CHAPTER 5

Transportation Model and its Variants

Set 5.1a

1

(a) False
(b) True
(c) True

2

(a) $\sum a_i = 25$, $\sum b_j = 31$
Add a dummy source whose supply amount is $31-25=6$ units

(b) $\sum a_i = 74$, $\sum b_j = 65$
add a dummy destination whose demand amount is $74-65=9$ units

3

Denver will be 150 cars short. Similarly, Miami will be 50 cars short of satisfying its demand

4

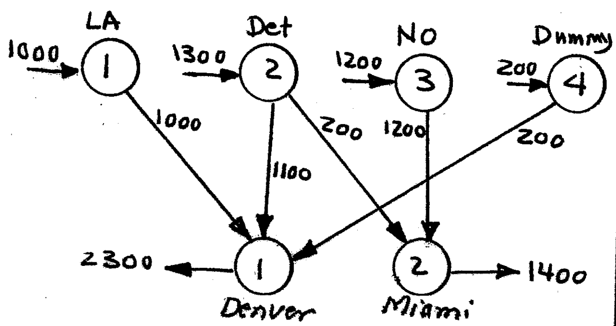
assign a very high cost M to the route from Detroit to Dummy

5

	Den 1	Miami 2	
LA 1	80: **1000**	M	**1000**
Det 2	100: **1100**	108: **200**	**1300**
NO 3	102	68: **1200**	**1200**
Dummy 4	200: **200**	300	**200**
	2300	**1400**	

use M = \$1000 in TORA

Optimum solution from TORA



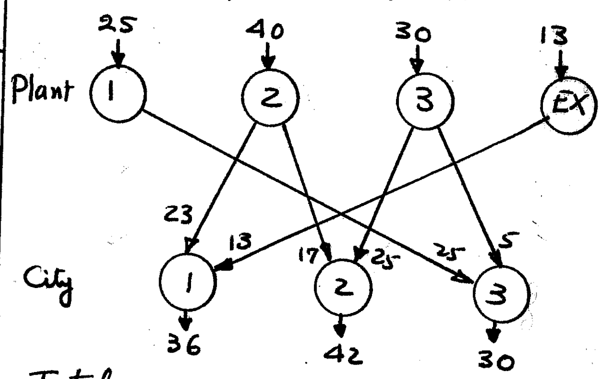
Denver is 200 cars short, Cost = \$33,200

6

(a)

	City 1	2	3	
Plant 1	600	700	400: **25**	**25**
Plant 2	320: **23**	300: **17**	350	**40**
Plant 3	500	480: **25**	450: **5**	**30**
Excess plant 4	1000: **13**	1000	M	**13**
	36	**42**	**30**	

(b) M = \$10,000 in TORA



Total cost = \$49,710

(c) City 1 excess cost = 13×1000 = \$13,000

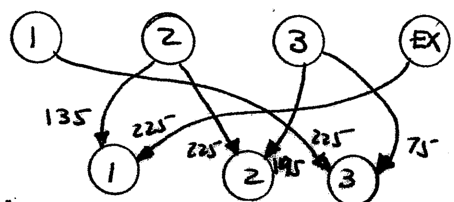
7

Assume units in 100,000 kWh

(a)

	City 1	2	3	
Plant 1	60	70	40: **225**	**225**
2	32: **135**	30: **225**	35	**360**
3	50	48: **195**	45: **75**	**270**
EX	100: **225**	100	M	**225**
	360	**420**	**300**	

(b)



Optimum cost = \$55,305

(c) City 1 excess cost = \$22,500

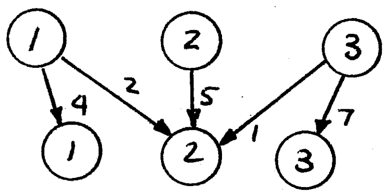
8

Unit transportation cost in thousand \$ per million gallons $= \left(\left(\frac{10¢}{1000} \times 10^6 \times \text{mileage}\right) \times \frac{1}{100}\right) \times \frac{1}{1000}$

$= \frac{\text{mileage}}{10}$

Distribution Area

	1	2	3	
Ref. 1	**4** (12)	**2** (18)	(M)	**6**
2	(30)	**5** (10)	(8)	**5**
3	(20)	**1** (25)	**7** (12)	**8**
	4	**8**	**7**	



Total cost = \$243,000

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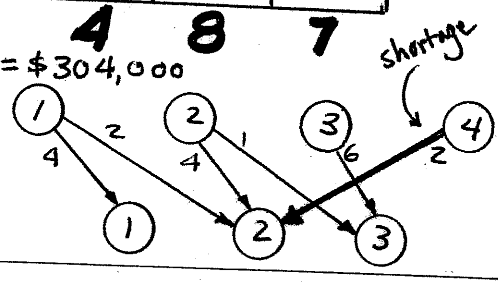
Unit cost in thousand \$ from Dummy source to distribution areas 2 or 3

$= \frac{5¢}{100} \times \frac{10^6}{10^3} = 50$ thousand \$/million gal

Distribution area

	1	2	3	
Ref. 1	**4** (12)	**2** (18)	(M)	**6**
2	(30)	**4** (10)	**1** (8)	**5**
3	(20)	(25)	**6** (12)	**6**
Dummy	(M)	**2** (50)	(50)	**2**
	4	**8**	**7**	

Cost = \$304,000



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Unit costs in thousand \$ per million gallons:

from refinery 1 to Dummy

$= \frac{\$1.50}{100} \times \frac{10^6}{10^3} = 15$

from refinery 2 to Dummy

$= \frac{\$2.20}{100} \times \frac{10^6}{10^3} = 22$

	1	2	3	Dummy	
Ref. 1	**4** (12)	**2** (18)	(M)	(15)	**6**
2	(30)	**5** (10)	(8)	(22)	**5**
3	(20)	**1** (25)	**4** (12)	**3** (0)	**8**
	4	**8**	**4**	**3**	

Refinery 3 diverts 3 million gallons for use within.

Total cost = \$207,000

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(a) Total supply = 150+200+250 = 600 crates

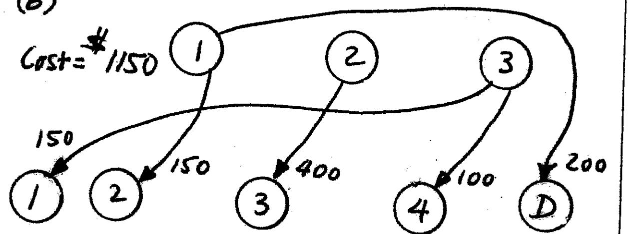
Total demand = 150+150+400+100 = 800 crates

(Potential overtime supply by each of orchards 1 & 2) = 800 − 600 = 200 crates

	1	2	3	4	Dummy	
Orch 1	(1)	(2) 150	(3)	(2)	(0) 200	150+200
2	(2)	(4)	(1) 400	(2)	(0)	200+200
3	(1) 150	(3)	(5)	(3) 100	(M)	250
	150	150	400	100	200	

(b)

Cost = \$1150



Problem has alternative optima.

(c) Orchad 1 = 0 overtime crates

Orchad 2 = 200 overtime crates

12

Supply/demand quantities are expressed in truckloads, determined by dividing the number of cars by 18 and rounding the result up, if necessary. For example, supply amount at center 1 is $\frac{400}{18} = 22.22$ or 23 truckloads. Expressing unit transportation costs in \$1000 per truck load, we get

	1	2	3	4	5	
1	2.5 (6)	3.75	5	3.5 (9)	.875 (8)	23
2	1.25	1.75 (3)	1.5 (9)	1.625	2	12
3	1	2.25 (9)	2.5	3.75	3.25	9
	6	12	9	9	8	

(b) alternative solution exists
Cost = \$92,500

13

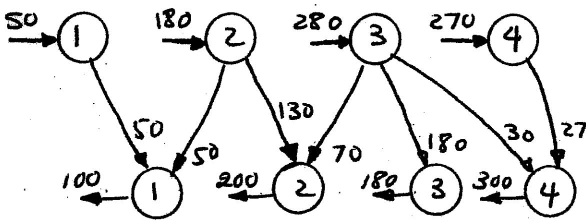
		Denver M1	Denver M2	Denver M3	Denver M4	Denver M1/2	Denver M3/4	Miami M1	Miami M2	Miami M3	Miami M4	Miami M1/3	Miami M2/4	
L.A.	M3			80 (400)			80 (120)			215 (180)		215		700
L.A.	M4				80 (300)		80				215		215	300
Det	M1	100 (500)				100		108				108		500
Det	M2		100 (450)			100 (70)			108 (75)				108 (5)	600
Det	M4				100 (180)		100 (100)				108 (95)		108 (25)	400
N.O.	M1	102 (130)				102 (50)		68 (540)				68 (80)		800
N.O.	M2		102			102			68 (400)				68	400
		630	450	400	480	120	220	540	475	180	95	80	30	

Optimum solution:

LA - Denver M4 = 300 cars
Det. - Denver M1 = 500 cars
Det. - Denver M2 = 450 cars
Det. - Denver M1/M2 = 70 cars
Det. - Miami M2 = 75 cars
Det. - Miami M2/4 = 5 cars
Det. - Denver M4 = 180 cars
Det. - Denver M3/4 = 100 cars
Det. - Miami M4 = 95 cars
Det. - Miami M2/4 = 25 cars
N.O. - Denver M1 = 130 cars
N.O. - Denver M1/2 = 50 cars
N.O. - Miami M1 = 540 cars
N.O. - Miami M1/3 = 80 cars
N.O. - Miami M2 = 400 cars
Total cost = \$343,620

1

	1	2	3	4	
1	40 (50)	40.4	40.7	41.4	50
2	42 (50)	40 (130)	40.3	41	180
3	44	42 (70)	40 (180)	40.7 (30)	280
4	46	44	42	40 (270)	270
	100	200	180	300	



Cost = $31,461

2

Least-cost starting solution.
(Problem has alternative optima.)

	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Disposal	
New	12 (24)	12 (12)	12 (2)	12	12	12	12	0 (86)	124
Mon		6	6	3 (6)	1 (18)	1	1	0	24
Tue			6 (12)	6	3	1	1	0	12
Wed				6 (14)	6	3	1	0	14
Thu					6	6	3 (20)	0	20
Fri						6 (14)	6	0 (4)	18
Sat							6 (2)	0 (12)	14
Sun								0 (22)	22
	24	12	14	20	18	14	22	124	

The given optimum solution is interpreted as summarized below.

Total cost = $804

continued...

Day	New	Sharpening Service: Overnite	2-day	3-day	Disposal
Mon	24	0	6	18	0
Tue	12	12	0	0	0
Wed	2	14	0	0	0
Thu	0	0	20	0	0
Fri	0	14	0	0	4
Sat	0	2	0	0	12
Sun	0	0	0	0	22

3

	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Disposal	
New	12 (24)	12	12 (8)	12	12	12	12	0 (92)	124
Mon		6	6.5	3 (12)	3.5	4	4.5	0	24
Tue			6 (6)	6.5	3 (6)	3.5	4	0	12
Wed				6 (8)	6.5	3 (6)	3.5	0	14
Thu					6 (12)	6.5	3 (8)	0	20
Fri						6 (8)	6.5	0 (10)	18
Sat							6 (14)	0	14
Sun								0 (22)	22
	24	12	14	20	18	14	22	124	

Day	New	Sharpening Service: Overnight	2-day	Disposal
Mon	24	12	12	0
Tue	0	6	6	0
Wed	8	8	6	0
Thu	0	12	8	0
Fri	0	8	0	10
Sat	0	14	0	0
Sun	0	0	0	22

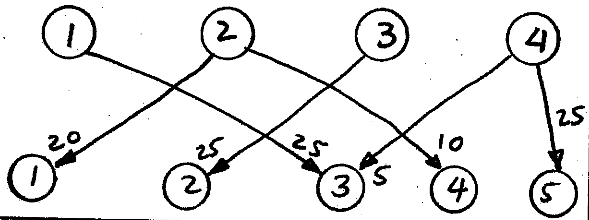
Total cost = $840
alternative solution exists

Set 5.2a

4

Machine \ Task	1	2	3	4	5	
1	10	2	3 (25)	15	9	25
2	5 (20)	10	15	2 (10)	4	30
3	15	5 (20)	14	7	15	20
4	20	15	13 (5)	M	8 (25)	30
	20	20	30	10	25	

Total cost = \$560



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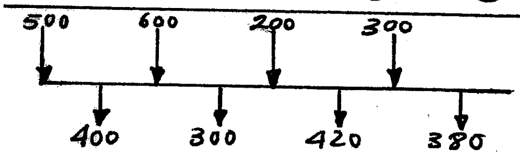


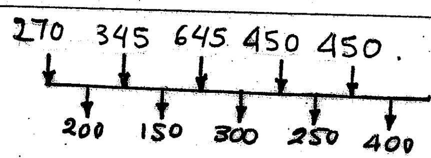
c: \$100 \$140 \$120 \$150
h: \$3 \$3 \$3 \$3

	1	2	3	4	Surplus	
1	100 (400)	103 (100)	106	M	0	500
2	M	140 (200)	143 (220)	146 (180)	0	600
3	M	M	120 (200)	123	0	200
4	M	M	M	150 (200)	0 (100)	300
	400	300	420	380	100	

Cost = \$190,040, alternative solution exists

Period	Capacity	Amt Prod.	Delivery
1	500	500	400 for 1 100 for 2
2	600	600	200 for 2 220 for 3 180 for 4
3	200	200	200 for 3
4	300	200	200 for 4

6



	1	2	3	4	5	Surplus	
R_1	100 (180)	104	108	112	116	0	180
O_1	150 (20)	154	158	162	166	0 (70)	90
R_2		96 (150)	100 (80)	104	108	0	230
O_2		144	148	152	156	0 (115)	115
R_3			116 (220)	120	124 (50)	0 (160)	430
O_3			174	178	182	0 (215)	215
R_4				102 (250)	106 (50)	0	300
O_4				153	157	0 (150)	150
R_5					106 (300)	0	300
O_5					159	0 (150)	150
	200	150	300	250	400	860	

Cost = \$137,720
alternative solution exists.

Period	Production schedule
1	Regular – 180 engines Overtime – 20 engines
2	Regular: 230 engines
3	Regular 270 engines
4	Regular 300 engines
5	Regular 300 engines

7

	1	2	3	4	5	6	Disposal	
New	200 / 200	210 / 180	226.5 / 140	231.53	243.1	255.26	0 / 878	1398
1		120	121.5 / 12	35 / 188	36.5	38	0	200
2			120 / 148	121.5	35 / 32	36.5	0	180
3				120 / 10	121.5	35 / 290	0	300
4					120 / 198	121.5	0	198
5						120	0 / 230	230
6							0 / 290	290
	200	180	300	198	230	290	1398	

Cost = \$170,698

alternative solution exists

Month	New	overhaul 1-day	overhaul 3-day	Disposal
1	200	12	188	0
2	180	148	32	0
3	140	10	290	0
4	0	198	0	0
5	0	0	0	230
6	0	0	0	290

8

(a) Use negative cost values

Loc \ Bidder	1	2	3	4	
1	-520	M	-650 / 10	-180	10
2	-210	-510 / 20	M	-430	20
3	-570	-495	-240	-710 / 30	30
Dummy	0 / 30	0 / 10	0 / 20	0	60
	30	30	30	30	

(b) Bidder 1 = 0 acre

Bidder 2 = 20 acres (location 1)

Bidder 3 = 10 acres (location 2)

Bidder 4 = 30 acres (location 3)

Set 5.3a

(a)

Northwest:

Cost = $42

5 ^{0}	1 ^{2}	^{1}	6
^{2}	4 ^{1}	3 ^{5}	7
^{2}	^{4}	7 ^{3}	7
5	5	10	

Least-cost:

Cost = $37

5 ^{0}	^{2}	1 ^{1}	6
^{2}	5 ^{1}	2 ^{5}	7
^{2}	^{4}	7 ^{3}	7
5	5	10	

Vogel:

Cost = $37

				Penalty	Penalty
5 ^{0}	^{2}	1 ^{1}	6	1	1
^{2}	5 ^{1}	^{5}	7	1	4
^{2}	^{4}	7 ^{3}	7	1	1
5	5	10			

Penalty 2 1 2 ← Step 1

Penalty — 1 2 ← Step 2

(b)

Northwest:

Cost = $94

7 ^{1}	^{2}	^{6}	7
3 ^{0}	9 ^{4}	^{2}	12
^{3}	1 ^{1}	10 ^{5}	11
10	10	10	

Least-Cost:

Cost = $61

^{1}	^{2}	7 ^{6}	7
10 ^{0}	^{4}	2 ^{2}	12
^{3}	10 ^{1}	1 ^{5}	11
10	10	10	

VAM:

Cost = $40

			Penalties		
7 ^{1}	^{2}	^{6}	1	1	1
2 ^{0}	^{4}	10 ^{2}	2	4	—
1 ^{3}	10 ^{1}	^{5}	2	2	2

Penalties
1 1 3 ← Step 1
2 1 — ← Step 2
2 1 — ← Step 3

(c)

Northwest:

Cost = $104

9 ^{5}	3 ^{1}	^{8}	12
^{2}	7 ^{4}	7 ^{0}	14
^{3}	^{6}	4 ^{7}	4
9	10	11	

Least-Cost

Cost = $38

2 ^{5}	10 ^{1}	^{8}	12
3 ^{2}	^{4}	11 ^{0}	14
4 ^{3}	^{6}	^{7}	4
9	10	11	

VAM:

Cost = $38

				Penalties	
2 ^{5}	10 ^{1}	^{8}	12	4	4
3 ^{2}	^{4}	11 ^{0}	14	2	2
4 ^{3}	^{6}	^{7}	4	3	3
9	10	11			

Penalties
1 3 7 ← Step 1
1 3 — ← Step 2

continued...

continued...

(i)

u \ v	0	2	6	
0	c=0: 5	c=2: 1 −	c=1: 5 +	6
−1	c=2: −3	c=1: 4 +	c=5: 5 −	9
−3	c=2: −5	c=4: −5	c=3: 5	5
	5	5	10	

u \ v	0	−3	1	
0	c=0: 5 −	c=2: −5	c=1: 1 +	6
4	c=2: 2 +	c=1: 5	c=5: 4 −	9
7	c=2: 0	c=4: −5	c=3: 5	5
	5	5	10	

u \ v	0	−1	1	
0	c=0: 1	c=2: −3	c=1: 5	6
2	c=2: 4	c=1: 5	c=5: −2	9
2	c=2: 0	c=4: −3	c=3: 5	5
	5	5	10	

cost = \$33
alternative solution exists

(ii)

u \ v	0	4	2	
0	c=0: 7	c=4: 1	c=2: 0	8
−1	c=2: −3	c=3: 5	c=4: −3	5
−2	c=1: −3	c=2: 0	c=0: 6	6
	7	6	6	

Problem has alternative optima. Cost = \$19
Note: If x_{23} were selected as the zero in place of x_{32}, solution would require one more iteration.

continued...

(iii)

u \ v	M	M−3	M−5	
0	c=M: 4 −	c=3: M−6 +	c=5: M−10	4
7−M	c=7: 1 +	c=4: 6 −	c=9: −7	7
11−M	c=1:	c=8: 0	c=6: 19	19
	5	6	19	

u \ v	6	3	1	
0	c=M: 6−M	c=3: 4	c=5: −4	4
1	c=7: 5 −	c=4: 2 +	c=9: −7	7
5	c=1: 10 +	c=8: 0 −	c=6: 19	19
	5	6	19	

u \ v	6	3	11	
0	c=M: 6−M	c=3: 4 −	c=5: 6 +	4
1	c=7: 5 −	c=4: 2 +	c=9: 3	7
−5	c=1: 0 +	c=8: −10	c=6: 19 −	19
	5	6	19	

u \ v	0	−3	5	
0	c=M: −M	c=3: −6	c=5: 4	4
7	c=7: 1 −	c=4: 6	c=9: 3 +	7
1	c=1: 4 +	c=8: −10	c=6: 15 −	19
	5	6	19	

u \ v	0	0	5	
	c=M: −M	c=3: −3	c=5: 4	4
	c=7: −3	c=4: 6	c=9: 1	7
	c=1: 5	c=8: −7	c=6: 14	19
	5	6	19	

Cost = \$142

continued...

(c)

Method	Nbr. of iterations (i)	(ii)	(iii)
NW	3	4	5
Least cost	2	2	2
Vogel	2	1	1

Least-cost starting solution:

2

u \ v	2	1	2	
	5: -3	1: 10	7: -5	10
	6: 70 −	4: 1 +	6: 10	80
	3: 5 +	2: 10 −	5: -2	15
	5: -3	3: -2	2: 40	40
	75	20	50	

u \ v	3	1	3	
0	5: -2	1: 10	7: -4	10
3	6: 60	4: 10	6: 10	80
0	3: 15	2: -1	5: -2	15
-1	5: -3	3: -3	2: 40	40
	75	20	50	

Destination 3 will be 40 units short. Optimum cost = $595

Least-cost starting solution:

3

u \ v	2	1	1	
0	5: -3	1: 10	7: -6	10
4	6: 30	4: 1	6: 50	80
1	3: 5	2: 10	5: -3	15
-2	0: 40	0: -1	M: -1-M	40
	75	20	50	

u \ v	3	1	3	
0	5: -2	1: 10	7: -4	10
3	6: 20	4: 10	6: 50	80
0	3: 15	2: -1	5: -2	15
-3	0: 40	0: -2	M: -M	40
	75	20	50	

Total cost = $515. Dest. 1 is 40 units short.

Vogel method:

4

1	2	1	5	0
3	4	5	M	1
2	3	3	3: 20	1
1	1	2	2	

1	2	1: 20		0
3	4	5		1
2	3	3		1
1	1	2		

3	4	5		1
2	3	3: 0		1
1	1	2		

3: 20	4: 20			1
2: 10	3			1
1	1			

$u \backslash v$	0	1	1	1	
0	-1 (cost 1)	-1 (cost 2)	**20** (cost 1)	-2 (cost 3)	**20**
3	**20** (cost 3)	**20** (cost 4)	-1 (cost 5)	$4-M$ (cost M)	**40**
2	**10** (cost 2)	(0) (cost 3)	**0** (cost 3)	**20** (cost 3)	**30**
	30	**20**	**20**	**20**	

Cost = \$240 — alternative solution exists

5

$u \backslash v$	2	5	10	
-2	15 (c_{11})	(c_{12})	(c_{13})	15
3	5 (c_{21})	25 (c_{22})	(c_{23})	30
5	(c_{31})	5 (c_{32})	80 (c_{33})	85
	20	30	80	

(a) $c_{ij} = u_i + v_j$ for basic x_{ij}

Thus,

$c_{11} = 2 - 2 = 0$

$c_{21} = 3 + 2 = 5$

$c_{22} = 3 + 5 = 8$

$c_{32} = 5 + 5 = 10$

$c_{33} = 5 + 10 = 15$

Cost $= 15 \times 0 + 5 \times 5 + 25 \times 8 + 5 \times 10 + 80 \times 15 = \$1475$

(b) $u_i + v_j - c_{ij} \leq 0$ for nonbasic x_{ij}

$-2 + 5 - c_{12} \leq 0 \implies c_{12} \geq 3$

$-2 + 10 - c_{13} \leq 0 \implies c_{13} \geq 8$

$3 + 10 - c_{23} \leq 0 \implies c_{23} \geq 13$

$5 + 2 - c_{31} \leq 0 \implies c_{31} \geq 7$

continued...

Problems 6 and 7 on next page

continued...

Set 5.3b

6

(a) For basic x_{ij}, $c_{ij} = u_i + v_j$.

$u \backslash v$	2	2	5	
1	$c_{11}=3$ (10)	$1+2\theta$	$1+3\theta$	10
−1	$2+\theta$	$c_{22}=1$ (20)	$c_{23}=4$ (20)	40
	10	20	20	

Cost $= 3\times 10 + 1\times 20 + 4\times 20 = \$130$

(b) For nonbasic x_{ij}: $u_i + v_j - c_{ij} \le 0$ to satisfy optimality. Hence

$$2+1-(1+2\theta) \le 0 \implies \theta \ge 1$$

$$5+1-(1+3\theta) \le 0 \implies \theta \ge 5/3$$

$$2-1-(2+\theta) \le 0 \implies \theta \ge -1$$

Take $\theta = \frac{5}{3}$ to yield $x_{13} = 0$ as the zero basic variable.

7

	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	
min Z =	1	1	2	6	5	1	
s.t.	1	1	1				≥ 5
				1	1	1	≥ 6
	1			1			≥ 2
		1			1		≥ 7
			1			1	≥ 1

$x_{ij} \ge 0$ for all i and j

Optimum LP solution using TORA:

$Z = 15$, $x_{11} = 2$, $x_{12} = 7$, $x_{23} = 6$

If we replace the first two constraints with equations, we get the optimum solution:

$Z = 27$, $x_{11} = 2$, $x_{12} = 3$,

$x_{22} = 4$, $x_{23} = 2$

The new solution is worse!

1

	u_1	u_2	u_3	v_1	v_2	v_3	v_4	
max	15	25	10	5	15	15	15	
s.t.								
	1			1				≤ 10
	1				1			≤ 2
	1					1		≤ 20
	1						1	≤ 11
		1		1				≤ 12
		1			1			≤ 7
		1				1		≤ 9
		1					1	≤ 20
			1	1				≤ 4
			1		1			≤ 14
			1			1		≤ 16
			1				1	≤ 18

From Table 5-25:

$u_1 = 0,\ u_2 = 5,\ u_3 = 7$

$v_1 = -3,\ v_2 = 2,\ v_3 = 4,\ v_4 = 11$

$$\text{Optimum } w = 15\times 0 + 25\times 5 + 10\times 7 + 5\times -3 + 15\times 2 + 15\times 4 + 15\times 11 = \$435$$

2

$$\text{minimize } z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i,\quad i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j,\quad j = 1, 2, \ldots, n$$

Next, consider

$$z' = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}+K)x_{ij}$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + K\sum_{i=1}^{m}\left(\sum_{j=1}^{n} x_{ij}\right)$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + K\sum_{i=1}^{m} a_i$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + K,\ K \text{ is a constant}$$

$$= z + K$$

This result shows that optimization using z and z' yield the same optimum values of x_{ij}.

To show why the dual values associated with a given primal basic solution are not unique, note that, for any constant K,

$$\begin{pmatrix}\text{Dual}\\ \text{Values}\end{pmatrix} = \begin{pmatrix}\text{Original basic}\\ \text{obj. coefficients}\\ + \\ K\end{pmatrix} \times \text{Inverse}$$

This means that even though the optimal primal solution is unique for all K, there are infinity of dual values, each corresponding to a given value of K.

The conclusion is that an arbitrary value assigned to one of the dual variables (e.g., $u_1 = 0$) implies a specific value for the constant K.

continued...

Set 5.4a

1

(a-i)

					Row min
3	8	2	10	3	2
8	7	2	9	7	2
6	4	2	7	5	2
8	4	2	3	5	2
9	10	6	9	10	6

1	6	0	8	1
6	5	0	7	5
4	2	0	5	3
6	2	0	1	3
3	4	0	3	4
Col min → 1	2	0	1	1

Assignment:
1-5
2-3
3-2
4-4
5-1
Cost = \$21

0	4	2	7	$\boxed{0}$
3	1	$\boxed{0}$	4	2
3	$\boxed{0}$	2	4	2
5	0	2	$\boxed{0}$	2
$\boxed{0}$	0	0	0	1

(a-ii)

3	9	2	2	7	2
6	1	5	6	6	1
9	4	7	10	3	3
2	5	4	2	1	1
9	6	2	4	6	2

1	7	0	1	5
5	0	4	5	5
6	1	4	7	0
1	4	3	1	0
7	4	0	2	4
Col min 1	0	0	1	0

continued...

$\boxed{0}$	7	0	0	5
4	$\boxed{0}$	4	4	5
5	1	4	6	$\boxed{0}$
0	4	3	$\boxed{0}$	0
6	4	$\boxed{0}$	1	4

Optimum:
1-1
2-2
3-5
4-4
5-3
Cost = \$11

2

5	5	M	2
7	4	2	3
9	3	5	M
7	2	6	7

3	3	M-2	0
5	2	0	1
6	0	2	M-3
5	0	4	5

(All entries are divided by 10 for convenience)

~~0~~	~~3~~	~~M-2~~	~~0~~
~~2~~	~~2~~	~~0~~	~~1~~
3	~~0~~	2	M-3
2	~~0~~	4	5

0	5	M-2	$\boxed{0}$
2	4	$\boxed{0}$	1
1	$\boxed{0}$	0	M-5
$\boxed{0}$	0	4	5

Optimum: 1-4, 2-3, 3-2, 4-1
Cost = \$140

3

Worker \ Job	1	2	3	4	5
1	50	50	M	20	0
2	70	40	20	30	0
3	90	30	50	M	0
4	70	20	60	70	0
5	60	45	30	80	0

Job 5 is dummy

	1	2	3	4	5
1	~~0~~	~~30~~	~~M-20~~	~~0~~	~~0~~
2	~~20~~	~~20~~	~~0~~	~~10~~	~~0~~
3	40	10	30	M-20	~~0~~
4	~~20~~	~~0~~	~~40~~	~~50~~	~~0~~
5	10	25	10	60	~~0~~

	1	2	3	4	5
1	0	30	M-20	$\boxed{0}$	10
2	20	20	$\boxed{0}$	10	10
3	30	0	20	M-30	$\boxed{0}$
4	20	$\boxed{0}$	40	50	10
5	$\boxed{0}$	15	0	50	0

Optimum:
1-4
2-3
3-5
4-2
5-1

Worker 3 is assigned to dummy job 5. Thus, worker 5 must replace worker 3.

4

Add a "dummy" operator with zero assignment cost to each job (including the fifth). The optimal solution will show the replacement by indicating which of the current jobs (1 thru 4) is assigned to the dummy operator. If the dummy operator is assigned to the new job, then the new job must assume lower priority to the current four jobs.

(all assignment cost are divided by 10 for convenience.)

$$
\begin{array}{cc|ccccc|l}
 & & & & \text{Job} & & & \\
 & & 1 & 2 & 3 & 4 & 5 & \\
\hline
 & 1 & 5 & 5 & M & 2 & 2 & \\
 & 2 & 7 & 4 & 2 & 3 & 1 & \\
\text{Operator} & 3 & 9 & 3 & 5 & M & 2 & \\
 & 4 & 7 & 2 & 6 & 7 & 8 & \\
 & 5 & 0 & 0 & 0 & 0 & 0 & \leftarrow \text{Dummy} \\
\hline
\end{array}
$$

$$
\begin{bmatrix}
3 & 3 & M-3 & 0 & 0 \\
6 & 3 & 1 & 2 & 0 \\
7 & 1 & 3 & M-2 & 0 \\
5 & 0 & 4 & 5 & 0 \\
0 & 0 & 0 & 0 &
\end{bmatrix}
$$

$$
\begin{bmatrix}
2 & 2 & M-4 & \boxed{0} & 0 \\
5 & 2 & \boxed{0} & 2 & 0 \\
6 & 0 & 2 & M-2 & \boxed{0} \\
5 & \boxed{0} & 4 & 6 & 7 \\
\boxed{0} & 0 & 0 & 1 & 0
\end{bmatrix}
$$

Optimum:
1 – 4
2 – 3
3 – 5
4 – 2
5 – 1

Since dummy operator is assigned to job 1, new job 5 has higher priority over job 1.

5

Define the following two sets:

Set 1: (DA,3), (DA,10), (DA,17), (DA,25)

Set 2: (AT,7), (AT,12), (AT,21), (AT,28).

The idea is to match one element from Set 1 with another element from Set 2. The matching automatically decides the date and location for the purchase of each ticket. For example, consider the following assignment:

(DA,3) – (AT,21)
(DA,10) – (AT,7)
(DA,17) – (AT,28)
(DA,25) – (AT,12)

This assignment can be interpreted as follows:

Ticket 1:	June 3	DA → AT
	June 21	AT → DA
Ticket 2:	June 7	AT → DA
	June 10	DA → AT
Ticket 3:	June 17	DA → AT
	June 28	AT → DA
Ticket 4:	June 12	AT → DA
	June 25	DA → AT

The complete assignment model is given below

	A,7	A,12	A,21	A,28
D,3	400	300	300	280
D,10	300	400	300	300
D,17	300	300	400	300
D,25	300	300	300	400

Optimum:

(D,3) – (A,28) (A,21) – (D,25)
(A,7) – (D,10) (A,12) – (D,17)

Problem has alternative optima.

continued...

6

Distance matrix in meters:

$$
\begin{array}{cc}
 & \text{candidate areas} \\
 & \begin{array}{cccc} a & b & c & d \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 50 & 50 & 95 & 45 \\ 30 & 30 & 55 & 65 \\ 70 & 50 & 25 & 55 \\ 100 & 60 & 55 & 25 \end{bmatrix}
\end{array}
$$

existing centers

A measure of the optimal assignment of new centers to candidate locations must reflect both distance and frequency of trips; that is

$$
\begin{array}{ccc}
 & \text{existing} & \text{candidate} \\
 & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} & \begin{array}{cccc} a & b & c & d \end{array} \\
\begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{array} & \begin{bmatrix} 10 & 7 & 0 & 11 \\ 2 & 1 & 8 & 4 \\ 4 & 9 & 6 & 0 \\ 3 & 5 & 2 & 7 \end{bmatrix} & \begin{bmatrix} 50 & 50 & 95 & 45 \\ 30 & 30 & 55 & 65 \\ 70 & 50 & 25 & 55 \\ 100 & 60 & 55 & 25 \end{bmatrix}
\end{array}
$$

new

$$
\begin{array}{cc}
 & \begin{array}{cccc} a & b & c & d \end{array} \\
\begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{array} & \begin{bmatrix} 1810 & 1370 & 1940 & 1180 \\ 1090 & 770 & 665 & 695 \\ 890 & 770 & 1025 & 1095 \\ 1140 & 820 & 995 & 745 \end{bmatrix}
\end{array}
$$

New

TORA optimum assignment:

I - d

II - c

III - a

IV - b

7

The ranking of the projects by the different teams can use the following numeric score

1 : Highest preference

1 : lowest preference

A tie in preference between two or more projects is indicated by assigning the projects the same score. For example, the scores

Project	1	2	3	4	5	6	7	8	9	10
Score	9	9	8	7	3	5	4	1	2	6

indicate that project 8 is the most preferred and projects 1 and 2 tie for the least preferred status.

For the development of the model, we use the following numeric designations for the projects

Project nbr.	Project name
1	Boeing-F15
2	Boeing-F18
3	Boeing Simulation
4	Cargil
5	Cobb-Vantress
6	ConAgra
7	Cooper
8	DaySpring (layout)
9	DaySpring (Materials)
10	JB Hunt
11	Raytheon
12	Tyson South
13	Tyson East
14	WAL-MART
15	Yellow

continued...

The following is a typical summary of preference scores submitted by the 11 teams:

Project	Team 1*	2	3	4	5	6*	7*	8	9*	10	11
1⊗	—	1	2	2	1	—	—	1	—	2	15
2⊗	—	1	3	1	2	—	—	1	—	10	12
3	1	2	5	3	2	13	5	1	4	15	1
4	2	3	6	4	10	5	14	2	1	4	14
5	3	5	4	5	9	4	12	3	3	13	13
6	3	4	2	5	9	8	12	1	2	1	13
7	4	6	1	12	8	9	10	2	5	2	5
8	5	6	7	14	7	9	10	4	6	3	15
9	7	8	9	14	7	1	1	15	1	15	1
10	7	9	12	15	6	3	9	5	4	7	5
11⊗	—	9	13	6	5	—	—	7	—	6	7
12	13	10	14	7	4	2	8	9	15	4	9
13	14	11	1	8	3	13	7	8	1	8	9
14	15	12	5	9	1	14	7	6	2	9	10
15	15	13	7	10	2	15	6	1	3	1	11

* Team does not meet citizenship requirements

⊗ project requiring US citizenship

The problem is modeled as an assignment model. Entries — are replaced by M, a very large value. The model is unbalanced. Thus, 4 artificial teams must be added to balance the model. In the end four projects will not be assigned.

TORA Solution:

Project	Team	Score
1	2	1
2	4	1
3	11	1
4	1	1
5	None	—
6	8	1
7	3	1
8	None	—
9	7	1
10	None	—
11	None	—
12	6	2
13	10	1
14	5	1
15	10	1
	Total score	13

Average score $= \frac{13}{11} = 1.18$

The average score is close to 1, meaning that all preferences are well met.

continued....

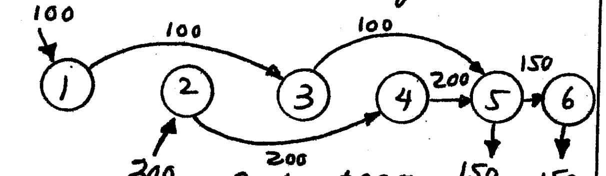
Set 5.5a

1

	3	4	5	6	
1	1 (100)	4	M	M	100
2	3	2 (200)	M	M	200
3	0 (200)	1	6 (100)	M	B
4	3	0 (100)	5 (200)	8	B
5	M	M	0 (150)	1 (150)	B
	B	B	150+B	150	

Let B = 300 units

Optimum solution using TORA:



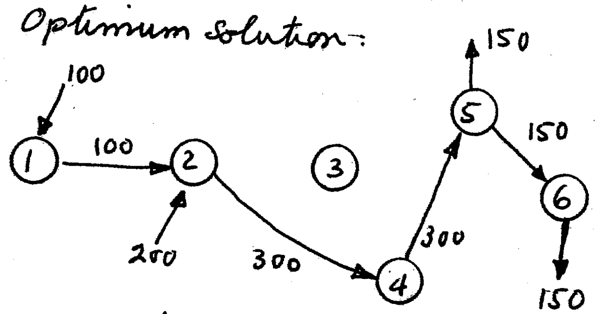
Cost = $2250

2

B = 300 units

	2	3	4	5	6	
1	1 (100)	5	4	M	M	100
2	0 (200)	3 (0)	2 (300)	M	M	200 + B
3	M	0 (300)	1	6 (0)	M	B
4	M	3	0	5 (300)	8	B
5	M	M	M	0 (150)	1 (150)	B
	B	B	B	150+B	150	

Optimum solution:

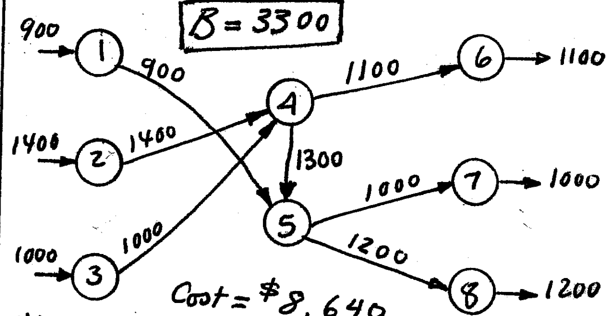


Cost = $2,350

3

	4	5	6	7	8	
1	1	.3 (900)	M	M	M	900
2	.8 (1400)	4.3	M	M	M	1400
3	2 (1000)	4.6	M	M	M	1000
4	0 (900)	.5 (1300)	.2 (1100)	4.5	6	B
5	M	0 (1100)	3	2.1 (1000)	1.9 (1200)	B
	B	B	1100	1000	1200	

$\boxed{B = 3300}$

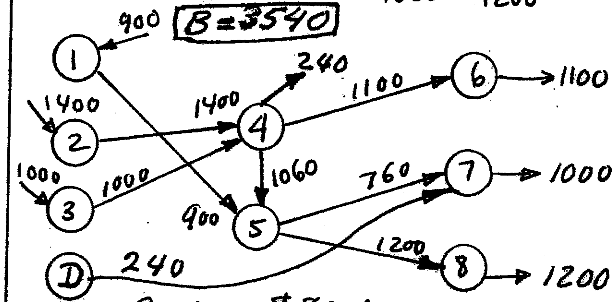


Cost = $8,640

(b)

	4	5	6	7	8	
1	1	.3 (900)	M	M	M	900
2	.8 (1400)	4.3	M	M	M	1400
3	2 (1000)	4.6	M	M	M	1000
4	0 (1380)	.5 (1060)	.2 (1100)	4.5	6	B
5	M	0 (1580)	3	2.1 (760)	1.9 (1200)	B
Dummy	0	0	0	0 (240)	0	240
	240+B	B	1100	1000	1200	

$\boxed{B = 3540}$



Cost = $8,016

4

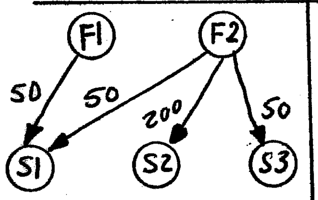
Cell entries: unit cost, with the circled allocation in parentheses.

	F1	F2	S1	S2	S3	Dum	
F1	0 (500)	6	7 (50)	8	9	0 (150)	200 + B
F2	6	0 (500)	5 (50)	4 (200)	3 (50)	0	300 + B
S1	7	2	0 (500)	5	1	0	B
S2	1	5	1	0 (500)	4	0	B
S3	8	9	7	6	0 (500)	0	B
	B	B	100 + B	200 + B	50 + B	150	

$\boxed{B = 500}$

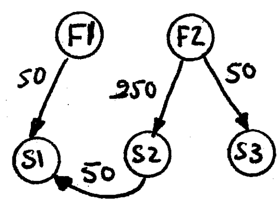
Alternative optima using TORA:

#1



Cost = \$1,550

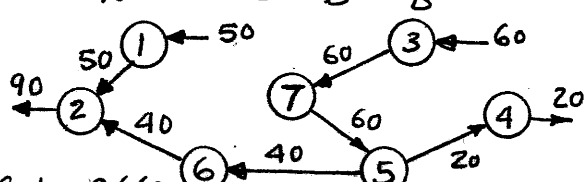
#2



5

assume that units of supply and demand are in thousand gallons. $\boxed{B = 110}$

	2	4	5	6	7	
1	20 (50)	M	M	M	3	50
3	M	30	M	M	9 (60)	60
5	M	2 (20)	0 (50)	4 (40)	10	B
6	8 (40)	M	4	0 (70)	M	B
7	40	M	10 (60)	M	0 (50)	B
	90	20	B	B	B	



Cost = 2,660,000 gallon miles

6

	2	3	4	5	6	7	
1	5	3 (1)	M	M	M	M	1
2	0 (1)	4	1	7	M	M	0+1
3	6	0	5	1	2 (1)	M	0+1
4	M	M	0 (1)	9	M	4	0+1
5	M	M	2	0 (1)	5	8	0+1
6	M	3	M	7	0	3 (1)	0+1
	0+1	0+1	0+1	0+1	0+1	1	

Optimum route using TORA:

$1 \rightarrow 3 \rightarrow 6 \rightarrow 7$

Distance = $3 + 2 + 3 = 8$

7

minimize z.

	x_{13}	x_{14}	x_{23}	x_{24}	x_{34}	x_{35}	x_{36}	x_{46}	x_{47}	x_{56}	x_{67}	
z=	3	4	2	5	7	8	6	4	9	5	3	
1	1	1										= 1000
2			1	1								= 1200
3	-1		-1		1	1	1					= 0
4		-1		-1	-1			1	1			= 0
5						-1				1		= -800
6							-1	-1		-1	1	= -900
7									-1		-1	= -500

Each node yields a constraint. The special characteristics of the model show that each column has exactly +1 and -1, with the remainder of the elements equal to zero.

8

x_{ij} = number of laborers hired at the <u>start</u> of period i and terminated at the <u>start</u> of period j.

Define nodes 1, 2, 3, 4, and 5 to correspond to the five months of the horizon. Node 6 is added to allow defining the variables x_{i6} that terminate at the end of the five-month planning horizon. The associated LP is defined below.

	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{23}	x_{24}	x_{25}	x_{26}	x_{34}	x_{35}	x_{36}	x_{45}	x_{46}	x_{56}	
	100	130	180	220	250	100	130	180	220	100	130	180	100	130	100	min
(1)	1	1	1	1	1											≥ 100
(2)		1	1	1	1	1	1	1	1							≥ 120
(3)			1	1	1		1	1	1	1	1	1				≥ 80
(4)				1	1			1	1		1	1	1	1		≥ 170
(5)					1				1			1		1	1	≥ 50

Let S_1, S_2, S_3, S_4, and S_5 be the surplus variables associated with constraints 1, 2, 3, 4, and 5, respectively. The LP after adding the surplus variables thus appears as

x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{23}	x_{24}	x_{25}	x_{26}	x_{34}	x_{35}	x_{36}	x_{45}	x_{46}	x_{56}	S_1	S_2	S_3	S_4	S_5	
100	130	180	220	250	100	120	180	220	100	130	180	100	130	100						min
1	1	1	1	1											−1					100
	1	1	1	1	1	1	1	1								−1				120
		1	1	1		1	1	1	1	1	1						−1			80
			1	1			1	1		1	1	1	1					−1		170
				1				1			1		1	1					−1	50

Next, perform the following transformation:

1. Leave equation (1) unchanged.
2. Replace equation (2) with (2) − (1).
3. Replace equation (3) with (3) − (2).
4. Replace equation (4) with (4) − (3).
5. Replace equation (5) with (5) − (4)
6. Add a new equation that equals −(5).

These transformations lead to the following LP

x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{23}	x_{24}	x_{25}	x_{26}	x_{34}	x_{35}	x_{36}	x_{45}	x_{46}	x_{56}	S_1	S_2	S_3	S_4	S_5	
100	130	180	220	250	100	130	180	220	100	130	180	100	130	100						min
1	1	1	1	1											−1					100
−1					1	1	1	1							1	−1				20
	−1				−1				1	1	1					1	−1			−40
		−1				−1			−1			1	1				1	−1		90
			−1				−1			−1		−1		1				1	−1	−120
				−1				−1			−1		−1	−1					1	−50

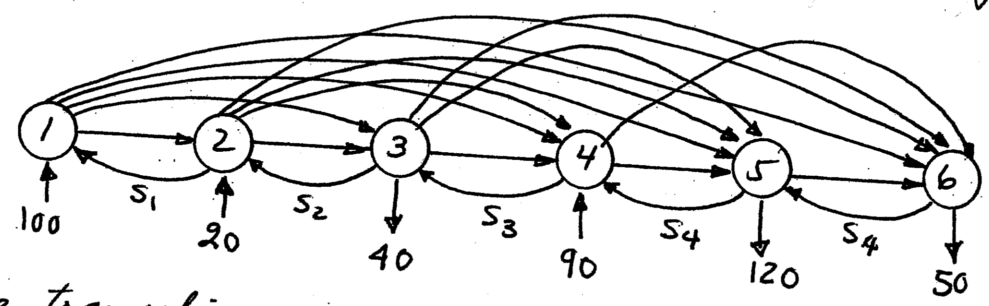
continued...

The last LP has the structure of a transshipment model (See Problem 7). Let

$S_1 = x_{21}$ $\quad S_3 = x_{43}$ $\quad S_5 = x_{65}$

$S_2 = x_{32}$ $\quad S_4 = x_{54}$

Then the LP above can be translated as a network as follows:

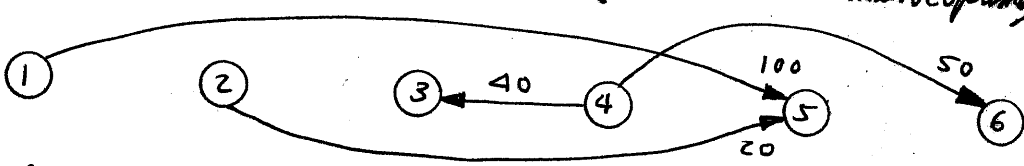


The transshipment model thus appears as

	1	2	3	4	5	6	
1	0	100	130	180	220	250	100 + B
2	0	0	100	130	180	220	20 + B
3	M	0	0	100	130	180	B
4	M	M	0	0	100	130	90 + B
5	M	M	M	0	0	100	B
6	M	M	M	M	0	0	B
	B	B	40 + B	B	120 + B	50 + B	

$B = 550$

The optimum solution from TORA is (Problem has alternative optima)



This solution can be interpreted as follows

1. Hire 100 laborers at the start of period 1 and terminate them at the start of period 5.
2. Hire 20 workers at the start of period 2 and terminate them at the start of period 5.
3. Hire 50 workers at the start of period 4 and terminate them at the start of period 6.

The solution satisfies the labor requirements exactly, except for period 3 where there is a surplus of 40 workers ($x_{43} = 40$).