CHAPTER 11

Deterministic Inventory Models

1

$y^* = \sqrt{\frac{2KD}{h}}$, $t_0 = \frac{y^*}{D}$, $TCU(y^*) = \sqrt{2KDh}$

a) $y^* = \sqrt{\frac{2 \times 100 \times 30}{.05}} = 346.4$ units

$t_0 = \frac{346.4}{30} = 11.55$ days

$TCU(y^*) = \frac{100 \times 30}{346.4} + \frac{.05 \times 346.4}{2} = \$17.32$

Policy: order 346.4 units whenever inventory drops to 207.2 units

Effective lead time = 6.91 days

b) $y^* = \sqrt{\frac{2 \times 50 \times 30}{.05}} \cong 245$ units

$t_0^* = \frac{245}{30} = 8.16$ days

$L_e = 5.51$ days

$TCU(y^*) = \frac{50 \times 30}{245} + \frac{.05 \times 245}{2} = \$12.25$

Policy: order 245 units whenever inventory drops to 165.15 units

c) $y^* = \sqrt{\frac{2 \times 100 \times 40}{.01}} = 894.4$ units

$t_0 = \frac{894.4}{40} = 22.36$ days

$L_e = 7.64$ days

$TCU(y^*) = \frac{100 \times 40}{894.4} + \frac{.01 \times 894.4}{2} = \$8.94$

Policy: Order 894.4 units whenever inventory drops to 305.57 units.

d) $y^* = \sqrt{\frac{2 \times 100 \times 20}{.04}} = 316.23$ units

$t_0^* = \frac{316.23}{20} = 15.81$ days

$L_e = 14.19$ days

$TCU(y^*) = \frac{100 \times 20}{316.23} + \frac{.04 \times 316.23}{2} = \$12.65$

Policy: Order 316.23 units whenever inventory drops to 283.8 units.

2

$D = 300$ lb/wk, $K = \$20$, $h = \$.03$/lb/day

(a) $TC/wk = \frac{KD}{y} + \frac{hy}{2}$

$= \frac{20 \times 300}{300} + \frac{7 \times .03 \times 300}{2} = \$51.50$

(b) $y^* = \sqrt{\frac{2 \times 20 \times 300}{(.03 \times 7)}} = 239$ lb

$t_0^* = \frac{239}{300/7} = .8$ wk

$TC/wk = \sqrt{2 \times 20 \times 300 \times .03 \times 7}$

$= \$50.20$

continued...

$L_e = 0$ days

Policy: Order 239 lb whenever inventory drops to zero level.

c) Cost difference $= 51.50 - 50.20$

$= \$1.30$

3

a) $h = \frac{.35}{7} = \$.05$/unit/day

$D = 50$ units/day, $K = \$20$

$y^* = \sqrt{\frac{2 \times 20 \times 50}{.05}} = 200$ units

$t_0 = \frac{200}{50} = 4$ days

$L = 7$ days, $L_e = 3$ days

$R = 3 \times 50 = 150$ units

Policy: Order 200 units whenever inventory drops to 150 units.

b) Optimum number of orders $= \frac{365}{4}$

$\cong 91$ orders

4

(a) Policy 1: $D = \frac{R}{L_e} = \frac{50}{10} = 5$ units/day

$\text{Cost/day} = \frac{KD}{y} + \frac{hy}{2}$

$= \frac{20 \times 5}{150} + \frac{.02 \times 150}{2} = \$2.17$

Policy 2: $D = \frac{75}{15} = 5$ units/day

$\text{Cost/day} = \frac{20 \times 5}{200} + \frac{.02 \times 200}{2} = \$2.50$

choose policy 1.

(b) $K = \$20$, $D = 5$ units/day

$h = \$.02$, $L = 22$ days

$y^* = \sqrt{\frac{2 \times 20 \times 5}{.02}} = 100$ units

$t_0 = \frac{100}{5} = 20$ days

$L_e = 22 - 20 = 2$ days

Reorder level $= 2 \times 5 = 10$ units

Order 100 units whenever the level drops to 10 units

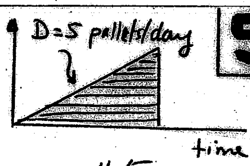
$\text{Cost/day} = \frac{20 \times 5}{100} + \frac{.02 \times 100}{2} = \$2.00$

5

$D = 5$ units/day

$h = \$0.10$/day

$K = \$100$

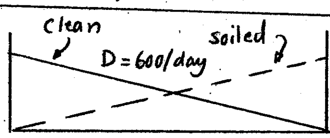


$$y^* = \sqrt{\frac{2 \times 5 \times 100}{.1}} = 100 \text{ pallets}$$

$$t_0 = \frac{100}{5} = 20 \text{ days}$$

Pick up 100 pallets every 20 days.

6



$$TC/day = \frac{K}{y/D} + \frac{h_1 y}{2} + \frac{h_2 y}{2} + .6D$$

$$= \frac{KD}{y} + (h_1 + h_2)\frac{y}{2} + .6D$$

$$y^* = \sqrt{\frac{2KD}{(h_1 + h_2)}} = \sqrt{\frac{2 \times 81 \times 600}{(.01 + .02)}} = 1800 \text{ towels}$$

$$t_0 = 1800/600 = 3 \text{ days}$$

$$\text{Cost/day} = \frac{81 \times 600}{1800} + \frac{.03 \times 1800}{2} = \$54$$

Optimal policy: Pick up soiled towels and deliver an equal batch of 1800 towels every 3 days

7

The basic assumption is that the employee will deposit sufficient funds in Europe to take advantage of the higher interest rate and periodically send lump sums to the US to take care of the obligations. This problem in the context of an application of the simple economic lot size formula with no shortages. The idea is that it may be more economical to hold funds longer in European banks to take advantage of their considerably higher interest rate. The cost of wiring funds from overseas (= \$50) may be regarded as the "setup" cost and the lost interest per dollar per year (=.065 - .015 = \$.05) can be treated as the "holding" cost. Using this information, the economic lot size formula will yield

$$\text{Deposit amount} = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2 \times 50 \times 12000}{.05}} = \$4899$$

$$\text{Time between deposits} = t_0 = \frac{4899}{12000} = .408 \text{ year}$$

$$= 4.9 \text{ months}$$

Optimal policy: Send \$4899 (≈\$5000) every 4.9 (≈ 5) months to the US. The first installment occurs at the start of the year

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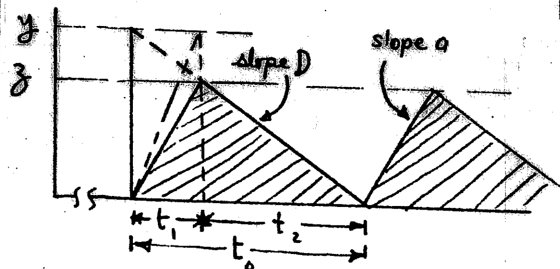

a) From the geometry of the figure,

$$z = t_1(a - D) = \frac{y}{a}(a - D) = y\left(1 - \frac{D}{a}\right)$$

b) $$TCU(y) = \frac{K + (z/2)\, t_0 \times h}{t_0}$$

$$= \frac{KD}{y} + \frac{h}{2}\left(1 - \frac{D}{a}\right)y$$

(c) $\frac{\partial TCU(y)}{\partial y} = 0$ gives

$$-\frac{KD}{y^2} + \frac{h}{2}\left(1 - \frac{D}{a}\right) = 0$$

$$y^* = \sqrt{\frac{2KD}{h\left(1 - \frac{D}{a}\right)}}$$

(d) $$\lim_{a \to \infty} \sqrt{\frac{2KD}{h\left(1 - \frac{D}{a}\right)}} = \sqrt{\frac{2KD}{h}}$$

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Alternative 1: Produce

$$y^* = \sqrt{\frac{2KD}{h\left(1 - \frac{D}{a}\right)}}$$

$$= \sqrt{\frac{2 \times 20 \times \frac{26000}{365}}{.02\left(1 - \frac{26000/365}{100}\right)}} = 703.7 \text{ units}$$

Total cost/day

$$= \frac{KD}{y^*} + \frac{h}{2}\left(1 - \frac{D}{a}\right)y^*$$

$$= \frac{200 \times \frac{2600}{365}}{703.7} + \frac{.02}{2}\left(1 - \frac{26000}{100 \times 365}\right) \times 703.7$$

$$= \$4.05 \text{ per day}$$

continued...

alternative 2: Buy

$$y^* = \sqrt{\frac{2KD}{h}}$$

$$= \sqrt{\frac{2 \times 15 \times \frac{26000}{365}}{.02}}$$

$$= 326.87 \text{ units}$$

Total cost/day

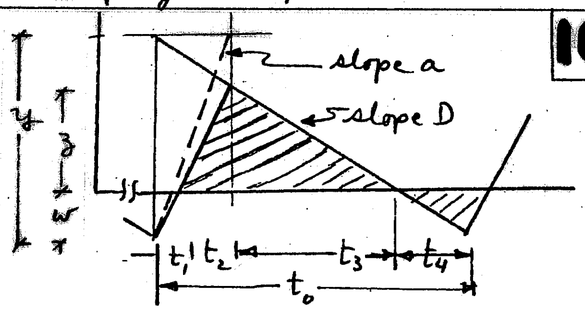
$$= \frac{KD}{y^*} + \frac{h}{2} y^*$$

$$= \frac{15 \times \frac{26000}{365}}{377.45} + \frac{.02}{2} \times 377.45$$

$$= \$6.54/\text{day}$$

The company should produce its own.

10



$$z = y(1 - \frac{D}{a}) - w$$

$$TCU(y,w) = \left[K + \frac{h\{y(1-\frac{D}{a}) - w\}^2 + pw^2}{2D(1-D/a)}\right] / t_0$$

$$= \frac{KD}{y} + \frac{h\{y(1-\frac{D}{a}) - w\}^2 + pw^2}{2y(1-D/a)}$$

Partial derivatives = 0 give

$$-\frac{KD}{y^2} + h\left(\frac{1}{2}(1-\frac{D}{a}) - \frac{w^2}{2y^2(1-D/a)}\right) - \frac{pw^2}{2y^2(1-\frac{D}{a})} = 0$$

$$h\left(\frac{w}{y(1-\frac{D}{a})} - 1\right) + \frac{pw}{y(1-D/a)} = 0$$

This gives,

$$y^* = \sqrt{\frac{2KD(p+h)}{ph(1-D/a)}}, \quad w^* = \sqrt{\frac{2KDh(1-\frac{D}{a})}{p(p+h)}}$$

1

EOQ before quantity dis-count = 1800 towels per Problem 6, Set 11.2a.

Total cost/day given batches of 1800 towels

$$= DC_1 + \frac{KD}{y} + \frac{h_1+h_2}{2}y$$

$$= 600\times.6 + \frac{81\times600}{1800} + \frac{.03\times1800}{2} = \$414$$

Total cost/day given batches of 2500 towels

$$= DC_2 + \frac{KD}{y} + \frac{(h_1+h_2)}{2}y$$

$$= 600\times.5 + \frac{81\times600}{2500} + \frac{.03\times2500}{2} = \$356.94$$

Take advantage of price discount.

2

$$y_m = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2\times100\times30}{.05}} = 346.41$$

$q = 500$ units

Because $y_m < q$, we need to compute Q.

$$TCU_1(y_m) = DC_1 + \frac{KD}{y_m} + \frac{hy_m}{2}$$

$$= 30\times10 + \frac{100\times30}{346.41} + \frac{.05\times346.41}{2}$$

$$= 317.32$$

The equation for computing Q is

$$Q^2 + \left(\frac{2(8\times30 - 317.32)}{.05}\right)Q + \frac{2\times100\times30}{.05} = 0$$

or

$$Q^2 - 3092.82Q + 120000 = 0$$

This yields $Q = 3053.52$ units.

Because $y_m < q < Q \Rightarrow y^* = q = 500$

$t_0 = \frac{500}{30} = 16.67$ days $\Rightarrow L_e = 4.33$

Order 500 units when inventory drop to 130.

3

$$y_m = \sqrt{\frac{2KD}{h}} = \sqrt{\frac{2\times50\times20}{.3}}$$

$$= 81.65 \text{ units}$$

Because $q > y_m$, we need to compute Q.

$$TCU_1(y_m) = 20\times25 + \frac{50\times20}{81.65} + \frac{.3\times81.65}{2}$$

$$= \$524.49$$

Q-equation:

$$Q^2 + \left(\frac{2(22.5\times20 - 524.49)}{.3}\right)Q + \frac{2\times50\times20}{.3} = 0$$

$$Q^2 - 496.63Q + 6666.67 = 0$$

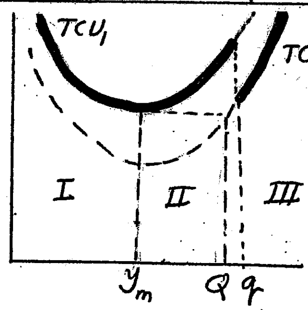
continued...

Thus, $Q = 482.83$

Because $y_m < q < Q \Rightarrow y^* = 150$

Order 150 units when inventory drops to 0

4



From the preceding figure, the dis-count is not advantageous if

$$TCU_1(y_m) \le TCU_2(q)$$

or

$$DC_1 + \frac{KD}{y_m} + \frac{hy_m}{2} \le DC_2 + \frac{KD}{q} + \frac{hq}{2}$$

or

$$20C_1 + \frac{50\times20}{81.65} + \frac{.3\times81.65}{2} \le 20C_2 + \frac{50\times20}{150} + \frac{.3\times150}{2}$$

Thus, the condition reduces to

$$C_1 - C_2 \le .23359$$

Let d = discount factor (<1).

Then $C_2 = (1-d)C_1$, $0 < d < 1$

Given $C_1 = 25$, we have

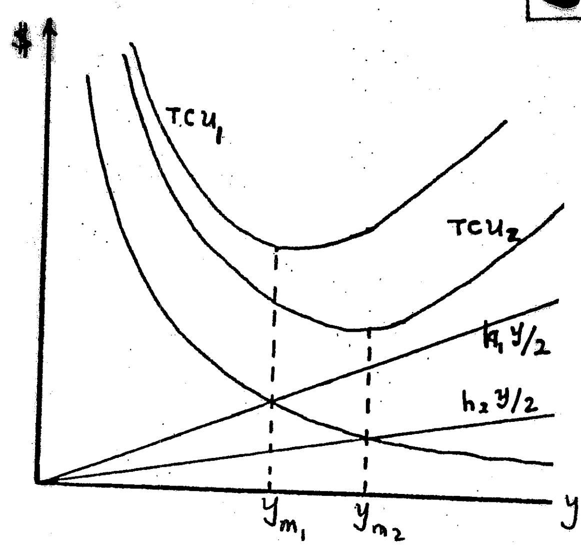
$$25d \le .233588$$

or

$$d \le .009344$$

Thus, no advantage if the % discount is $\le .9344\%$ ($\simeq 1\%$)

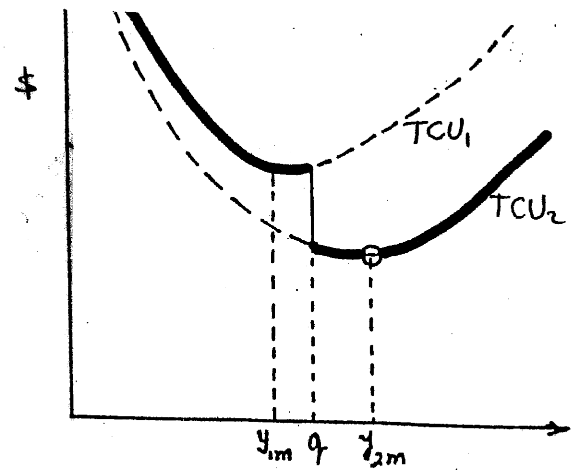
5



$$TCU_1(y) = \frac{KD}{y} + \frac{h_1 y}{2}$$

$$TCU_2(y) = \frac{KD}{y} + \frac{h_2 y}{2}$$

Case 1: $q < y_{2m}$



Solution:

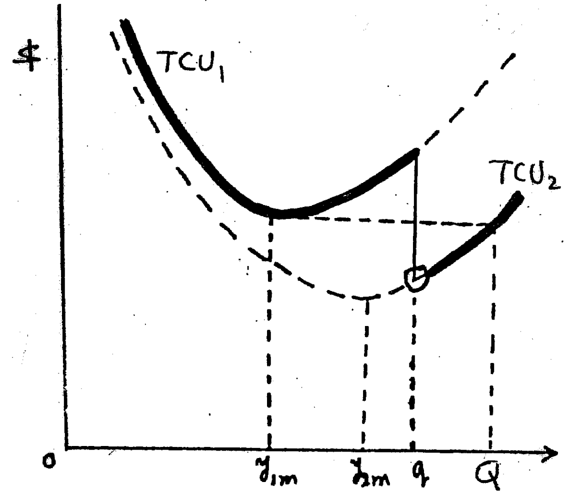
$$y^* = y_{2m}$$

$$TCU(y^*) = TCU_2(y_{2m})$$

Case 2: $y_{2m} < q \le Q$

The value of Q is determined from the equation:

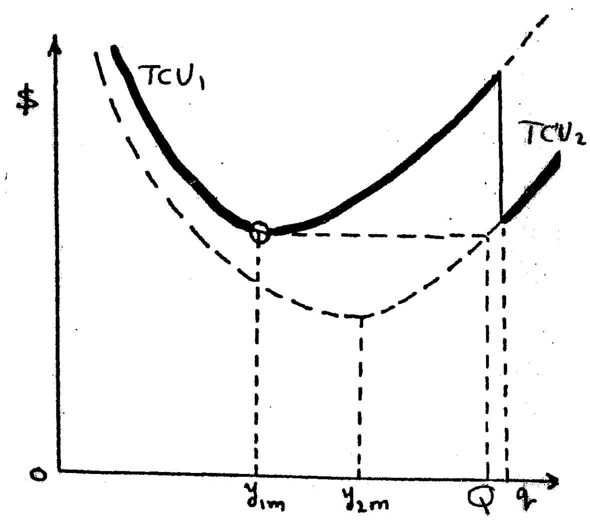
$$TCU_1(y_{1m}) = TCU_2(Q)$$



Solution: $y^* = q$

$$TCU(y^*) = TCU_2(q)$$

Case 3: $y_{2m} < Q < q$



Solution: $y^* = y_{1m}$, $TCU(y^*) = TCU_1(y_{1m})$

$$TCU(y^*) = \begin{cases} TCU_2(y_{2m}), & q < y_{2m} \\ TCU_2(q), & y_{2m} < q \le Q \\ TCU_1(y_{1m}), & y_{2m} < Q < q \end{cases}$$

continued...

1

See file ampl11.3c-1.txt.
AMPL model will not converge unless K_iD_i/y_i is replaced with $K_iD_i/(y_i+\varepsilon)$, where $\varepsilon>0$ and very small.

SOLUTION:
Total cost = 568.11
$y_1 = 4.42$
$y_2 = 6.87$
$y_3 = 4.12$
$y_4 = 7.20$
$y_5 = 5.80$

2

See file ampl11.3c-2.txt.
New constraint:
$(1/2)(y_1 + y_2 + y_3) \leq 25$

SOLUTION:
Total cost = 10.42
$y_1 = 10.83$
$y_2 = 16.85$
$y_3 = 22.32$

3

See file ampl11.3c-3.txt.
New constraint:
Average inventory for item i = $y_i/2$.
$(1/2)(100y_1 + 55y_2 + 100y_3) \leq 1000$

SOLUTION:
Total cost = 14.31
y1 = 5.58
y2 = 7.90
y3 = 10.07

4

See file ampl11.3c-4.txt.
AMPL model will not converge unless K_iD_i/y_i is replaced with $K_iD_i/(y_i+\varepsilon)$, where $\varepsilon>0$ and very small.

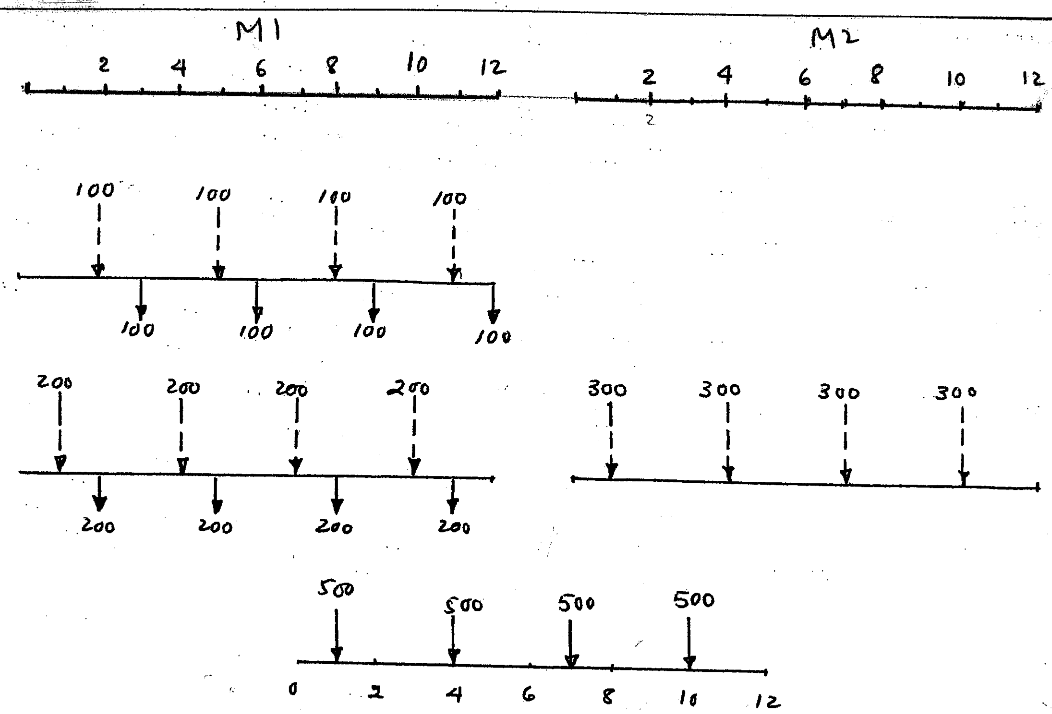
New constraint:
$365(10/y_1 + 20/y_2 + 5/y_3 + 10/y_4) \leq 150$

SOLUTION:
Total cost = 54.71
y1 = 155.30
y2 = 118.81
y3 = 74.36
y4 = 90.09

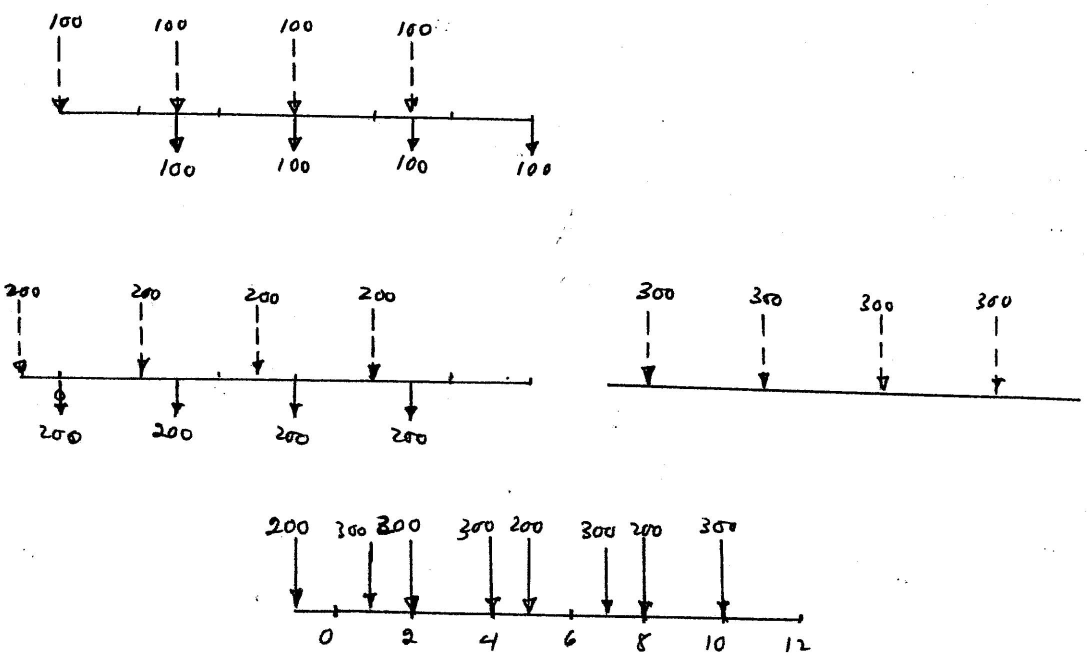
Set 11.4a

I

(a)


M1
M2
2 4 6 8 10 12
100 100 100 100
100 100 100 100
200 200 200 200
200 200 200 200
300 300 300 300
500 500 500 500
0 2 4 6 8 10 12

(b)


100 100 100 100
100 100 100 100
200 200 200 200
200 200 200 200
300 300 300 300
200 300 200 300 200 300 200 300
0 2 4 6 8 10 12

1

	1	2	3	4	Surplus	
R_1	90 (5)	5.1	5.25	5.37	0	90
O_1	10 (7.5)	30 (7.6)	10 (7.75)	7.87	0	50
R_2	—	100 (3)	3.15	3.27	0	100
O_2	—	60 (4.5)	4.65	4.77	0	60
R_3	—	—	120 (4)	4.12	0	120
O_3	—	—	80 (6)	6.12	0	80
R_4	—	—	—	110 (1)	0	110
O_4	—	—	—	50 (1.5)	20 (0)	70
	100	190	210	160	20	

2

(a)

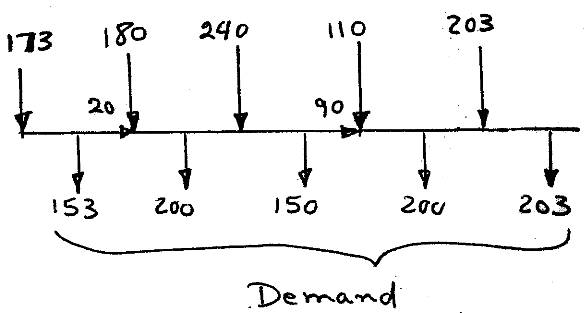
	1	2	3	4	Surplus	
I	11 (1)	1.3	1.65	1.85	0	11
II	2	1 (2.3)	3 (2.65)	2.85	0	4
III	5	5.3	5.65	5 ⑤ (5.85)	5 ⓪ (0)	10
I	X	3 (2)	2.25	2.55	0	3
II	X	4	11 (4.35)	1 (4.55)	0	12
III	X	6	6.35	⑤ (6.55)	10 ⑤ (0)	10
I	X	X	3 (2)	2.2	0	3
II	X	X	5	8 (5.2)	0	8
III	X	X	7	7.2	4 (0)	4
IV	X	X	10	10.2	10 (0)	10
I	X	X	X	3 (3)	0	3
II	X	X	X	8 (4)	0	8
III	X	X	X	4 (5)	0	4
IV	X	X	X	7	10 (0)	10
	11	4	17	29	39	

(b) Additional 10 units are produced as shown by the circled entries in period 4. The problem has alternative solutions.

3

		1	2	3	4	5	Surplus	
1	R	100 (4)	4.5	5	5.5	6	0	100
	O	50 (6)	6.5	7	7.5	8	0	50
	S	3 (7)	20 (7.5)	8	8.5	9	7 (0)	30
2	R	X	40 (4)	4.5	5	6	0	40
	O	X	60 (6)	6.5	7	7.5	0	60
	S	X	80 (7)	7.5	8	8.5	0	80
3	R	X	X	90 (4)	4.5	5	0	90
	O	X	X	60 (6)	20 (6.5)	7	0	80
	S	X	X	7	70 (7.5)	8	0	70
4	R	X	X	X	60 (4)	4.5	0	60
	O	X	X	X	50 (6)	6.5	0	50
	S	X	X	X	7	7.5	20 (0)	20
5	R	X	X	X	X	70 (4)	0	70
	O	X	X	X	X	50 (6)	0	50
	S	X	X	X	X	83 (7)	17 (0)	100
		153	200	150	200	203	44	

Solution summary

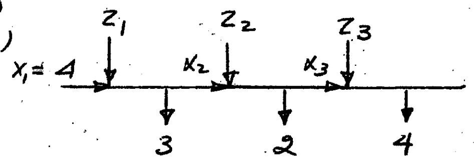


1

(a) No, because inventory should not be held needlessly at the end of planning horizon

(b)

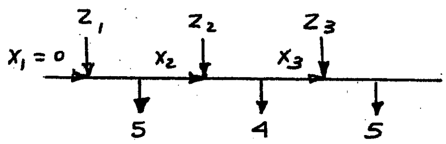
(i)



$0 \le z_1 \le 5, \quad 1 \le z_2 \le 5, \quad 0 \le z_3 \le 4$

$x_1 = 4, \quad 1 \le x_2 \le 6, \quad 0 \le x_3 \le 4$

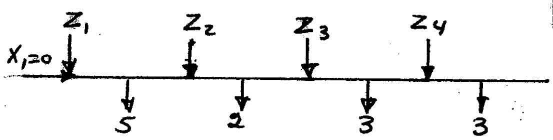
(ii)



$5 \le z_1 \le 14, \quad 0 \le z_2 \le 9, \quad 0 \le z_3 \le 5$

$x_1 = 0, \quad 0 \le x_2 \le 9, \quad 0 \le x_3 \le 5$

2



Stage 1: $f_1(x_2) = \min_{z_1 = D_1 + x_2} \{K_1 + C_1(z_1) + h_1 x_2\}$

where $C_i(z_i) = \begin{cases} 1z_i, & 0 \le z_i \le 6 \\ 2z_i, & z_i \ge 7 \end{cases} \quad i = 1, 2, \dots, 4$

$K_1 = 5, h_1 = 1$

x_2 \ z_1	5	6	7	8	9	10	11	12	13	Opt. Sol. f_1	z_1
0	10									10	5
1		12								12	6
2			15							15	7
3				18						18	8
4					21					21	9
5						24				24	10
6							27			27	11
7								30		30	12
8									33	33	13

Stage 2:

$f_2(x_3) = \min_{0 \le z_2 \le D_2 + x_3} \{K_2 + C_2(z_2) + h_2 x_3 + f_1(x_3 + D_2 - z_2)\}$

$0 \le z_2 \le 8, \quad 0 \le x_3 \le 6, \quad D_2 = 2$

$K_2 = 7, h_2 = 1$

x_3 \ z_2	0	1	2	3	4	5	6	7	8	Opt. Sol. f_2	z_2
0	15	20	19							15	0
1	19	24	22	21						19	0
2	23	28	26	24	23					23	0,4
3	27	32	30	28	26	25				25	5
4	31	36	34	32	30	28	27			27	6
5	35	40	38	36	34	32	30	30		30	6
6	39	44	42	40	38	36	34	33	33	33	7,8

Stage 3: $0 \le z_3 \le 6, \quad 0 \le x_4 \le 3, \quad D_3 = 3$

$K_3 = 9, h_3 = 1$

x_4 \ z_3	0	1	2	3	4	5	6	Opt. Sol. f_3	z_3
0	25	33	30	27				25	0
1	28	36	35	32	29			28	0
2	32	39	38	37	34	31		31	5
3	36	43	41	40	39	36	33	33	6

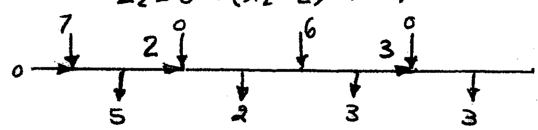
Stage 4: $0 \le z_4 \le 3, \quad x_5 = 0, \quad D_4 = 3$

$K_4 = 7, h_4 = 1$

x_5 \ z_4	0	1	2	3	Opt. Sol. f_4	z_4
0	33	39	37	35	33	6

Solution:

$(x_5 = 0) \rightarrow z_4 = 0 \rightarrow (x_4 = 3) \rightarrow z_3 = 6 \rightarrow (x_3 = 0) \rightarrow$
$z_2 = 0 \rightarrow (x_2 = 2) \rightarrow z_1 = 7$



Total cost = \$33

continued...

3

$$f_1(x_2) = \min_{0 \le z_1 \le D_1+x_2}\left\{C_1(z_1)+K_1+h_1\left(\frac{x_1+z_1+x_2}{2}\right)\right\}$$

$$= \min_{0 \le z_1 \le D_1+x_2}\left\{K_1+c_1(z_1)+h_1\left(x_2+\frac{D_1}{2}\right)\right\}$$

$$f_i(x_{i+1}) = \min_{0 \le z_i \le D_i+x_{i+1}}\left\{K_i+C_i(z_i)+h_i\left(x_{i+1}+\frac{D_i}{2}\right) + f_{i-1}(x_{i+1}+D_i-z_i)\right\}$$

4

$$f_n(x_n) = \min_{z_n+x_n=D_n}\{K_n+C_n(z_n)\}$$

$$f_i(x_i) = \min_{D_i \le x_i+z_i \le D_1+\dots+D_n}\{K_i+C_i(z_i)+h_i(x_i+z_i-D_i) + f_{i+1}(x_i+z_i-D_i)\}$$

Stage 3: $D_3=4$, $0 \le x_3 \le 4$

x_3	$z_3=0$	1	2	3	4	Opt. Sol. f_3	z_3
0					56	56	4
1				36		36	3
2			26			26	2
3		16				16	1
4	0					0	0

Stage 2: $D_2=2$

x_2	$z_2=0$	1	2	3	4	5	6	Opt. Sol. f_2	z_2
0			83	76	89	102	109	76	3
1		73	66	69	82	89		66	2
2	56	56	59	62	69			56	0, 1
3	39	49	52	49				34	0
4	32	42	39					32	0
5	25	29						25	0
6	12							12	0

continued...

Stage 1: $D_1=3$

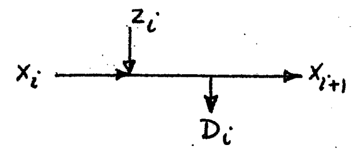
x_1	$z_1=2$	3	4	5	6	7	8	Opt. Sol. f_1	z_1
1	99	100	111	115	129	193	151	99	2

Solution:

$(x_1=1) \to z_1=2 \to (x_2=0) \to z_2=3 \to (x_3=1) \to z_3=3$

Cost = \$99

5



$$\text{Average inventory} = \frac{x_i+z_i+x_{i+1}}{2} = \frac{x_i+z_i+x_i+z_i-D_i}{2} = x_i+z_i-\frac{D_i}{2}$$

Replace $h_i(x_i+z_i-D_i)$ with $h_i(x_i+z_i-\frac{D_i}{2})$ in the backward formulation of problem 4.

Set 11.4d

1

Period 1:

	A	B	C	D	E	F	G	H	I	N	O	P	Q	R	S	T	U	V
1		Wagner-Whitin (Forward) Dynamic Programming Inventory Model																
2		Number of periods, N=			4	Current period=			1	<<Mac			Optimum Solution Summary					
3	I	Period	1	2	3	4												
4	N	c(1 to 4)=	2	2	2	2												
5	P	K(1 to 4)=	98	114	185	70												
6	U	h(1 to 4)=	1	1	1	1							x	f	z	x	f	z
7	T	D(1 to 4)=	0	22	90	67					Current			period 1				
8		Are z1 values correct?			yes	yes	yes	yes			optimum		0	0	0			
9		Period 0		z1=	0	22	112	179			Period 1		22	164	22			
10		f0		C1(z1)=	0	142	322	456			f1	z1	112	434	112			
11	S		x2=	0	0	1111111	1111111	1111111			0	0	179	635	179			
12	T		x2=	22	1111111	164	1111111	1111111			164	22						
13	A		x2=	112	1111111	1111111	434	1111111			434	112						
14	G		x2=	179	1111111	1111111	1111111	635			635	179						
15	E																	

Period 2:

	A	B	C	D	E	F	G	H	I	N	O	P	Q	R	S	T	U	V
1		Wagner-Whitin (Forward) Dynamic Programming Inventory Model																
2		Number of periods, N=			4	Current period=			2	<<Mac			Optimum Solution Summary					
3	I	Period	1	2	3	4												
4	N	c(1 to 4)=	2	2	2	2												
5	P	K(1 to 4)=	98	114	185	70												
6	U	h(1 to 4)=	1	1	1	1							x	f	z	x	f	z
7	T	D(1 to 4)=	0	22	90	67					Current			period 1			period 2	
8		Are z2 values correct?			yes	yes	yes	yes			optimum		0	0	0	0	158	22
9		Period 1		z2=	0	22	112	179			Period 2		22	164	22	90	428	112
10		f1		C2(z2)=	0	158	338	472			f2	z2	112	434	112	157	629	179
11	S	0	x3=	0	164	158	1111111	1111111			158	22	179	635	179			
12	T	164	x3=	90	524	1111111	428	1111111			428	112						
13	A	434	x3=	157	792	1111111	1111111	629			629	179						
14	G	635																
15	E																	

Period 3:

	A	B	C	D	E	F	G	H	I	N	O	P	Q	R	S	T	U	V
1		Wagner-Whitin (Forward) Dynamic Programming Inventory Model																
2		Number of periods, N=			4	Current period=			3	<<Mac			Optimum Solution Summary					
3	I	Period	1	2	3	4												
4	N	c(1 to 4)=	2	2	2	2												
5	P	K(1 to 4)=	98	114	185	70												
6	U	h(1 to 4)=	1	1	1	1							x	f	z	x	f	z
7	T	D(1 to 4)=	0	22	90	67					Current			period 1			period 2	
8		Are z3 values correct?			yes	yes	yes				optimum		0	0	0	0	158	22
9		Period 2		z3=	0	90	157				Period 3		22	164	22	90	428	112
10		f2		C3(z3)=	0	365	499				f3	z3	112	434	112	157	629	179
11	S	158	x4=	0	428	523	1111111				428	0	179	635	179			
12	T	428	x4=	67	696	111111	724				696	0		period 3				
13	A	629											0	428	0			
14	G												67	696	0			
15	E																	

Period 4:

V13 = 67

	A	B	C	D	E	F	G	H	I	N	O	P	Q	R	S	T	U	V
1		Wagner-Whitin (Forward) Dynamic Programming Inventory Model																
2		Number of periods, N=			4	Current period=			4	<<Mac			Optimum Solution Summary					
3	I	Period	1	2	3	4												
4	N	c(1 to 4)=	2	2	2	2												
5	P	K(1 to 4)=	98	114	185	70												
6	U	h(1 to 4)=	1	1	1	1							x	f	z	x	f	z
7	T	D(1 to 4)=	0	22	90	67					Current			period 1			period 2	
8		Are z4 values correct?			yes	yes					optimum		0	0	0	0	158	22
9		Period 3		z4=	0	67					Period 4		22	164	22	90	428	112
10		f3		C4(z4)=	0	204					f4	z4	112	434	112	157	629	179
11	S	428	x5=	0	696	632					632	67	179	635	179			
12	T	696												period 3			period 4	
13	A												0	428	0	0	632	67
14	G												67	696	0			

Optimum solution:

z_4	x_4	z_3	x_3	z_2	x_2	z_1
67	0	0	90	112	0	0

Cost = \$632

2

Stage 1: $D_1 = 150,\ x_1 = 50$

x_2	z_1 = 100	200	220	260	330	420	550	730	870	920	Opt. Sol. f_1	z_1^*
0	700										700	100
100		1400									1400	200
120			1540								1540	220
160				1820							1820	260
230					2310						2310	330
320						2940					2940	420
450							3850				3850	550
630								5110			5110	730
770									6090		6090	870
820										6940	6940	920

Stage 2: $D_2 = 100$

x_3	z_2 = 0	100	120	160	230	320	450	630	770	820	Opt. Sol. f_2	z_2^*
0	1400	1400									1400	100
20	1564		1540								1540	120
60	1880			1820							1820	160
130	2440				2310						2310	230
220	3160					2940					2940	320
350	4200						3850				3850	450
530	5690							5110			5110	630
670	6760								6090		6090	770
720	7160									6940	6940	820

Stage 3: $D_3 = 20$

x_4	z_3 = 0	20	60	130	220	350	530	670	720	Opt. Sol. f_3	z_3
0	1540	1580								1540	0
40	1900		1820							1820	60
110	2530			2240						2240	130
200	3340				2780					2780	220
330	4510					3560				3560	350
510	6130						4640			4640	530
650	7390							5480		5480	670
700	7840								5780	5780	720

Stage 4: $D_4 = 40$

x_5	z_4 = 0	40	110	200	330	510	650	750	Opt. Sol. f_4	z_4
0	1820	1900							1820	0
70	2310		2250						2250	110
160	2990			2700					2700	200
290	3850				3350				3350	330
470	5110					4250			4250	510
610	6090						4950		4950	650
660	6440							5200	5200	700

Stage 5: $D_5 = 70$

x_6	z_5 = 0	70	160	290	470	610	660	Opt. Sol. f_5	z_5
0	2250	2440						2250	0
90	2380		3160					2380	0
220	3790			4200				3790	0
400	5050				5640			5050	0
540	6030					6760		6030	0
590	6380						7160	6390	0

continued…

Stage 6: $D_6 = 90$

x_7	$z_6 = 0$	90	220	400	540	590	Opt. Sol. f_6	z_6
0	2880	3110					2880	0
130	4180		4600				4180	0
310	5980			6580			5980	0
450	7380				8120		7380	0
500	7880					8670	7880	0

Stage 7: $D_7 = 130$

x_8	$z_7 = 0$	130	310	450	500	Opt. Sol. f_7	z_7
0	4180	3700				3700	130
180	6160		4600			4600	310
320	7700			5300		5300	450
370	8250				5550	5550	500

Stage 8: $D_8 = 180$

x_9	$z_8 = 0$	180	320	370	Opt. Sol. f_8	z_8
0	4600	4720			4600	0
140	5860		5840		5840	320
190	6310			6240	6240	370

Stage 9: $D_9 = 140$

	$z_9 = 0$	140	190	Opt. Sol. f_9	z_9
0	5840	5180		5180	140
50	6340		5380	5380	190

Stage 10: $D_{10} = 50$

x_{11}	$z_{10} = 0$	50	Opt. Sol. f_{10}	z_{10}
0	5380	5780	5380	0

Solution:

Period	Order amount
1	100
2	120
3	0
4	200
5	0
6	0
7	310
8	0
9	190
10	0

Minimum cost = \$5380

Period 1:

Wagner-Whitin (Forward) Dynamic Programming Inventory Model

Number of periods, N= 5 | Current period= 1

Period	1	2	3	4	5
c(1 to 5) =	10	10	10	10	10
K(1 to 5) =	80	70	60	80	60
h(1 to 5) =	1	1	1	1	1
D(1 to 5) =	50	70	100	30	60

Period 0		z1=	50	120	220	250	310	f1	z1
10		C1(z1)=	580	1280	2280	2580	3180		
	x2=	0	580	1111111	1111111	1111111	1111111	580	50
	x2=	70	1111111	1350	1111111	1111111	1111111	1350	120
	x2=	170	1111111	111111	2450	1111111	1111111	2450	220
	x2=	200	1111111	111111	1111111	2780	1111111	2780	250
	x2=	260	1111111	111111	1111111	1111111	3440	3440	310

Optimum Solution Summary

	x	f	z
Current optimum Period 1		period 1	
	0	580	50
	70	1350	120
	170	2450	220
	200	2780	250
	260	3440	310

Period 2:

Wagner-Whitin (Forward) Dynamic Programming Inventory Model

Number of periods, N= 5 | Current period= 2

Period	1	2	3	4	5
c(1 to 5) =	10	10	10	10	10
K(1 to 5) =	80	70	60	80	60
h(1 to 5) =	1	1	1	1	1
D(1 to 5) =	50	70	100	30	60

Period 1		z2=	0	70	170	200	260	f2	z2
f1		C2(z2)=	0	770	1770	2070	2670		
580	x3=	0	1350	1350	1111111	1111111	1111111	1350	0
1350	x3=	100	2550	111111	2450	1111111	1111111	2450	170
2450	x3=	130	2910	111111	1111111	2780	1111111	2780	200
2780	x3=	190	3630	111111	1111111	1111111	3440	3440	260
3440									

Optimum Solution Summary

	x	f	z
Current optimum Period 2		period 1	
	0	580	50
	70	1350	120
	170	2450	220
	200	2780	250
	260	3440	310
		period 2	
	0	1350	0
	100	2450	170
	130	2780	200
	190	3440	260

Period 3:

Wagner-Whitin (Forward) Dynamic Programming Inventory Model

Number of periods, N= 5 | Current period= 3

Period	1	2	3	4	5
c(1 to 5) =	10	10	10	10	10
K(1 to 5) =	80	70	60	80	60
h(1 to 5) =	1	1	1	1	1
D(1 to 5) =	50	70	100	30	60

Period 2		z3=	0	100	130	190	f3	z3
f2		C3(z3)=	0	1060	1360	1960		
1350	x4=	0	2450	2410	1111111	1111111	2410	100
2450	x4=	30	2810	111111	2740	1111111	2740	130
2780	x4=	90	3530	111111	1111111	3400	3400	190
3440								

Optimum Solution Summary

	x	f	z	x	f	z
Current optimum Period 3		period 1			period 3	
	0	580	50	0	2410	100
	70	1350	120	30	2740	130
	170	2450	220	90	3400	190
	200	2780	250			
	260	3440	310			
		period 2				
	0	1350	0			
	100	2450	170			
	130	2780	200			
	190	3440	260			

Period 4:

Wagner-Whitin (Forward) Dynamic Programming Inventory Model

Number of periods, N= 5 | Current period= 4

Period	1	2	3	4	5
c(1 to 5) =	10	10	10	10	10
K(1 to 5) =	80	70	60	80	60
h(1 to 5) =	1	1	1	1	1
D(1 to 5) =	50	70	100	30	60

Period 3		z4=	0	30	90	f4	z4
f3		C4(z4)=	0	380	980		
2410	x5=	0	2740	2790	1111111	2740	0
2740	x5=	60	3460	111111	3450	3450	90
3400							

Optimum Solution Summary

	x	f	z	x	f	z
Current optimum Period 4		period 1			period 3	
	0	580	50	0	2410	100
	70	1350	120	30	2740	130
	170	2450	220	90	3400	190
	200	2780	250		period 4	
	260	3440	310	0	2740	0
		period 2		60	3450	90
	0	1350	0			
	100	2450	170			
	130	2780	200			
	190	3440	260			

continued...

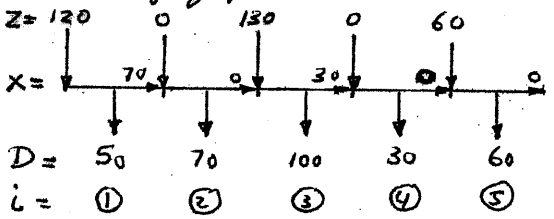
Set 11.4d

Period 5:

Wagner-Whitin (Forward) Dynamic Programming Inventory Model							
Number of periods, N=	5	Current period=	5	<<Max			
Period	1	2	3	4	5		
c(1 to 5)=	10	10	10	10	10		
K(1 to 5)=	80	70	60	80	60		
h(1 to 5)=	1	1	1	1	1		
D(1 to 5)=	50	70	100	30	60		
Are z5-values correct?		yes	yes				
Period 4		z5=	0	60			
f4		C5(z5)=	0	660			
2740	x6=	0	3450	3400			
3450							

Current			Optimum Solution Summary					
		x	f	z	x	f	z
			period 1			period 3	
optimum		0	580	50	0	2410	100
Period 5		70	1350	120	30	2740	130
f5	z5	170	2450	220	90	3400	190
3400	60	200	2780	250		period 4	
		260	3440	310	0	2740	0
			period 2		60	3450	90
		0	1350	0		period 5	
		100	2450	170	0	3400	60
		130	2780	200			
		190	3440	260			

Summary of optimum solution:



Cost = $ 3400

4

Period 1:

Wagner-Whitin (Forward) Dynamic Programming Inventory Model								
Number of periods, N=		6		Current period=	1	<<Max		
Period		1	2	3	4	5	6	
c(1 to 6)=		2	2	2	2	2	2	
K(1 to 6)=		20	17	10	18	5	50	
h(1 to 6)=		1	1	1	3	1	1	
D(1 to 6)=		10	15	7	20	13	25	
Are z-values correct?		yes	yes	yes	yes	yes	yes	
Period 0		z1=	10	25	32	52	65	90
f0		C1(z1)=	40	70	84	124	150	200
	x2=	0	40	111111	111111	111111	111111	111111
	x2=	15	111111	85	111111	111111	111111	111111
	x2=	22	111111	111111	106	111111	111111	111111
	x2=	42	111111	111111	111111	166	111111	111111
	x2=	55	111111	111111	111111	111111	205	111111
	x2=	80	111111	111111	111111	111111	111111	280

Current		Optimum Solution Summary					
		x	f	z	x	f	z
			period 1				
optimum		0	40	10			
Period 1		15	85	25			
f1	z1	22	106	32			
40	10	42	166	52			
85	25	55	205	65			
106	32	80	280	90			
166	52						
205	65						
280	90						

Period 2

Wagner-Whitin (Forward) Dynamic Programming Inventory Model							
periods, N=	6		Current period=	2	<<Max		
1	2	3	4	5	6		
2	2	2	2	2	2		
20	17	10	18	5	50		
1	1	1	3	1	1		
10	15	7	20	13	25		
s correct?	yes	yes	yes	yes	yes	yes	
z2=	0	15	22	42	55	80	
C2(z2)=	0	47	61	101	127	177	
x3=	0	85	87	111111	111111	111111	111111
x3=	7	113	111111	108	111111	111111	111111
x3=	27	193	111111	111111	168	111111	111111
x3=	40	245	111111	111111	111111	207	111111
x3=	65	345	111111	111111	111111	111111	282

Current		Optimum Solution Summary					
		x	f	z	x	f	z
			period 1				
optimum		0	40	10			
Period 2		15	85	25			
f2	z2	22	106	32			
85	0	42	166	52			
108	22	55	205	65			
168	42	80	280	90			
207	55		period 2				
282	80	0	85	0			
		7	108	22			
		27	168	42			
		40	207	55			
		65	282	80			

Period 3:

Wagner-Whitin (Forward) Dynamic Programming Inventory Model								
Number of periods, N=		6		Currant period=	3	<<Max		
Period		1	2	3	4	5	6	
c(1 to 6)=		2	2	2	2	2	2	
K(1 to 6)=		20	17	10	18	5	50	
h(1 to 6)=		1	1	1	3	1	1	
D(1 to 6)=		10	15	7	20	13	25	
Are z3-values correct?			yes	yes	yes	yes	yes	
Period 2		z3=	0	7	27	40	65	
f2		C3(z3)=	0	24	64	90	140	
85	x4=	0	108	109	111111	111111	111111	
108	x4=	20	188	111111	169	111111	111111	
168	x4=	33	240	111111	111111	208	111111	
207	x4=	58	340	111111	111111	111111	283	
282								

Current		Optimum Solution Summary					
		x	f	z	x	f	z
			period 1			period 3	
optimum		0	40	10	0	108	0
Period 3		15	85	25	20	169	27
f3	z3	22	106	32	33	208	40
108	0	42	166	52	58	283	65
169	27	55	205	65			
208	40	80	280	90			
283	65		period 2				
		0	85	0			
		7	108	22			
		27	168	42			
		40	207	55			
		65	282	80			

Period 4:

Wagner-Whitin (Forward) Dynamic Programming Inventory Model							
Number of periods, N=		6		Current period=	4	<<Max	
Period		1	2	3	4	5	6
c(1 to 6)=		2	2	2	2	2	2
K(1 to 6)=		20	17	10	18	5	50
h(1 to 6)=		1	1	1	3	1	1
D(1 to 6)=		10	15	7	20	13	25
Are z4-values correct?			yes	yes	yes	yes	
Period 3		z4=	0	20	33	58	
f3		C4(z4)=	0	58	84	134	
108	x5=	0	169	166	111111	111111	
169	x5=	13	247	111111	231	111111	
208	x5=	38	397	111111	111111	356	
283							

Current		Optimum Solution Summary					
		x	f	z	x	f	z
			period 1			period 3	
optimum		0	40	10	0	108	0
Period 4		15	85	25	20	169	27
f4	z4	22	106	32	33	208	40
166	20	42	166	52	58	283	65
231	33	55	205	65		period 4	
356	58	80	280	90	0	166	20
			period 2		13	231	33
		0	85	0	38	356	58
		7	108	22			
		27	168	42			
		40	207	55			
		65	282	80			

Period 5:

Wagner-Whitin (Forward) Dynamic Programming Inventory Model							
Number of periods, N=		6		Current period=	5	<<Max	
Period		1	2	3	4	5	6
c(1 to 6)=		2	2	2	2	2	2
K(1 to 6)=		20	17	10	18	5	50
h(1 to 6)=		1	1	1	3	1	1
D(1 to 6)=		10	15	7	20	13	25
Are z5-values correct?			yes	yes	yes		
Period 4		z5=	0	13	38		
f4		C5(z5)=	0	31	81		
166	x6=	0	231	197	111111		
231	x6=	25	381	111111	272		
356							

Current		Optimum Solution Summary					
		x	f	z	x	f	z
			period 1			period 3	
optimum		0	40	10	0	108	0
Period 5		15	85	25	20	169	27
f5	z5	22	106	32	33	208	40
197	13	42	166	52	58	283	65
272	38	55	205	65		period 4	
		80	280	90	0	166	20
			period 2		13	231	33
		0	85	0	38	356	58
		7	108	22		period 5	
		27	168	42	0	197	13
		40	207	55	25	272	38
		65	282	80			

Period 6

Wagner-Whitin (Forward) Dynamic Programming Inventory Model							
Number of periods, N=		6		Current period=	6	<<Max	
Period		1	2	3	4	5	6
c(1 to 6)=		2	2	2	2	2	2
K(1 to 6)=		20	17	10	18	5	50
h(1 to 6)=		1	1	1	3	1	1
D(1 to 6)=		10	15	7	20	13	25
Are z6-values correct?			yes	yes			
Period 5		z6=	0	25			
f5		C6(z6)=	0	100			
197	x7=	0	272	297			
272							

Current		Optimum Solution Summary					
		x	f	z	x	f	z
			period 1			period 3	
optimum		0	40	10	0	108	0
Period 6		15	85	25	20	169	27
f6	z6	22	106	32	33	208	40
272	0	42	166	52	58	283	65
		55	205	65		period 4	
		80	280	90	0	166	20
			period 2		13	231	33
		0	85	0	38	356	58
		7	108	22		period 5	
		27	168	42	0	197	13
		40	207	55	25	272	38
		65	282	80		period 6	
					0	272	0

Optimum: Cost = $272

$z_1 = 10$, $z_2 = 22$, $z_3 = 0$

$z_4 = 20$, $z_5 = 38$, $z_6 = 0$

continued...

1

$i=1$, $K_1=250$:

Period, t	D_t	TC(1,t)	TCU(1,t)
1	60	250	250/1 = 250
2	70	250+1×70 = 320	320/2 = 160*
3	80	320+2×80 = 480	480/3 = 160*
4	90	480+3×90 = 750	750/4 = 187.50

Produce 60+70+80 = 210 for 1, 2, and 3

$i=4$, $K_4=300$

Period, t	D_t	TC(4,t)	TCU(4,t)
4	90	300	300/1 = 300
5	85	300+85 = 385	385/2 = 192.5
6	80	385+2×80 = 545	545/3 = 181.67
7	75	545+3×75 = 770	770/4 = 192.5

Produce 90+85+80 = 255 for 4, 5, and 6

$i=7$, $K_7=\$250$:

Period t	D_t	TC(7,t)	TCU(7,t)
7	75	250	250/1 = 250
8	70	250+70 = 320	320/2 = 160
9	65	320+2×65 = 450	450/3 = 150
10	60	450+3×60 = 630	630/4 = 157.50

Produce 75+70+65 = 210 for 7, 8, and 9

$i=10$, $K_{10}=\$250$:

Period t	D_t	TC(10,t)	TCU(10,t)
10	60	250	250/1 = 250
11	55	250+1×55 = 305	305/2 = 152.50
12	50	305+2×50 = 405	405/3 = 135

Produce 60+55+50 = 165 for 10, 11, and 12

2

$i=1$, $K=200$:

t	D_t	TC(1,t)	TCU(1,t)
1	100	200	200/1 = 200
2	120	200+144 = 344	344/2 = 172
3	50	344+2×1.2×50 = 464	464/3 = 154.67
4	70	464+3×1.2×70 = 716	716/4 = 179

$i=4$, $K=\$200$:

t	D_t	TC(4,t)	TCU(4,t)
4	70	200	200/1 = 200
5	90	200+1.2×90 = 308	308/2 = 154
6	105	308+2×1.2×105 = 560	560/3 = 186.67

$i=6$, $K=\$200$:

t	D_t	TC(6,t)	TCU(6,t)
6	105	200	200/1 = 200
7	115	200+1.2×115 = 338	338/2 = 169
8	95	338+2×1.2×95 = 566	566/3 = 188.67

$i=8$, $K=\$200$:

t	D_t	TC(8,t)	TCU(8,t)
8	95	200	200/1 = 200
9	80	200+1.2×80 = 296	296/2 = 148
10	85	296+2×1.2×85 = 500	500/3 = 166.67

$i=10$, $K=\$200$:

t	D_t	TC(10,t)	TCU(10,t)
10	85	200	200/1 = 200
11	100	200+1.2×100 = 320	320/2 = 160
12	110	320+2×1.2×110 = 584	548/3 = 194.67

Schedule:

Produce	For periods
270	1, 2, and 3
160	4, and 5
220	6 and 7
175	8 and 9
185	10 and 11
110	12

Continued...