# Mishkin ch.4: Interest Rates

## Summary

1. Three key concepts:

**Present Value** **Yield to Maturity** **Total Return**

2. Know how to work with the key concepts:

**Task in Exams: Problem solving**

3. Applications:

Real versus nominal rates
Inflation-indexed bonds
PV of stocks: Discounted dividends
Tracking total returns

# Fundamentals

- Present Value:

$$PV_t = \frac{Payment_{t+1}}{(1+i)} + \frac{Payment_{t+2}}{(1+i)^2} + \ldots + \frac{Payment_{t+N}}{(1+i)^N}$$

i = discount rate = interest rate used to discount future payments

- Yield to Maturity (or simply: Yield) = Particular discount rate i that solves

$$P_B = PV_t = \frac{Payment_{t+1}}{(1+i)} + \frac{Payment_{t+2}}{(1+i)^2} + \ldots + \frac{Payment_{t+N}}{(1+i)^N}$$

One-period example (Treasury Bill promises $10,000 at maturity)

$$P_B = \frac{10000}{(1+i)} \quad \Rightarrow \quad (1+i) = \frac{10000}{P_B} \Rightarrow i = \frac{10000 - P_B}{P_B}$$

## Application: Coupon Bonds

- Defined by: Maturity date => Years to maturity = N
  Coupon = C
  Face Value = F => Coupon rate = C/F
- Market data: Price = $P_B$ or $P_{Bt}$ => Yield to maturity = i or $i_t$

We use subscript t (for time) when multiple dates are involved.
Yield or "interest rate" without modifiers refers to yield to maturity.

- Common problems: Compute yield from price. Compute price from yield.

## Example

- Bond quote from WSJ:

| Date: | 4/9/2018 | | | |
|---|---|---|---|---|
| Security | Maturity | Coupon Rate | Price | Yield |
| Treasury note | 3/31/2021 | 1.25% | 96.63 | 2.431% |

*Question: How is the yield calculated?*

## Example

| Date: | 4/9/2018 | | | |
|---|---|---|---|---|
| Security | Maturity | Coupon Rate | Price | Yield |
| Treasury note | 3/31/2021 | 1.25% | 96.63 | 2.431% |

- Task #1: Set up the present value equation

  About 3 years to maturity => N = 3

  Price is per F=100 face value => C = (Coupon Rate) * 100 = 1.25

  Payment at dates t+1,…,t+N-1: Coupon = C

  Payment at maturity date t+N: Face value + final coupon = F+C

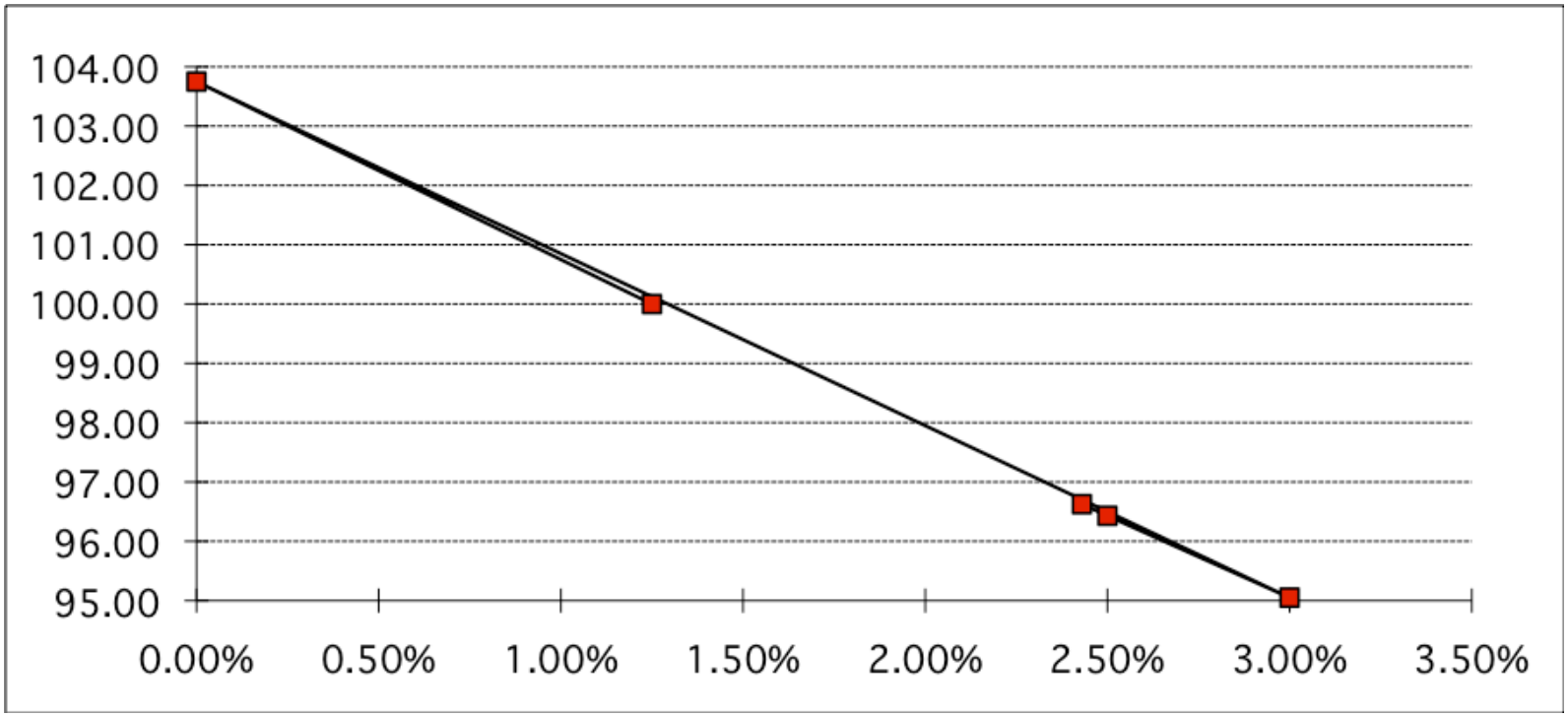
$$PV = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \frac{C+F}{(1+i)^3} = \frac{1.25}{(1+i)} + \frac{1.25}{(1+i)^2} + \frac{101.25}{(1+i)^3}$$

- Task #2: Solve the present value equation

  Cubic equation – solve numerically by exploiting that PV is declining in i.

*Knowing how this works is essential to understanding yield quotes.*

| Worksheet | Discount | PV | Graph each pair |
|---|---|---|---|
| try coupon: | 1.25% | 100.00 | -> par value |
| try low: | 0.00% | 103.75 | -> high PV => yield too low |
| try high: | 3.00% | 95.05 | -> low PV => yield too high |
| try between: | 2.50% | 96.43 | -> yield too high |
| try again: | 2.43% | 96.63 | -> close enough |



# The Total Return

(a.k.a: Return, Rate of Return)

- Definition:

$$\mathbf{RET} = \frac{\mathbf{Payment} + \mathbf{P_{t+1}} - \mathbf{P_t}}{\mathbf{P_t}}$$

- Measured over a specific time period t to t+1:

  $P_t$ = Price at the start; known.

  $P_{t+1}$ = Price at the end; often unknown at time t.

  Payment = current yield or other payout during the time period t to t+1; assumed known

- Can be computed for ANY financial asset – not only bonds

- Components:

$$\text{Current Yield} = \frac{\text{Payment}}{P_t}$$

$$\text{Capital Gain} = \frac{P_{t+1} - P_t}{P_t} = \frac{\text{Change in Price}}{\text{Initial Price}}$$

Application in Mishkin:

Total return on a coupon bond for one year:

$$RET = \frac{C + P_{Bt+1} - P_{Bt}}{P_{Bt}}$$

Mishkin's formula is a special case: Payment = C. Period = one year

Remember the general principle:

$$RET = \frac{Payment}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

e.g. if period = D days, then: Payment = (Annualized current yield) * D/365.
if asset = real estate, then: Payment = Rental income minus expenses.

• Caution: Yields are usually annualized. But current yield in RET refers to the period over which RET is calculated – may require conversion.

## Illustrations in Mishkin

# 1. Price – Yield Relation

**Table 1** Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

| Price of Bond ($) | Yield to Maturity (%) |
|---|---|
| 1,200 | 7.13 |
| 1,100 | 8.48 |
| 1,000 | 10.00 |
| 900 | 11.75 |
| 800 | 13.81 |

# 2. Key Linkages: Yield Change - Price Change - Return

**Table 2** One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

| (1) Years to Maturity When Bond Is Purchased | (2) Initial Current Yield (%) | (3) Initial Price ($) | (4) Price Next Year* ($) | (5) Rate of Capital Gain (%) | (6) Rate of Return [col (2) + col (5)] (%) |
|---|---|---|---|---|---|
| 30 | 10 | 1,000 | 503 | −49.7 | −39.7 |
| 20 | 10 | 1,000 | 516 | −48.4 | −38.4 |
| 10 | 10 | 1,000 | 597 | −40.3 | −30.3 |
| 5 | 10 | 1,000 | 741 | −25.9 | −15.9 |
| 2 | 10 | 1,000 | 917 | −8.3 | +1.7 |
| 1 | 10 | 1,000 | 1,000 | 0.0 | +10.0 |

- Lesson: Price responses increase with maturity.

  Quantitative measure of price-sensitivity: $\%\Delta P = -$ **Duration** $* \Delta i/(1+i)$

  Discount bonds have duration = maturity; coupon bonds have duration < maturity.

  Optional reading: Online Appendix to Ch.4

# Real Returns and Real Interest Rates

• **Real interest rate** = Nominal interest rate minus expected inflation

$$r = i - \pi^e$$

- Traditional measurement: Find nominal interest rates & estimate expected inflation

  Problem: expectations are not directly observable.

- Better approach: exploit yields on inflation-protected securities (in U.S. since 1997)

• **Treasury Inflation-Protected Securities (TIPS)**:

- Face value is fixed in real terms: $F_r = \$100$
- Nominal face value varies with CPI: $F = F_r * (CPI)$
- Nominal coupon varies with CPI: $C = (\text{Coupon rate}) * F$
- If Price = Face value, then: nominal return ≈ (Coupon rate) + (Change in CPI)

  => **Real yield to maturity = real return ≈ Coupon rate**

  => Interpret coupon rate as real interest rate.

- If Price <> Face value, real return differs from coupon rate & not easy to compute.

  => Rely on published sources for real yields to maturity (e.g. WSJ)

• Remember: **Quoted yields on TIPS are direct measures of REAL interest rates.**

• Expected inflation: compute $\pi^e = i - r$ from observed Treasury yields (i and r).

## Other Applications

- Present values of corporate stocks:

  - Payment = Dividend. Example of infinitely-lived asset.

$$PV_t = \frac{Dividend_{t+1}}{1+i} + \frac{Dividend_{t+2}}{(1+i)^2} + \ldots$$

  - More on stocks in Mishkin ch.7.

- British consols = coupon bonds without repayment date.

  - Simple present value formula from math of geometric sums:

$$P_B = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \ldots = \frac{C}{i}$$

  - Illustrates negative price-yield relationship.

## More on Bond Quotes: Review and Caveats

- Most of the time, economists use published or online quotes to obtain yields.
- Traders sometimes work with yields without mentioning prices.
  Then $P_B$ = PV is implied. Everyone knows.

- Bond quotes have two parts:
  (1) Identify the security: issuer, maturity date, coupon = fixed data.
  (2) Market information: price, yield, time of quote = changes over time.

- Implied items that also change: current yield, remaining maturity.
- Assumptions to check:
  Prices are usually per $100 face value; but sometimes per $1000.
  Prices are may be decimal or fractional (e.g. "$100:8" = 100\frac{8}{32} = 100.25$)
  *Good sources should have legends or footnotes to confirm interpretation*

- Simplifications for this class:
  Disregard discounting over fractional periods; usually maturity = whole years
  Disregard lumpiness in coupons; treat payments as smooth over the year
  Disregard "accrued interest" – use prices are as quoted "clean"