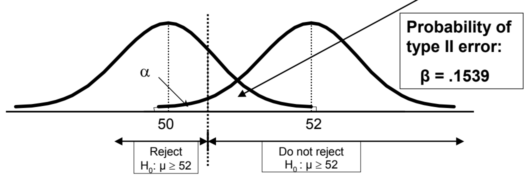
Calculating β

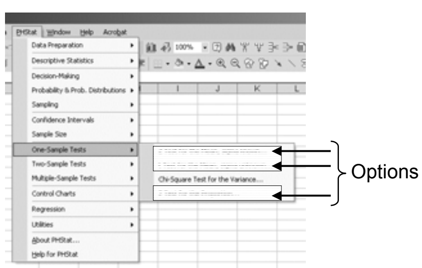
(continued)

- Suppose n = 64 , σ = 6 , and α = .05

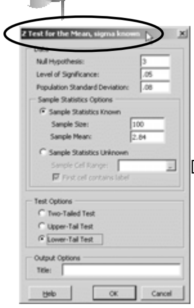
$$P(\bar{x} \geq 50.766 \mid \mu = 50) = P\left(z \geq \frac{50.766 - 50}{6/\sqrt{64}}\right) = P(z \geq 1.02) = .5 - .3461 = .1539$$

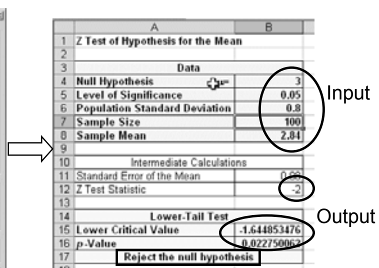


Using PHStat



Sample PHStat Output





	A	B
1	Z Test of Hypothesis for the Mean	
2		
3	Data	
4	Null Hypothesis	3
5	Level of Significance	0.05
6	Population Standard Deviation	0.8
7	Sample Size	100
8	Sample Mean	2.84
9		
10	Intermediate Calculations	
11	Standard Error of the Mean	0.08
12	Z Test Statistic	-2
13		
14	Lower-Tail Test	
15	Lower Critical Value	-1.644853476
16	p-Value	0.022750062
17	Reject the null hypothesis	
18		

Chapter Summary

- Addressed hypothesis testing methodology
- Performed z Test for the mean (σ known)
- Discussed p–value approach to hypothesis testing
- Performed one-tail and two-tail tests . . .

Chapter Summary

(continued)

- Performed t test for the mean (σ unknown)
- Performed z test for the proportion
- Discussed type II error and computed its probability

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