CHAPTER 6

Network Models

Set 6.1a

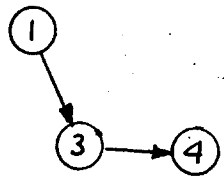
1

(i)

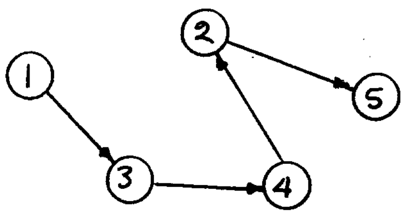
(a) Path: 1-3-4-2

(b) Cycle: 1-3-4-5-1

(c) Tree



(d) Spanning tree:

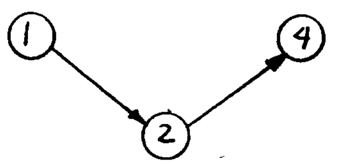


(ii)

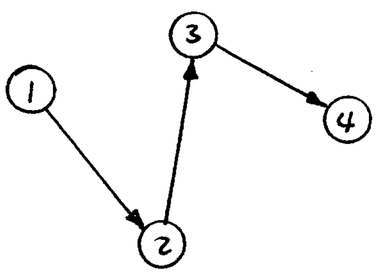
(a) Path: 1-2-3

(b) Cycle: 1-2-3-1

(c) Tree



(d) Spanning Tree:



2

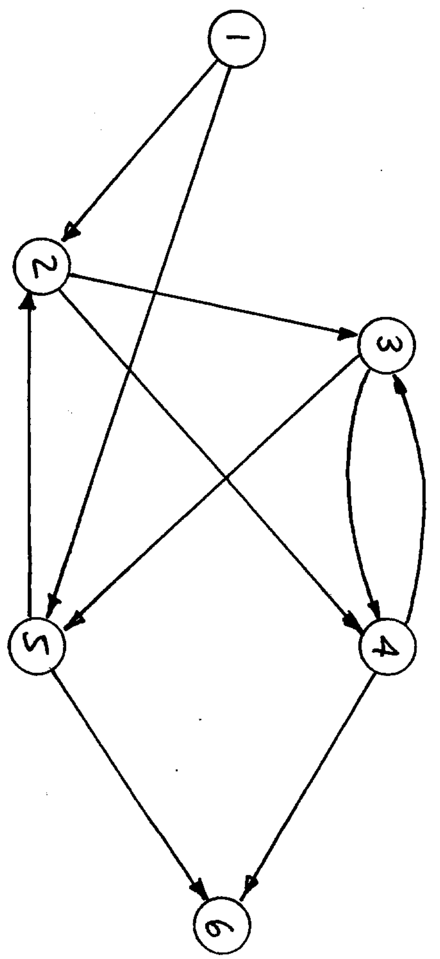
(i) $N = \{1, 2, 3, 4, 5\}$

$A = \{1\text{-}2, 1\text{-}3, 2\text{-}5, 3\text{-}4, 3\text{-}5, 4\text{-}2, 4\text{-}5, 5\text{-}1\}$

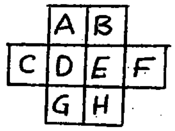
(ii) $N = \{1, 2, 3, 4\}$

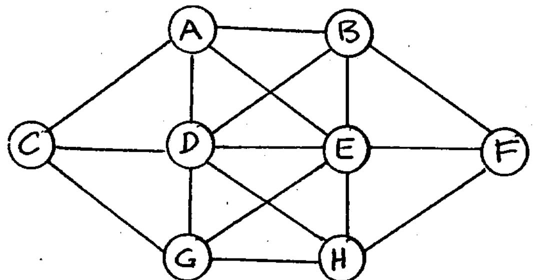
$A = \{1\text{-}2, 1\text{-}3, 2\text{-}3, 2\text{-}4, 3\text{-}4\}$

3

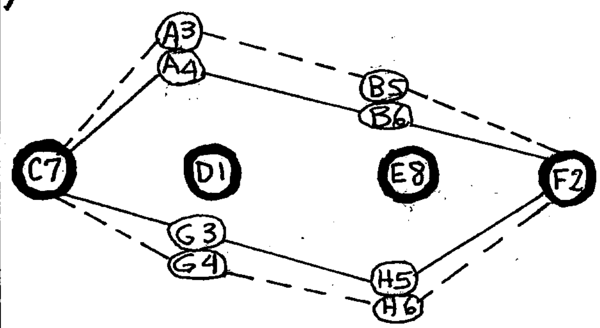


4





The network shows that nodes connected by an arc cannot hold consecutive numbers. Nodes D and E each has 6 emanating arcs, whereas all the remaining nodes have at most 4 emanating arcs. Because 1 and 8 each can have 6 nonconsecutive neighbors (namely, 1-3, 1-4, 1-5, 1-6, 1-7, 1-8 or 8-6, 8-5, 8-4, 8-3, 8-2, 8-1) and no other number has this property, 1 and 8 <u>must</u> be assigned to D and E. Letting D=1 and E=8, we <u>must</u> assign C=7 and F=2 because 2 and 7 can't be assigned anywhere else without violating the sequence condition. Next, we have the following possibilities:



Two possible solutions indicated by the solid and dashed arcs:

Switch D=1 and E=8 to two mirror arrangements.

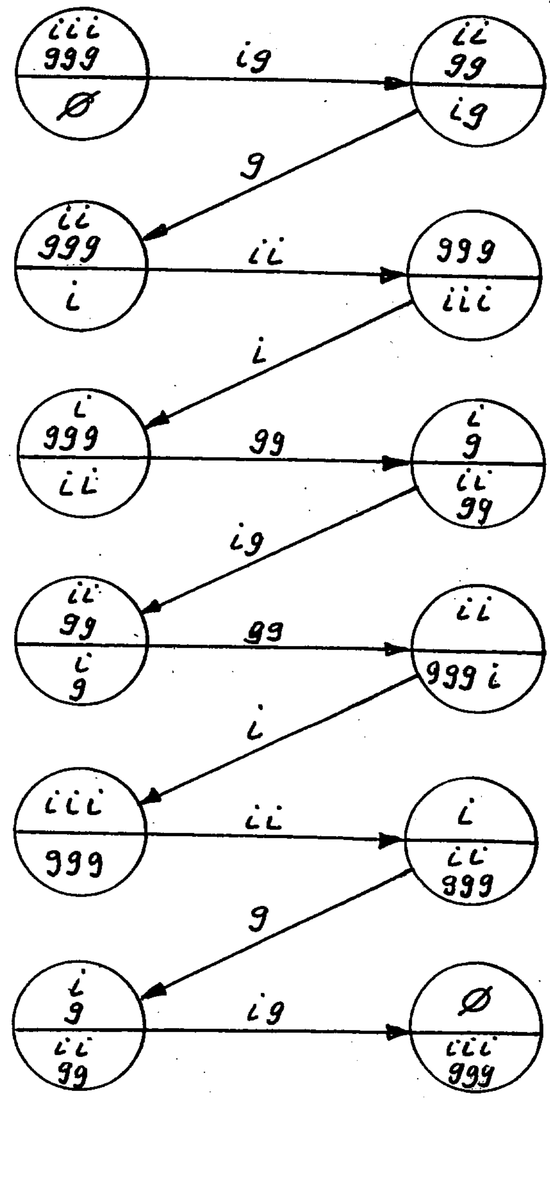
5

Let

$i \equiv$ inmate

$g \equiv$ guard

For each node, top half represents the number of i's and g's on the mainland side. The bottom half is that of Alcatraz.



Set 6.2a

1

Spanning tree length = 16

0.	Start at node N5
1.	Connect N2 to N5: Length = 3.
2.	Connect N1 to N2: Length = 1.
3.	Connect N4 to N2: Length = 4.
4.	Connect N6 to N4: Length = 3.
5.	Connect N3 to N4: Length = 5.

2

(a) Spanning tree length = 14

0.	Start at node N1
1.	Connect N2 to N1: Length = 1.
2.	Connect N5 to N2: Length = 3.
3.	Connect N6 to N5: Length = 2.
4.	Connect N4 to N6: Length = 3.
5.	Connect N3 to N4: Length = 5.

(b) Spanning tree length = 21

0.	Start at node N1
1.	Connect N2 to N1: Length = 1.
2.	Connect N4 to N2: Length = 4.
3.	Connect N6 to N4: Length = 3.
4.	Connect N3 to N4: Length = 5.
5.	Connect N5 to N4: Length = 8.

(c) Spanning tree length = 16

0.	Start at node N1
1.	Connect N2 to N1: Length = 1.
2.	Connect N5 to N2: Length = 3.
3.	Connect N6 to N2: Length = 4.
4.	Connect N4 to N6: Length = 3.
5.	Connect N3 to N4: Length = 5.

(d) Spanning tree length = 20

0.	Start at node N1
1.	Connect N3 to N1: Length = 5.
2.	Connect N4 to N3: Length = 5.
3.	Connect N6 to N4: Length = 3.
4.	Connect N2 to N4: Length = 4.
5.	Connect N5 to N2: Length = 3.

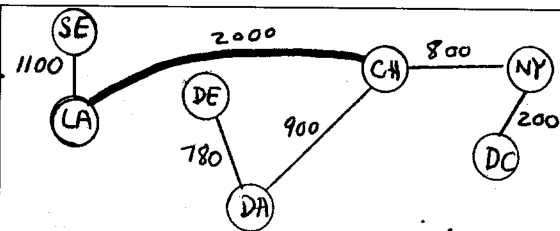
(e) Spanning tree length = 13

0.	Start at node N1
1.	Connect N2 to N1: Length = 1.
2.	Connect N5 to N2: Length = 3.
3.	Connect N3 to N5: Length = 2.
4.	Connect N4 to N2: Length = 4.
5.	Connect N6 to N4: Length = 3.

(f) Spanning tree length = 21

0.	Start at node N1
1.	Connect N2 to N1: Length = 1.
2.	Connect N4 to N2: Length = 4.
3.	Connect N6 to N4: Length = 3.
4.	Connect N3 to N4: Length = 5.
5.	Connect N5 to N4: Length = 8.

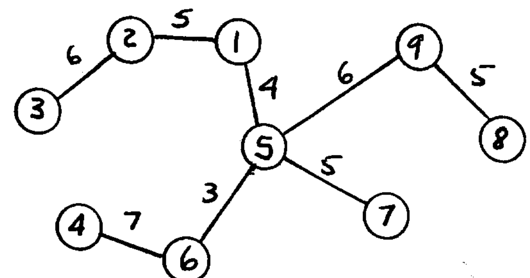
3



Total length = 5780 miles

4

Order of selection: 1-5, 5-6, 1-2, 5-7, 2-3, 5-9, 9-8, 6-4

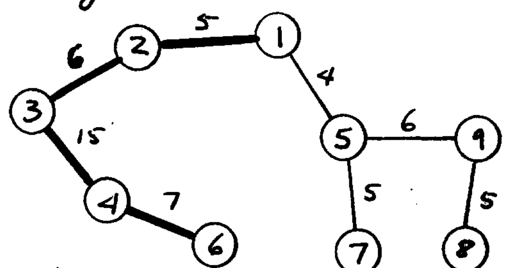


Total length = 41

5

Set length of arcs 3-5, 5-3, 4-5, 5-4, 4-7, 7-4, 5-6, and 6-5 to ∞

Total length = 53



High pressure Low pressure

6

(a) $d_{ij} = 1 - \dfrac{n_{ij}}{n_{ij} + m_{ij}}$

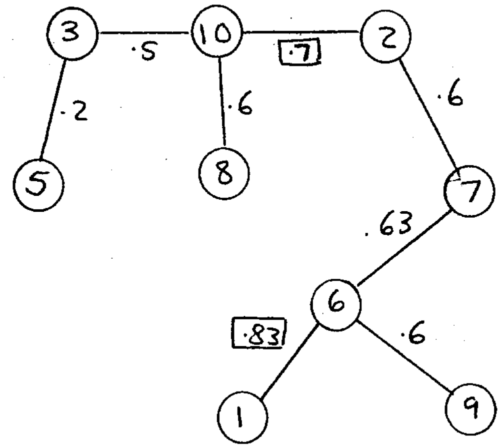
$i-j$	n_{ij}	m_{ij}	d_{ij}
1-2	0	10	1
1-3	0	6	1
1-4	0	8	1
1-5	0	7	1
1-6	1	5	.83
1-7	0	8	1
1-8	0	5	1
1-9	0	4	1
1-10	0	7	1

continued...

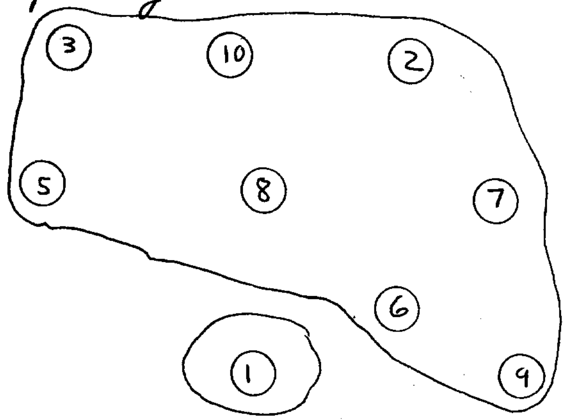
i-j	n_{ij}	m_{ij}	d_{ij}
2-3	1	10	.91
2-4	5	4	.44
2-5	1	11	.92
2-6	1	11	.92
2-7	4	6	.6
2-8	2	7	.78
2-9	0	10	1
2-10	3	7	.7
3-4	0	10	1
3-5	4	1	.2
3-6	2	5	.71
3-7	2	6	.75
3-8	1	5	.83
3-9	1	4	.8
3-10	3	3	.5
4-5	1	9	.9
4-6	0	11	1
4-7	3	6	.67
4-8	0	9	1
4-9	0	8	1
4-10	1	9	.9
5-6	2	6	.75
5-7	2	7	.78
5-8	1	6	.86
5-9	1	5	.83
5-10	3	4	.57
6-7	3	5	.63
6-8	1	6	.86
6-9	2	3	.60
6-10	1	8	.89
7-8	0	9	1
7-9	1	6	.86
7-10	1	9	.9
8-9	1	3	.75
8-10	2	4	.67
9-10	1	5	.83

continued...

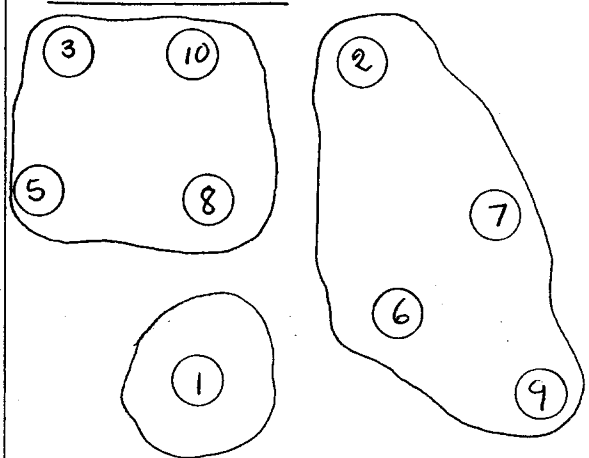
(b) Spanning tree



(c) A 2-cell solution is formed by removing the highest link in the minimal spanning tree.



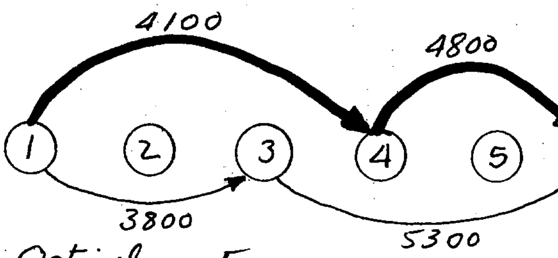
3-cell solution:



continued...

Set 6.3a

1

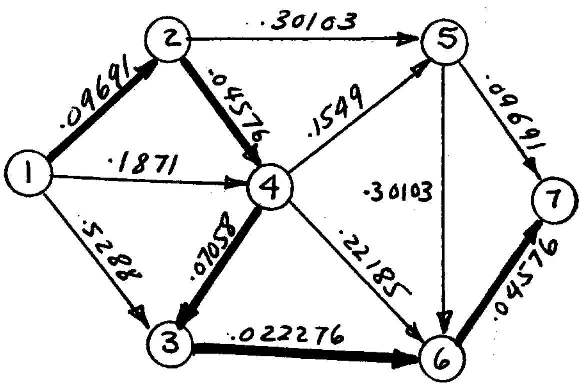


Optimal route: 1–4–6

Decision: Buy new cars in 2001 & 2004.

2

$$\max(p_1 p_2 \cdots p_n)$$
$$\equiv \max(\log p_1 + \log p_2 + \cdots + \log p_n)$$
$$\equiv \min(-\log p_1 - \log p_2 - \cdots - \log p_n)$$



Optimum solution by TORA:

1 – 2 – 4 – 3 – 6 – 7

$$\sum_{i=1}^{7} -\log p_i = .281286. \text{ Thus,}$$

$$\sum_{i=1}^{7} \log p_i = -.281286.$$

Hence,

$$p = 10^{-.28128} = .52326$$

5

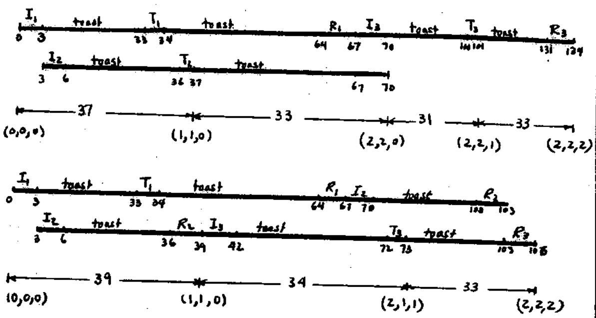
Define

(i, j, k) = number of sides toasted of slices 1, 2, and 3

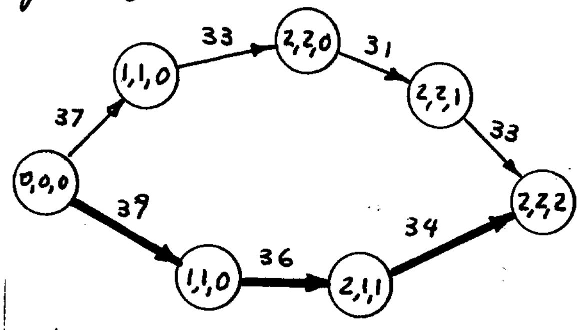
The two time charts below provides a summary of the times between the successive nodes.

Problem 4 on p. 6-7

continued...



The associated network is thus given as



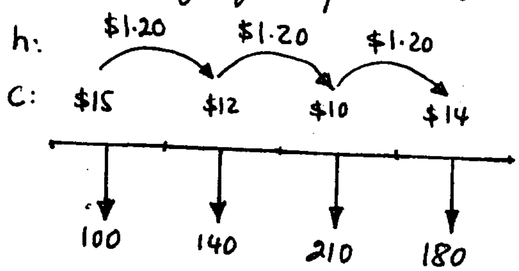
The optimal sequence is $(0,0,0) \rightarrow (1,1,0) \rightarrow (2,1,1) \rightarrow (2,2,2)$. It is interpreted as follows:

- Toast both sides of slice 1 successively (without interruption) in side A.
- Toast side 1 of slice 2 in side B, then remove slice 2.
- Toast both sides of slice 3 in side B
- Toast side 2 of slice 2 in side A after slice 1 is toasted.

Total time = 106 seconds.

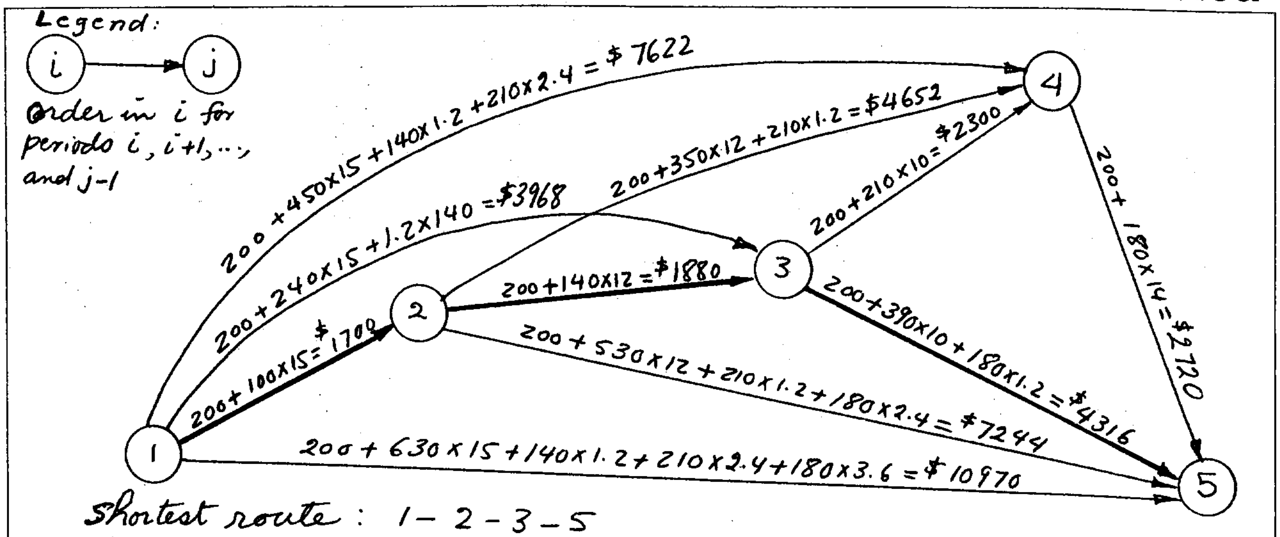
3

Summary of the problem data



Setup cost = $200

continued...



Shortest route: 1 – 2 – 3 – 5

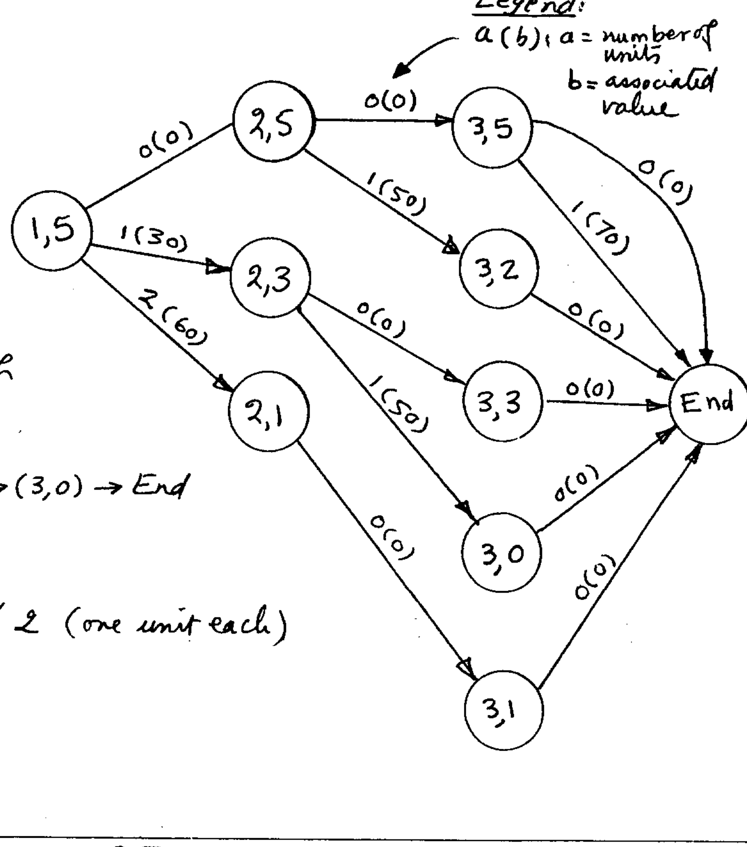
Interpretation of the solution: order 100 units in Period 1, 140 units in Period 2, and 390 units in Period 3. Total cost = $7896

4

Define node (i, v), where i is the item number and v is the volume remaining before item i is selected. Each arc represents a feasible value of the number of units of item i.

Item i	1	2	3
Volume/unit	2	3	4
Value/unit	30	50	70

Total available volume = 5 ft^3



The objective is to determine the longest path between (1,5) and (End).

Longest path: (1,5) → (2,3) → (3,0) → End

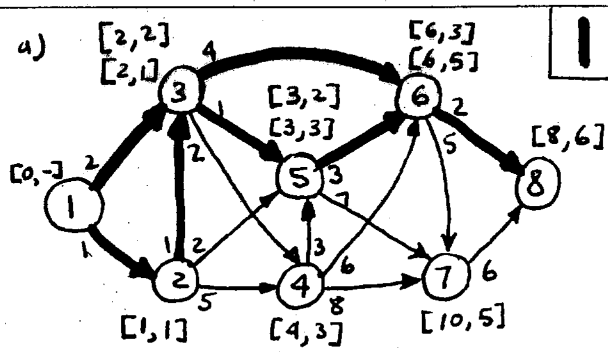
Interpretation of the solution:

Select items 1 and 2 (one unit each)

Total value = 80

1

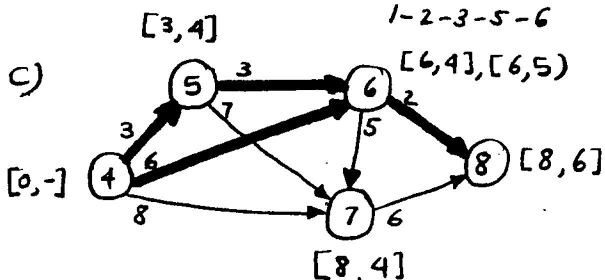
a)



shortest distance = 8:
alternative routes: 1-3-6-8
1-2-3-6-8
1-3-5-6-8
1-2-3-5-6-8

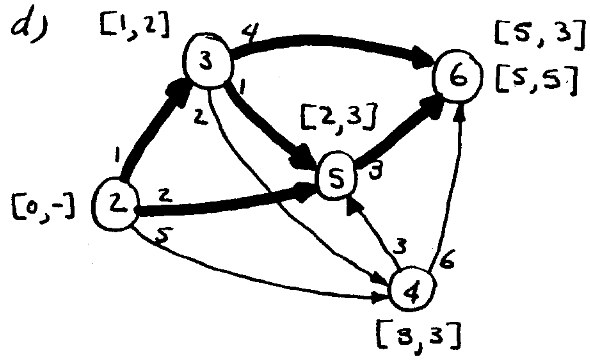
b) From part (a), shortest distance between ① and ⑥ is 6.
alternative routes: 1-3-6
1-3-5-6
1-2-3-6
1-2-3-5-6

c)



shortest distance = 8
alternative routes: 4-5-6-8
4-6-8

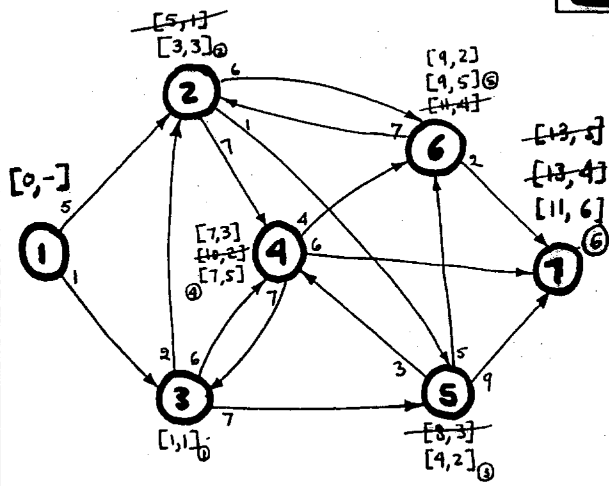
d)



shortest distance = 5

Alternative routes = $\begin{cases} 2-3-6 \\ 2-3-5-6 \\ 2-5-6 \end{cases}$

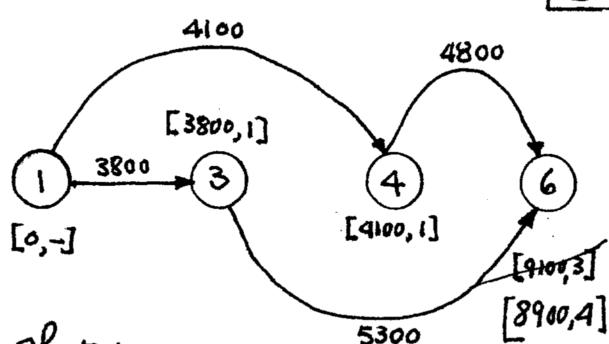
2



Shortest routes:		Length
1-2:	1-3-2	3
1-3:	1-3	1
1-4:	{1-3-4, 1-3-2-5-4}	7
1-5:	1-3-2-5	4
1-6:	{1-3-2-5-6, 1-3-2-6}	9
1-7:	{1-3-2-5-6-7, 1-3-2-6-7}	11

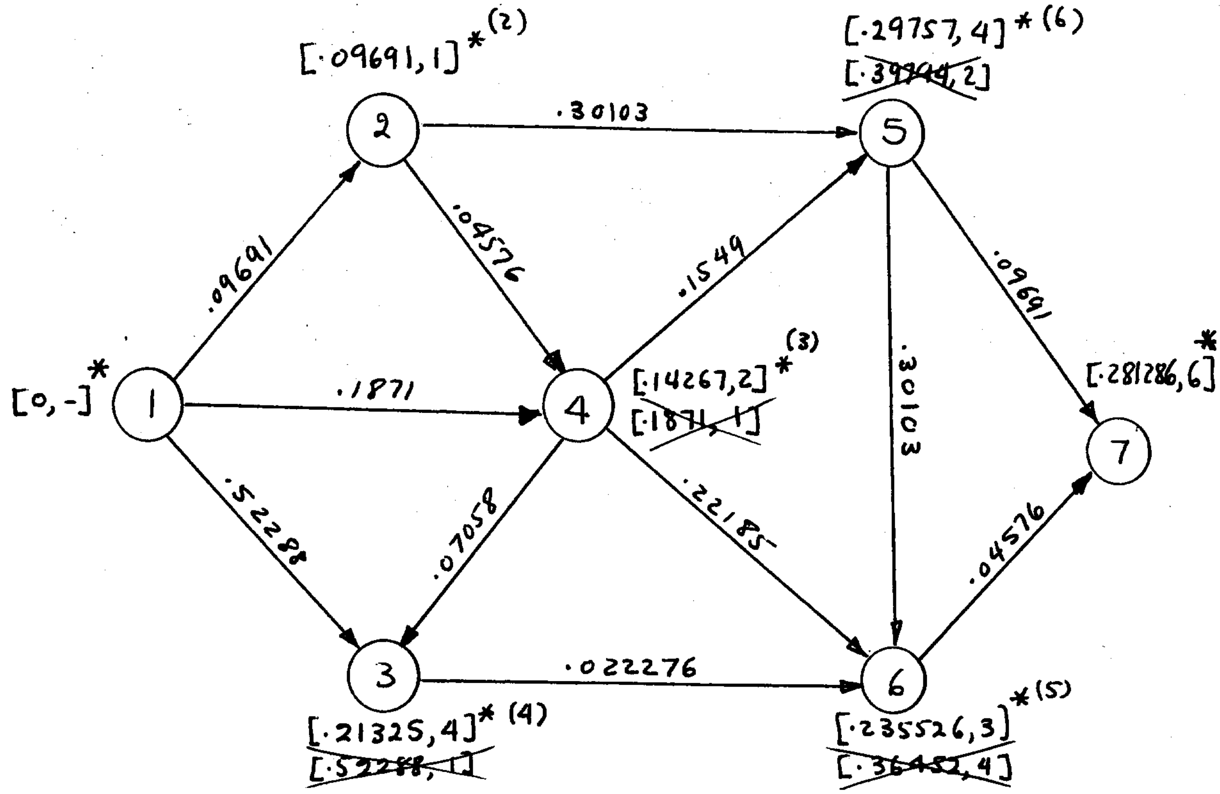
3

(a)



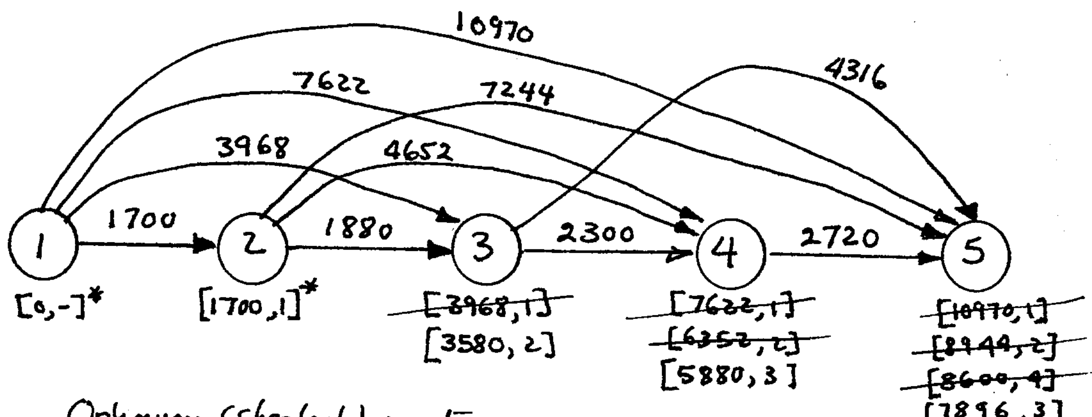
Shortest route: 1-4-6. Cost = \$8900
Buy in 2001 & 2004

3(b)



Solution: 1-2-4-3-5-6, Route value = .281286
Probability = $10^{-.281286}$ = .52326

3(c)



Optimum (shortest) route: 1-2-3-5

Solution: Order in 1 for 1
Order in 2 for 2
Order in 3 for 3 and 4

Iteration 5

Array D5

	N1:	N2:	N3:	N4:	N5:	N6:	N7:
N1:		4.00	3.00	9.00	6.00	7.00	12.00
N2:	4.00		1.00	5.00	2.00	3.00	8.00
N3:	3.00	1.00		6.00	3.00	4.00	9.00
N4:	9.00	5.00	6.00		3.00	4.00	3.00
N5:	6.00	2.00	3.00	3.00		1.00	6.00
N6:	7.00	3.00	4.00	1.00	1.00		4.00
N7:	12.00	8.00	9.00	3.00	6.00	4.00	

Array S5

	N1:	N2:	N3:	N4:	N5:	N6:	N7:
N1:		3	3	3	3	5	4
N2:	3		3	4	5	5	4
N3:	1	2		2	2	5	4
N4:	3	2	2		5	5	7
N5:	3	2	2	4		6	4
N6:	5	5	5	4	5		4
N7:	4	4	4	4	4	6	

Iteration 6

Array D6

	N1:	N2:	N3:	N4:	N5:	N6:	N7:
N1:		4.00	3.00	8.00	6.00	7.00	11.00
N2:	4.00		1.00	4.00	2.00	3.00	7.00
N3:	3.00	1.00		5.00	3.00	4.00	8.00
N4:	9.00	5.00	6.00		3.00	4.00	3.00
N5:	6.00	2.00	3.00	2.00		1.00	5.00
N6:	7.00	3.00	4.00	1.00	1.00		4.00
N7:	11.00	7.00	8.00	3.00	5.00	4.00	

Array S6

	N1:	N2:	N3:	N4:	N5:	N6:	N7:
N1:		3	3	6	3	5	6
N2:	3		3	6	5	5	6
N3:	1	2		6	2	5	6
N4:	3	2	2		5	5	7
N5:	3	2	2	6		6	6
N6:	5	5	5	4	5		4
N7:	6	6	6	4	6	6	

(a) $\boxed{1-7}$ distance = 11

1-6-7 ⇒ 1-5-6-7 ⇒ 1-3-5-6-7 ⇒
1-3-2-5-6-7 ⇒ 1-3-2-5-6-4-7

(b) $\boxed{7-1}$ distance = 11

7-6-1

7-6-5-1

7-6-5-3-1

7-6-5-2-3-1

(c) $\boxed{6-7}$ distance = 4

6-4-7

3

Iteration 0

Array D0

	N1:	N2:	N3:	N4:	N5:	N6:
N1:		700.00	200.00	infinity	infinity	infinity
N2:	infinity		300.00	200.00	infinity	400.00
N3:	200.00	300.00		700.00	600.00	infinity
N4:	infinity	200.00	700.00		300.00	100.00
N5:	infinity	infinity	600.00	300.00		500.00
N6:	infinity	400.00	infinity	100.00	500.00	

Array S0

	N1:	N2:	N3:	N4:	N5:	N6:
N1:		2	3	4	5	6
N2:	1		3	4	5	6
N3:	1	2		4	5	6
N4:	1	2	3		5	6
N5:	1	2	3	4		6
N6:	1	2	3	4	5	

Iteration 1

Array D1

	N1:	N2:	N3:	N4:	N5:	N6:
N1:		700.00	200.00	infinity	infinity	infinity
N2:	infinity		300.00	200.00	infinity	400.00
N3:	200.00	300.00		700.00	600.00	infinity
N4:	infinity	200.00	700.00		300.00	100.00
N5:	infinity	infinity	600.00	300.00		500.00
N6:	infinity	400.00	infinity	100.00	500.00	

Array S1

	N1:	N2:	N3:	N4:	N5:	N6:
N1:		2	3	4	5	6
N2:	1		3	4	5	6
N3:	1	2		4	5	6
N4:	1	2	3		5	6
N5:	1	2	3	4		6
N6:	1	2	3	4	5	

Iteration 2

Array D2

	N1:	N2:	N3:	N4:	N5:	N6:
N1:		700.00	200.00	900.00	infinity	1100.00
N2:	infinity		300.00	200.00	infinity	400.00
N3:	200.00	300.00		500.00	600.00	700.00
N4:	infinity	200.00	500.00		300.00	100.00
N5:	infinity	infinity	600.00	300.00		500.00
N6:	infinity	400.00	700.00	100.00	500.00	

Array S2

	N1:	N2:	N3:	N4:	N5:	N6:
N1:		2	3	2	5	2
N2:	1		3	4	5	6
N3:	1	2		2	5	2
N4:	1	2	2		5	6
N5:	1	2	3	4		6
N6:	1	2	2	4	5	

Iteration 3

Array D3

	N1:	N2:	N3:	N4:	N5:	N6:
N1:		500.00	200.00	700.00	800.00	900.00
N2:	500.00		300.00	200.00	900.00	400.00
N3:	200.00	300.00		500.00	600.00	700.00
N4:	700.00	200.00	500.00		300.00	100.00
N5:	800.00	900.00	600.00	300.00		500.00
N6:	900.00	400.00	700.00	100.00	500.00	

Array S3

	N1:	N2:	N3:	N4:	N5:	N6:
N1:		3	3	3	3	3
N2:	3		3	4	3	6
N3:	1	2		2	5	2
N4:	3	2	2		5	6
N5:	3	3	3	4		6
N6:	3	2	2	4	5	

continued...

Set 6.3c

Iteration 4

Array D4

	N1:	N2:	N3:	N4:	N5:	N6:
N1:		500.00	200.00	700.00	800.00	800.00
N2:	500.00		300.00	200.00	500.00	300.00
N3:	200.00	300.00		500.00	600.00	600.00
N4:	700.00	200.00	500.00		300.00	100.00
N5:	800.00	500.00	600.00	300.00		400.00
N6:	800.00	300.00	600.00	100.00	400.00	

Array S4

	N1:	N2:	N3:	N4:	N5:	N6:
N1:		3	3	3	3	4
N2:	3		3	4	4	4
N3:	1	2		2	5	4
N4:	3	2	2		5	6
N5:	3	4	3	4		4
N6:	4	4	4	4	4	

Iteration 5

Array D5

	N1:	N2:	N3:	N4:	N5:	N6:
N1:		500.00	200.00	700.00	800.00	800.00
N2:	500.00		300.00	200.00	500.00	300.00
N3:	200.00	300.00		500.00	600.00	600.00
N4:	700.00	200.00	500.00		300.00	100.00
N5:	800.00	500.00	600.00	300.00		400.00
N6:	800.00	300.00	600.00	100.00	400.00	

Array S5

	N1:	N2:	N3:	N4:	N5:	N6:
N1:		3	3	3	3	4
N2:	3		3	4	4	4
N3:	1	2		2	5	4
N4:	3	2	2		5	6
N5:	3	4	3	4		4
N6:	4	4	4	4	4	

Shortest routes:

From	To	Distance	Route
1	2	500.00	1- 3- 2
1	3	200.00	1- 3
1	4	700.00	1- 3- 2- 4
1	5	800.00	1- 3- 5
1	6	800.00	1- 3- 2- 4- 6
2	1	500.00	2- 3- 1
2	3	300.00	2- 3
2	4	200.00	2- 4
2	5	500.00	2- 4- 5
2	6	300.00	2- 4- 6
3	1	200.00	3- 1
3	2	300.00	3- 2
3	4	500.00	3- 2- 4
3	5	600.00	3- 5
3	6	600.00	3- 2- 4- 6
4	1	700.00	4- 2- 3- 1
4	2	200.00	4- 2
4	3	500.00	4- 2- 3
4	5	300.00	4- 5
4	6	100.00	4- 6
5	1	800.00	5- 3- 1
5	2	500.00	5- 4- 2
5	3	600.00	5- 3
5	4	300.00	5- 4
5	6	400.00	5- 4- 6
6	1	800.00	6- 4- 2- 3- 1
6	2	300.00	6- 4- 2
6	3	600.00	6- 4- 2- 3
6	4	100.00	6- 4
6	5	400.00	6- 4- 5

continued...

4

Iteration 0

Array D0

	N1:joe	N2:bob	N3:kay	N4:jim	N5:rae	N6:kim
N1:joe		1.00	infinity	infinity	infinity	1.00
N2:bob	infinity		1.00	infinity	infinity	infinity
N3:kay	infinity	1.00		1.00	1.00	infinity
N4:jim	infinity	infinity	1.00		infinity	infinity
N5:rae	infinity	infinity	infinity	infinity		1.00
N6:kim	1.00	1.00	infinity	infinity	infinity	

Array S0

	N1:joe	N2:bob	N3:kay	N4:jim	N5:rae	N6:kim
N1:joe		2	3	4	5	6
N2:bob	1		3	4	5	6
N3:kay	1	2		4	5	6
N4:jim	1	2	3		5	6
N5:rae	1	2	3	4		6
N6:kim	1	2	3	4	5	

Iteration 1

Array D1

	N1:joe	N2:bob	N3:kay	N4:jim	N5:rae	N6:kim
N1:joe		1.00	infinity	infinity	infinity	1.00
N2:bob	infinity		1.00	infinity	infinity	infinity
N3:kay	infinity	1.00		1.00	1.00	infinity
N4:jim	infinity	infinity	1.00		infinity	infinity
N5:rae	infinity	infinity	infinity	infinity		1.00
N6:kim	1.00	1.00	infinity	infinity	infinity	

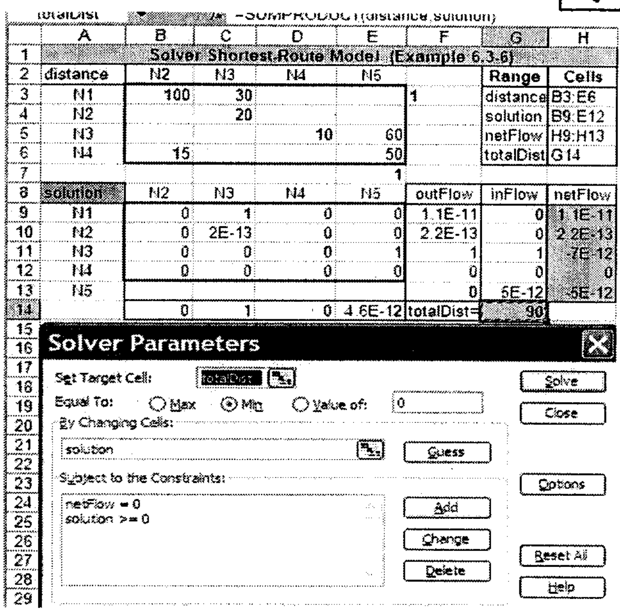
Array S1

	N1:joe	N2:bob	N3:kay	N4:jim	N5:rae	N6:kim
N1:joe		2	3	4	5	6
N2:bob	1		3	4	5	6
N3:kay	1	2		4	5	6
N4:jim	1	2	3		5	6
N5:rae	1	2	3	4		6
N6:kim	1	2	3	4	5	

continued...

1

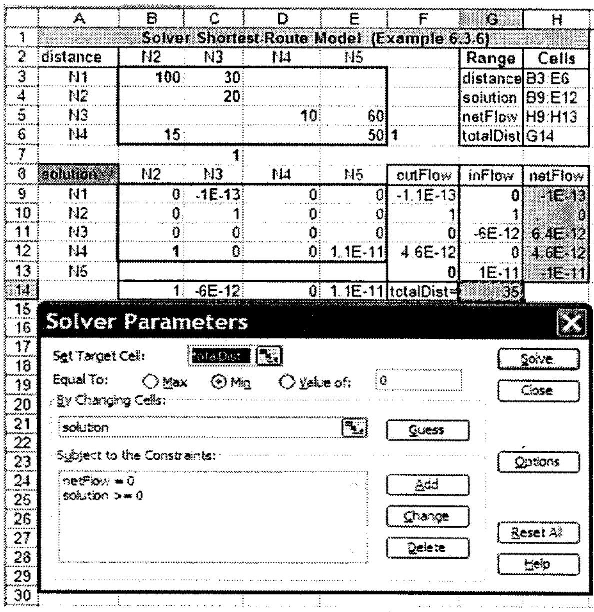
(a)



totalDist = SUMPRODUCT(distance,solution)

	A	B	C	D	E	F	G	H
1	Solver Shortest-Route Model (Example 6.3-6)							
2	distance	N2	N3	N4	N5		Range	Cells
3	N1	100	30			1	distance	B3:E6
4	N2		20				solution	B9:E12
5	N3			10	60		netFlow	H9:H13
6	N4	15			50		totalDist	G14
7					1			
8	solution	N2	N3	N4	N5	outFlow	inFlow	netFlow
9	N1	0	1	0	0	1.1E-11	0	1.1E-11
10	N2	0	2E-13	0	0	2.2E-13	0	2.2E-13
11	N3	0	0	0	1	1	1	-7E-12
12	N4	0	0	0	0	0	0	0
13	N5					0	5E-12	-5E-12
14		0	1	0	4.6E-12	totalDist=	90	

(b)



	A	B	C	D	E	F	G	H
1	Solver Shortest-Route Model (Example 6.3-6)							
2	distance	N2	N3	N4	N5		Range	Cells
3	N1	100	30				distance	B3:E6
4	N2		20				solution	B9:E12
5	N3			10	60		netFlow	H9:H13
6	N4	15			50	1	totalDist	G14
7			1					
8	solution	N2	N3	N4	N5	outFlow	inFlow	netFlow
9	N1	0	-1E-13	0	0	-1.1E-13	0	-1E-13
10	N2	0	1	0	0	1	1	0
11	N3	0	0	0	0	0	-6E-12	6.4E-12
12	N4	1	0	0	1.1E-11	4.6E-12	0	4.6E-12
13	N5					0	1E-11	-1E-11
14		1	-6E-12	0	1.1E-11	totalDist=	35	

2

```
param n;
param start;
param end;
param p{1..n,1..n} default 0;
param rhs{i in 1..n}=if i=start then 1 else (if i=end then -1 else
0);

var x{i in 1..n,j in 1..n:p[i,j]>0}>=0;
var outFlow{i in 1..n}=sum{j in 1..n:p[i,j]>0}x[i,j];
var inFlow{j in 1..n}=sum{i in 1..n:p[i,j]>0}x[i,j];
var logProb=sum{i in 1..n}sum{j in 1..n:p[i,j]>0}-log(p[i,j])*x[i,j];
var prob=2.718^-logProb;

minimize z: sum {i in 1..n, j in 1..n:p[i,j]>0}-log(p[i,j])*x[i,j];
subject to limit {i in 1..n}: outFlow[i]-inFlow[i]=rhs[i];

data;
param n:=7;
param start:=4;
param end:=7;

param p:
 1 2 3 4  5 6 7:=
1 . .8 .3 .65 . . .
2 . . . .9 .5 . .
3 . . . . . .95 .
4 . . .85 . .7 . .
5 . . . . . .5 .8
6 . . . . . . .9;

solve;
display z,logProb,prob, x;
```

1

(a) Surplus capacities:

2-3 : 40 - 0 = 40 units
2-5 : 30 - 20 = 10 units
4-3 : 5 - 0 = 5 units

All other arcs have zero surplus capacities.

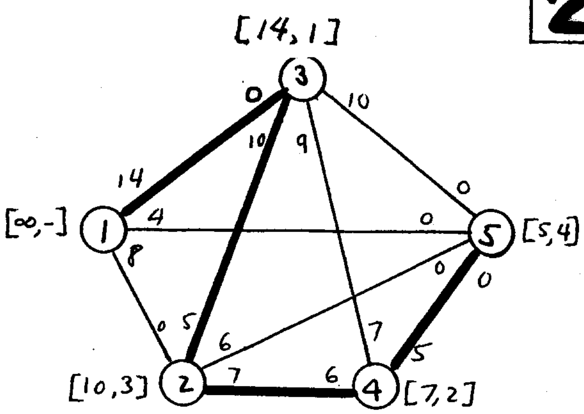
(b)

Flow through node 2 = 20 units
Flow through node 3 = 30 units
Flow through node 4 = 20 units

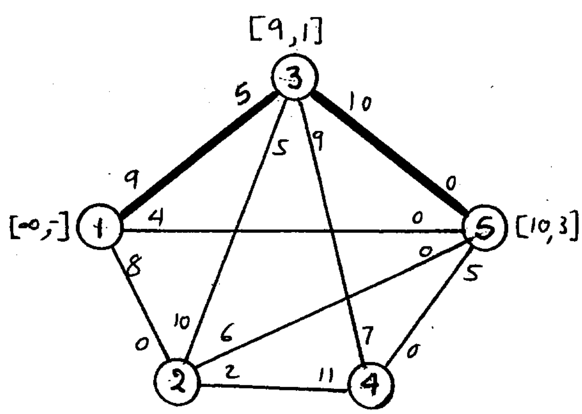
(c)

No, because the arcs out of node 1 have zero surplus capacity

2

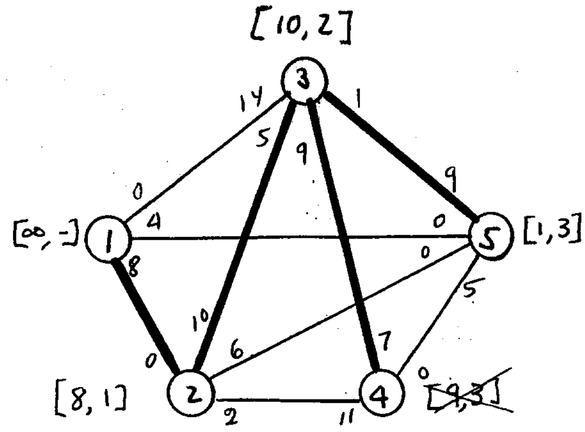


f_1 = 5 units

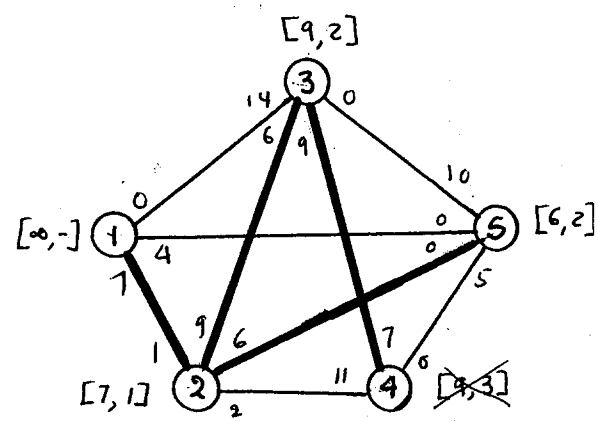


f_2 = 9 units

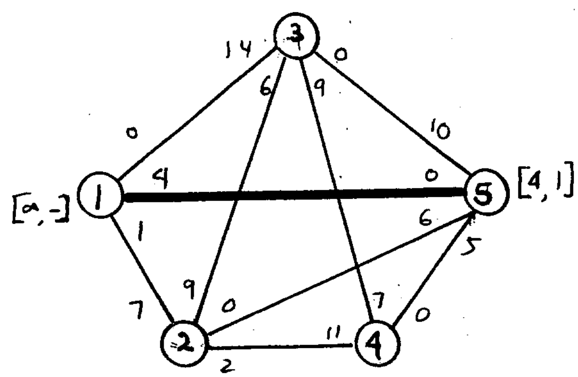
continued...



f_3 = 1 unit



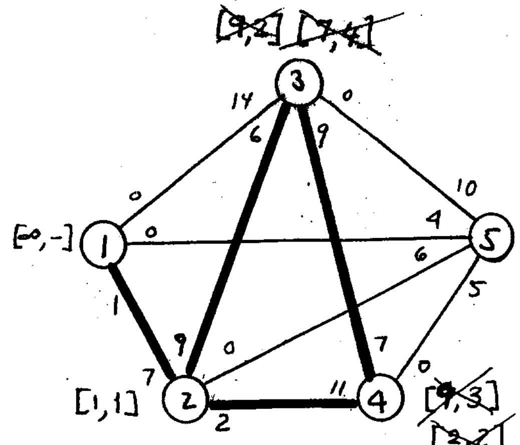
f_4 = 6 units



f_5 = 4 units

continued...

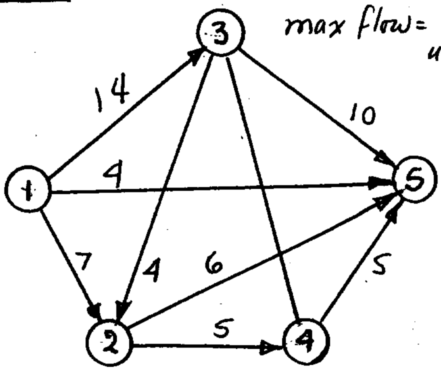
Set 6.4b



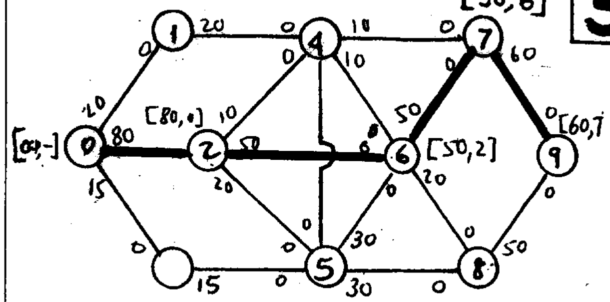
No breakthrough

Solution:

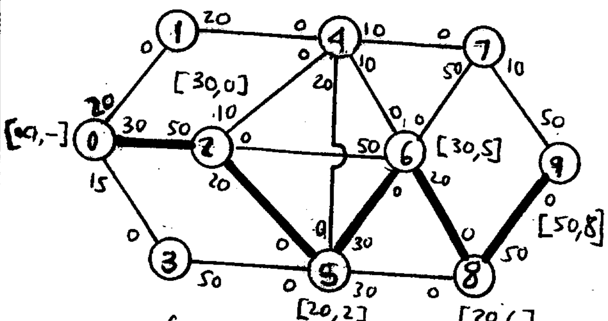
max flow = 25 units



3

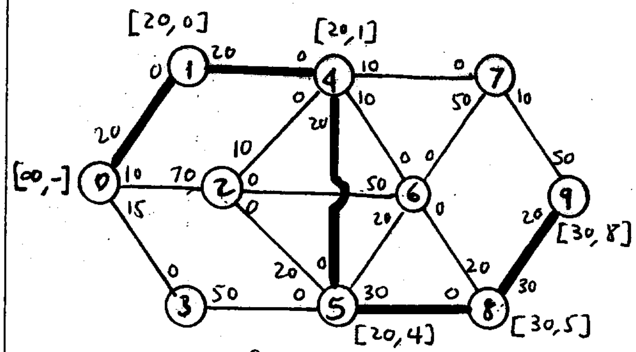


$f_1 = 50$ units

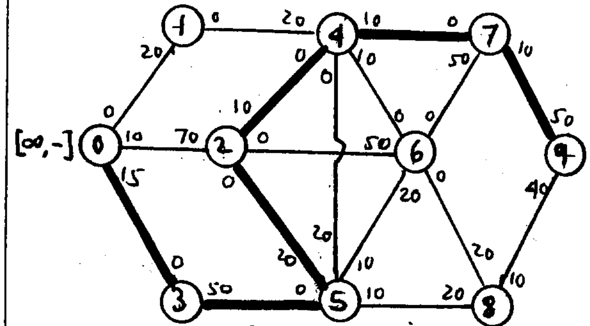


$f_2 = 20$ units

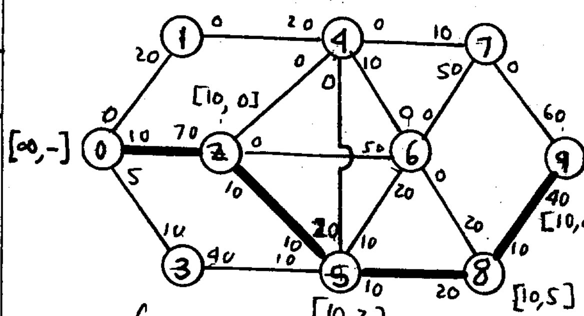
continued...



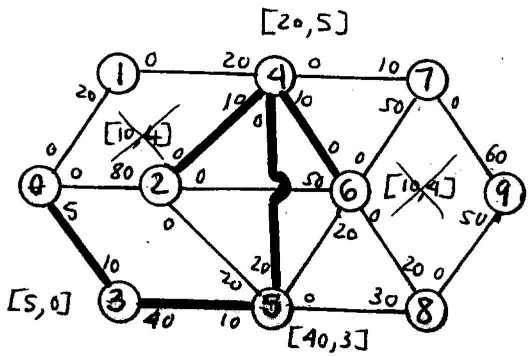
$f_3 = 20$ units



$f_4 = 10$ units



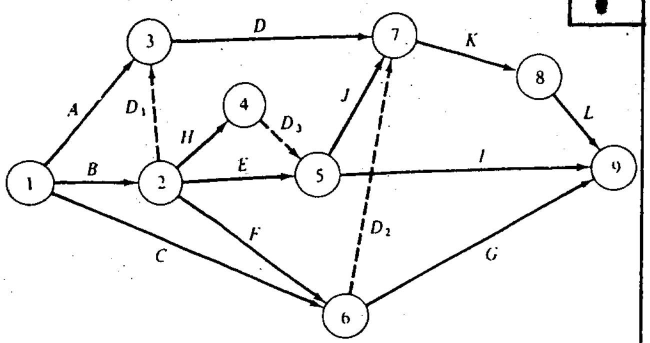
$f_5 = 10$ units



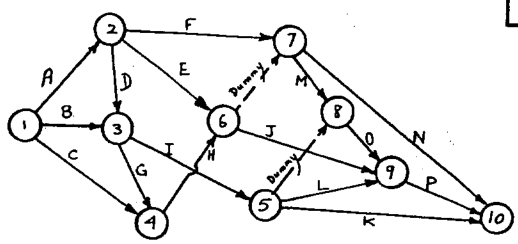
continued...

Set 6.5a

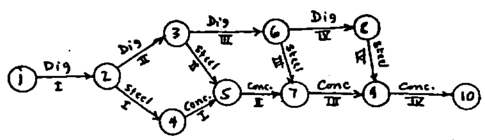
1



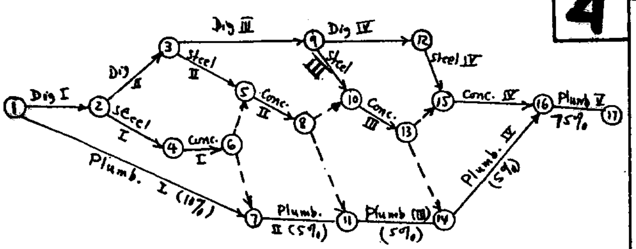
2



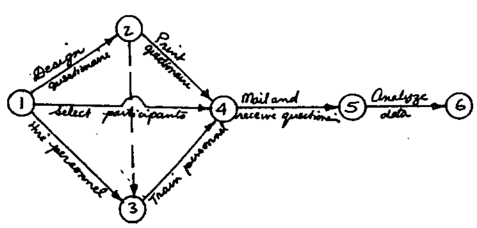
3



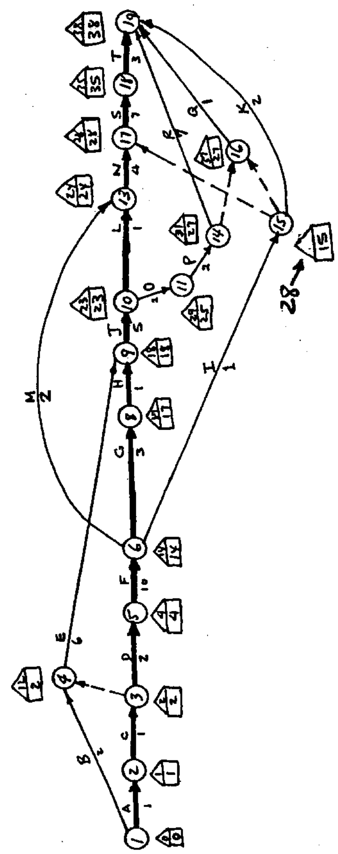
4



5

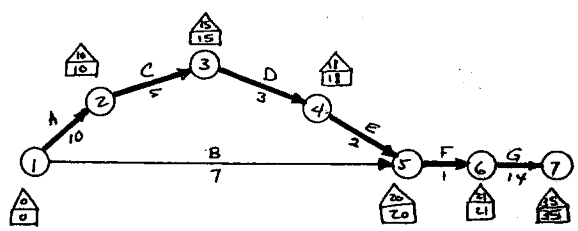


6



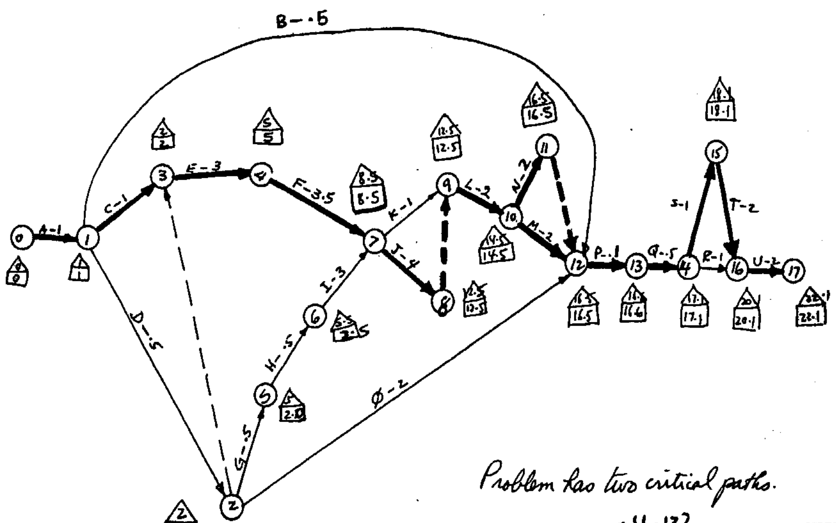
Critical path = 1-2-3-5-6-8-10-13-17-18-19
Duration = 38 days

7



Critical path : 1-2-3-4-5-6-7
Duration : 35 days

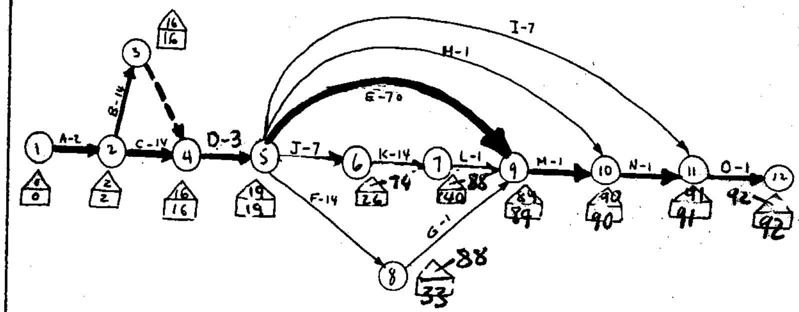
9



Problem has two critical paths.

0-1-3-4-7-8-9-10 → 11-12 / 12 } 13-14-15-16-17

8

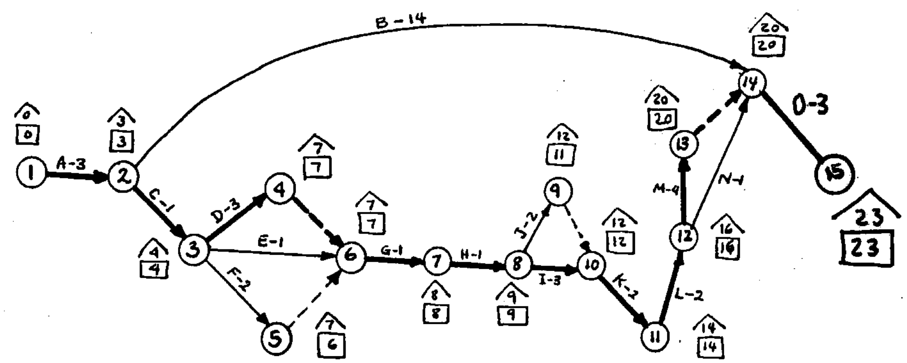


critical paths:

1-2-3-4-5-9-10-11-12

1-2-4-5-9-10-11-12

10

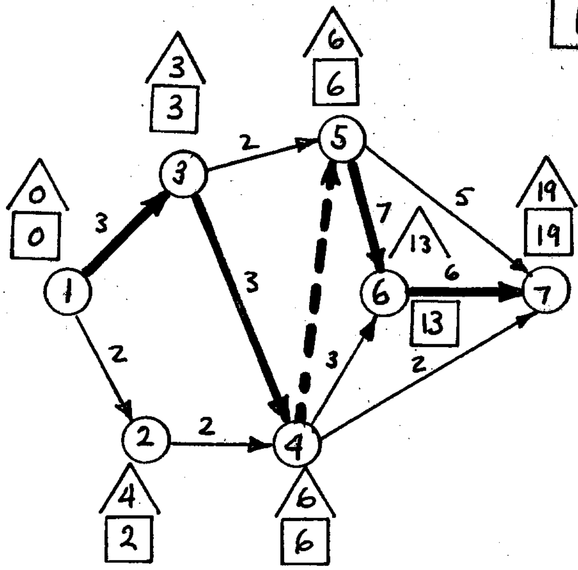


Critical path: 1-2-3-4-6-7-8-10-11-12-13-14-15

Duration: 20 days

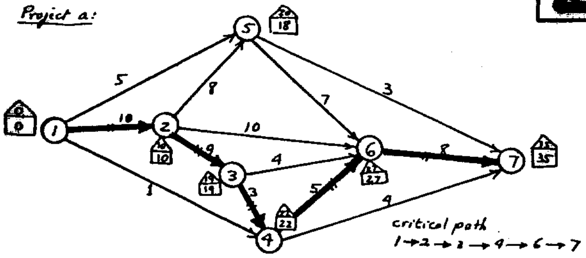
Set 6.5b

1

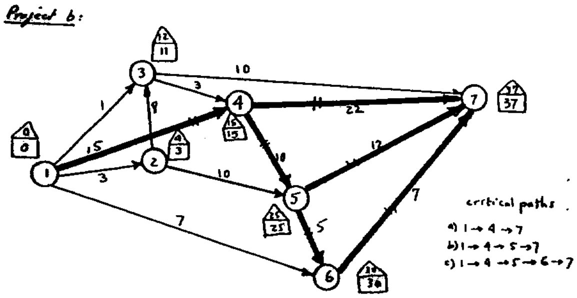


2

Project a:



Project b:



3

See solution to Problem 6, Set 6.6a

4

See solution to Problem 8, Set 6.6a

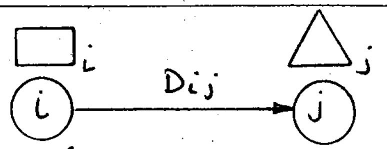
5

See solution to Problem 9, Set 6.6a

6

See solution to Problem 10, Set 6.6a

1



Earliest completion of $(i,j) = \square_i + D_{ij}$

Latest start of $(i,j) = \Delta_j - D_{ij}$

2

Both floats are zero by definition

3

(a) $FF = 10$, $TF = 10$, $D = 4$
maximum delay = 10

(b) $FF = 5$, $TF = 10$, $D = 4$
maximum delay = 5

(c) $FF = 0$, $TF = 10$, $D = 4$
maximum delay = 0

4

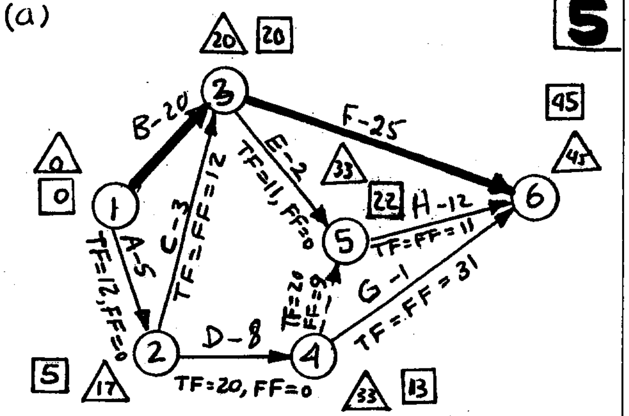
(a) For B: $TF = 5$, $FF = 2$
Because $FF = 2$, a delay of 1 has no effect on succeeding activities.
For C: Starting at time 5 implies no delay. Thus, the earliest start time for E and F is time 8.

(b) For B: Delay = 3, $FF = 2$. Thus, the start of E and F must be delayed by at least $3 - 2 = 1$.
For C: Delay $= 7 - 5 = 2$, $FF = 0$. Thus, start of E and F must be delayed by at least 2.
For B & C combined: Start of E and F must be delayed by $\max(1, 2) = 2$.

(c) Delay in B = 6. Because FF of B = 2, the start of E and F must be delayed by 4. Next, a delay of 4 in E will delay critical H by 1 because $FF_E = 3$. Also, a delay of 4 in E will not impact other activities in the project. Thus, the proposed delay in B will delay the entire project by 1 (because of the delay in critical H).

5

(a)



(b) Red-flagged activities are A, D, and E.

(c) $FF_A = 0$: $\text{Delay}_A = 5$ will delay each of C and D by 5.

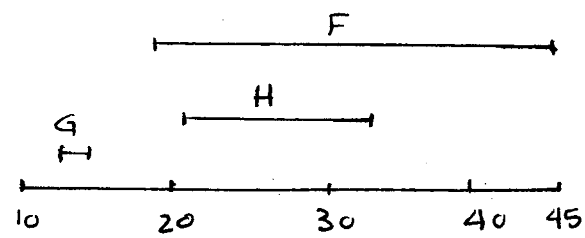
$FF_D = 0$: $\text{Delay}_D = 5$ will delay G by 5.

$FF_C = 12$: $\text{Delay}_C = 5$ does not affect other activities

Conclusion: Start of C, D, and G is delayed by 5.

Note: If you use TORA to experiment with the effect of $\text{Delay}_A = 5$, the chart will only show a delay in C and D; but not in G. The effect of C and D on succeeding activities must be done manually. To effect that after $\text{Delay}_A = 5$ is implemented, select C with $\text{delay}_C = 0$ and D with $\text{delay}_D = 0$. $\text{Delay}_C = 0$ produces no action, but $\text{delay}_D = 0$ will delay G and Dummy properly to match $\text{delay}_A = 5$.

(d)



Two unit of equipment are required.

6

*** CPM SOLUTION ***

Title: (a)

Size: 7 nodes x 13 activities

Activity	Duration	Earliest start	Earliest Compl.	Latest start	Latest compl.	Total float	Free float
c 1-2	10.0	0.0	10.0	0.0	10.0	0.0	0.0
1-4	1.0	0.0	1.0	21.0	22.0	21.0	21.0
1-5	5.0	0.0	5.0	15.0	20.0	15.0	13.0
c 2-3	9.0	10.0	19.0	10.0	19.0	0.0	0.0
2-5	8.0	10.0	18.0	12.0	20.0	2.0	0.0
2-6	10.0	10.0	20.0	17.0	27.0	7.0	7.0
c 3-4	3.0	19.0	22.0	19.0	22.0	0.0	0.0
3-6	4.0	19.0	23.0	23.0	27.0	4.0	4.0
c 4-6	5.0	22.0	27.0	22.0	27.0	0.0	0.0
4-7	4.0	22.0	26.0	31.0	35.0	9.0	9.0
5-6	7.0	18.0	25.0	20.0	27.0	2.0	2.0
5-7	3.0	18.0	21.0	32.0	35.0	14.0	14.0
c 6-7	8.0	27.0	35.0	27.0	35.0	0.0	0.0

*** CPM SOLUTION ***

Title: (b)

Size: 7 nodes x 13 activities

Activity	Duration	Earliest start	Earliest Compl.	Latest start	Latest compl.	Total float	Free float
1-2	3.0	0.0	3.0	1.0	4.0	1.0	0.0
1-3	1.0	0.0	1.0	11.0	12.0	11.0	10.0
c 1-4	15.0	0.0	15.0	0.0	15.0	0.0	0.0
1-6	7.0	0.0	7.0	23.0	30.0	23.0	23.0
2-3	8.0	3.0	11.0	4.0	12.0	1.0	0.0
2-5	10.0	3.0	13.0	15.0	25.0	12.0	12.0
3-4	3.0	11.0	14.0	12.0	15.0	1.0	1.0
3-7	10.0	11.0	21.0	27.0	37.0	16.0	16.0
c 4-5	10.0	15.0	25.0	15.0	25.0	0.0	0.0
c 4-7	22.0	15.0	37.0	15.0	37.0	0.0	0.0
c 5-6	5.0	25.0	30.0	25.0	30.0	0.0	0.0
c 5-7	12.0	25.0	37.0	25.0	37.0	0.0	0.0
c 6-7	7.0	30.0	37.0	30.0	37.0	0.0	0.0

Project (a):

Red flagged activities:

(1-5), TF = 15, FF = 13

(2-5), TF = 2, FF = 0

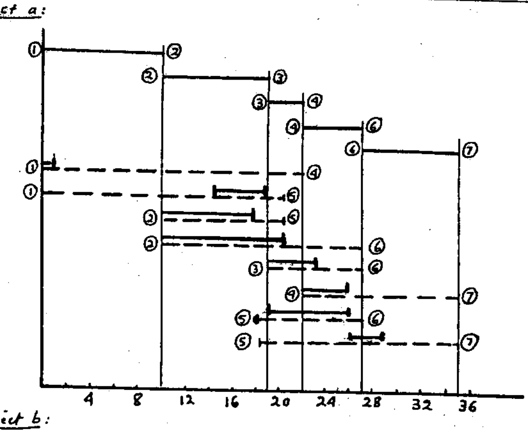
Project (b):

The following activities are red-flagged:

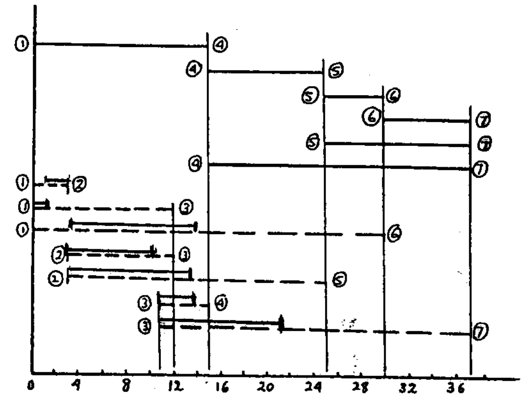
Activity	TF	FF
1-2	1	0
1-3	11	10
2-3	1	0

continued...

Project a:



Project b:



In project (a), note the delay in the start of activity 5-6 to account for the effect of starting (1-5) at time 14.

1

	x_{12}	x_{13}	x_{24}	x_{34}	x_{35}	x_{45}	x_{46}	x_{47}	x_{56}	x_{57}	x_{67}		
Maximize z =	3	3	2	3	2	0	3	2	7	5	6		
Node 1	-1	-1										=	-1
Node 2	1		-1									=	0
Node 3		1		-1	-1							=	0
Node 4			1	1		-1	-1	-1				=	0
Node 5					1	1			-1	-1		=	0
Node 6							1		1		-1	=	0
Node 7								1		1	1	=	1

Optimal:

$x_{13} = x_{34}$ x_{45} $x_{56} = x_{67} = 1$

$Z = 19$

2

(a)

	x_{12}	x_{14}	x_{15}	x_{23}	x_{25}	x_{26}	x_{34}	x_{36}	x_{46}	x_{47}	x_{56}	x_{57}	x_{67}		
Maximize z =	10	1	5	9	8	10	3	4	5	4	7	3	8		
Node 1	-1	-1	-1											=	-1
Node 2	1			-1	-1	-1								=	0
Node 3				1			-1	-1						=	0
Node 4		1					1		-1	-1				=	0
Node 5			1		-1				1		-1	-1		=	0
Node 6						1		1			1		-1	=	0
Node 7										1		1	1	=	1

Optimum: $x_{12} = x_{23} = x_{34} = x_{46} = x_{67} = 1$, $Z = 35$

(b)

	x_{12}	x_{13}	x_{14}	x_{16}	x_{23}	x_{25}	x_{34}	x_{37}	x_{45}	x_{47}	x_{56}	x_{57}	x_{67}		
Maximize z =	3	1	15	7	8	10	3	10	10	22	5	12	7		
Node 1	-1	-1	-1											=	-1
Node 2	1			-1	-1	-1								=	0
Node 3		1			1		-1	-1						=	0
Node 4			1				1		-1	-1				=	0
Node 5						1			1		-1	-1		=	0
Node 6				1						1	1		-1	=	0
Node 7								1				1	1	=	1

Optimum: $x_{14} = x_{47} = 1$

$x_{14} = x_{45} = x_{57} = 1$

$x_{14} = x_{45} = x_{56} = x_{67} = 1$

} alternative optima $Z = 37$

Set 6.5e

1

Project (a)

Title:

Activity	Mean Duration	Variance
1 - 2	4.00	0.11
1 - 4	2.83	0.25
1 - 5	3.83	0.25
2 - 3	5.00	0.11
2 - 5	8.17	0.25
2 - 6	9.50	0.69
3 - 4	10.00	5.44
3 - 6	4.00	0.11
4 - 6	7.67	1.00
4 - 7	6.17	0.25
5 - 6	10.67	1.00
5 - 7	6.00	0.44
6 - 7	4.00	0.11

Title:

Node	Longest Path	Path Mean	Path Std. Dev.
2	1- 2	4.00	0.33
3	1- 2- 3	9.00	0.47
4	1- 2- 3- 4	19.00	2.38
5	1- 2- 5	12.17	0.60
6	1- 2- 3- 4- 6	26.67	2.58
7	1- 2- 3- 4- 6- 7	30.67	2.60

Event	Latest occurrence time, LC	P{occurrence time ≤ LC}
2	4	.5
3	9	.5
4	19	.5
5	16	1.0
6	26.67	.5
7	30.67	.5

LC is determined by carrying out CPM calculations using average duration time

Example of probability calculations:

For node 5:

$$P\{T \leq 16\} = P\{z \leq \frac{16-12.17}{.6}\}$$
$$= P\{z \leq 6.38\} \simeq 1$$

continued...

Project (b)

Title:

Activity	Mean Duration	Variance
1 - 2	2.83	0.25
1 - 3	6.83	0.25
1 - 4	7.17	0.25
1 - 6	2.00	0.11
2 - 3	4.00	0.11
2 - 5	8.00	0.11
3 - 4	15.00	2.78
3 - 7	13.00	0.11
4 - 5	12.17	0.69
4 - 7	10.00	0.44
5 - 6	8.33	0.44
5 - 7	4.33	1.00
6 - 7	6.00	0.11

Title:

Node	Longest Path	Path Mean	Path Std. Dev.
2	1- 2	2.83	0.50
3	1- 3	6.83	0.50
4	1- 3- 4	21.83	1.74
5	1- 3- 4- 5	34.00	1.93
6	1- 3- 4- 5- 6	42.33	2.04
7	1- 3- 4- 5- 6- 7	48.33	2.07

Event	Latest occurrence time, LC	P{Occurrence time ≤ LC}
2	2.83	.5
3	6.83	.5
4	21.83	.5
5	34.00	.5
6	42.33	.5
7	48.33	.5

All events happen to fall on the critical path (using average durations). This is the reason all probabilities = .5