Heat transfer introduction

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1 HEAT TRANSFER

1.1 Introduction

What is Heat Transfer

Heat transfer is the process of the movement of energy due to a temperature difference. The calculations we are interested in include determining the final temperatures of materials and how long it takes for these materials to reach these temperatures. This can help inform the level of insulation required to ensure heat is not lost from a system. Typically, heat loss is proportional to a temperature gradient (driving force or potential).

Heat transfer can be achieved by conduction, convection or radiation.

Conduction

Conduction is the form of heat that exists due to direct contact without movement. A temperature gradient within a substance causes a flow of energy from a hotter to colder region. These gradients can exist in solids, liquids and gases; provided there is no movement in the fluid phases, *i.e.* fluids which are not well mixed. Over time the temperature difference will reduce and approach thermal equilibrium (same temperature). Conduction occurs in a solid, liquid or gas; provided there is no bulk movement.

Examples of conduction include the end of a metal rod placed in a fire heating up from one end to the other, hot coffee heating through the mug or ice-cream cooling the bowl it is placed in.

Convection

Convection is the transfer of heat due to the bulk movement of fluids. As such convection only applies to heat transfer within a fluid or between a solid and a fluid but not the heat transfer within a solid. This heat transfer is achieved by the movement of molecules within the fluid. The term convection can refer to either mass transfer and/or heat transfer. Typically, when referred to as "convection", heat transfer is meant.

Convection is the sum of advection and diffusion:
Advection is the heat transported by large-scale movement of currents in the fluid; and
Diffusion is the random Brownian motion of individual particles in the fluid.

Examples of convection include the effect of hot air rising and falling (convection currents) or the large scale convection currents of the atmosphere and oceans.

Radiation

Radiation is the transfer of energy due to electromagnetic waves when thermal energy is converted by the movement of the charges of electrons and protons in the material. When a body radiates, the energy comes from the entire depth of the body, not just the surface. Radiation does not require a temperature gradient. A person standing some distance from the source will still feel the effects of the heat, *e.g.* a person near a fire is heated by the fire, not by the air surrounding them.

Examples include infra-red radiation such as, an incandescent light bulb emitting visible-light, the infrared radiation emitted by a common household radiator or electric heater, as well as the sun heating the earth.

1.2 Conductive Heat Transfer

Fourier's Law

Conduction is governed by Fourier's Law: The energy flux (rate of energy transfer per unit area; W/m) is proportional to the temperature gradient.

$$q' \propto \frac{\Delta T}{\Delta x} \qquad \textbf{1.1}$$

where:

q' = Heat flux (W/m^2)

ΔT = Temperature difference (K)

x = direction in which there is a temperature gradient (direction of heat transfer) (m)

Including a proportionality constant, this can be written as:

$$q' = -k\frac{dT}{dx} \qquad \textbf{1.2}$$

where:

k = thermal conductivity (W/(m.K))

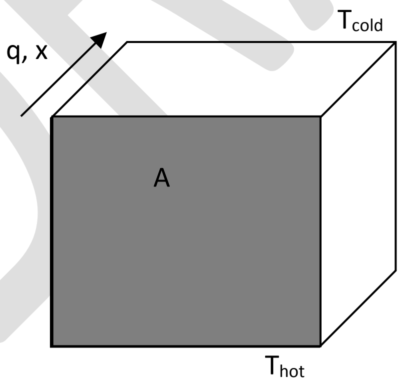
Or more conveniently:

$$\dot{q} = -k.A\frac{dT}{dx} \qquad \textbf{1.3}$$

where:

$\dot{q}$ = Heat (W)

A = Cross sectional area (m^2)



<u>Assumptions</u>:

- T_{hot} is at the same temperature across entire Area
- Use area perpendicular to the direction of heat transfer
- x is the direction from hot to cold (direction of temperature gradient

Thermal Conductivity

The constant k, is the thermal conductivity of the material through which heat is transferring. Simply, thermal conductivity is the property of a material to conduct heat. Materials with a good heat transfer ability (*e.g.* metals) have a high conductivity, while those with a poor conductivity (gases) have a low k value.

Some examples of thermal conductivities are given in

Table 1.1: Typical Thermal Conductivities

Material	Thermal conductivity (W/(m.K))
Air	0.025
Wood	0.04 - 0.4
Hollow Fill Fibre Insulation	0.042
Alcohols and oils	0.1 - 0.21
Polypropylene	0.25
Mineral oil	0.138
Rubber	0.16
LPG	0.23 - 0.26
Cement, Portland	0.29
Water (liquid)	0.6
Thermal grease	0.7 - 3
Thermal epoxy	1 - 7
Glass	1.1
Soil	1.5
Concrete, stone	1.7
Ice	2
Stainless steel	12.11 ~ 45.0
Lead	35.3
Aluminium	237 (pure) 120 — 180 (alloys)
Gold	318
Copper	401
Silver	429

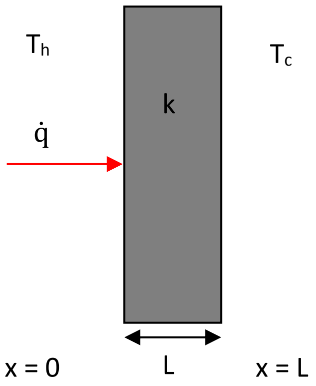
In reality, k is not a constant but dependent on temperature:
$k = k_0 + \beta T$, therefore need to integrate with respect to temperature

Good conductors have high thermal conductivity, *e.g.* copper.
Poor conductors (*i.e.* good insulators) have low thermal conductivity, *e.g.* asbestos.

Conduction Through a Wall

Case 1: Conduction through a single slab

Consider the heat transfer across a single slab.



where:

T_h: Hot temperature on one side of the slab (K)
T_c: Cooler temperature on other side of the slab (K)
$\dot{q}$: Heat Flow (W)
L: Length of the slab (m)
x: Direction in which heat flows (m)
k: Thermal conductivity of the slab (W/(m.K))

From Fourier's Law:

$$\dot{q} = -k.A\frac{dT}{dx} \qquad \textbf{1.3}$$

Assumptions:

- Area through slab is constant
- k is constant
- Steady state conditions
- Energy in one face of the slab = Energy out other face of the slab

This is a separable differential equation:

Rearranging:

$$\frac{\dot{q}}{-kA}dx = dT \qquad \textbf{1.4}$$

Therefore:

$$\int_{x=0}^{x=L}\frac{\dot{q}}{-kA}dx = \int_{T_h}^{T_c}dT \qquad \textbf{1.5}$$

Since none of $\dot{q}$, k or A are functions of temperature or x, we can remove these from the integral:

$$\frac{\dot{q}}{-kA}\int_{x=0}^{x=L}dx = \int_{T_h}^{T_c}dT \qquad \textbf{1.6}$$

And integrate:

$$\frac{\dot{q}}{-kA}(L-0) = (T_c - T_h) \qquad \textbf{1.7}$$

Therefore:

$$\dot{q} = \frac{kA(T_h - T_c)}{L} \qquad \textbf{1.8}$$

Example:

An aluminium plate (k = 215 W/m°C) is heated to 300°C. If the heat flux is 8.6 MW/m², how hot is the other face, if the metal is 5 mm thick?

Solution:

$q' = -k\frac{dT}{dx}$

Assuming constant k and A:

$q' = \frac{k(T_h - T_c)}{L}$

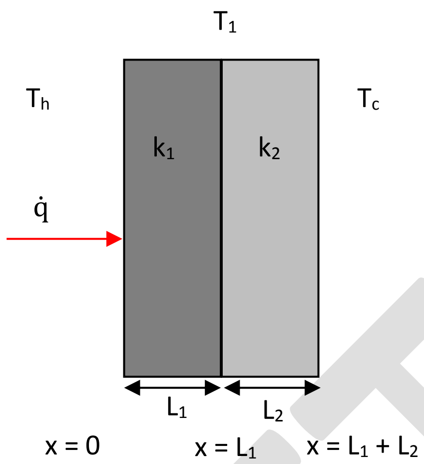
$\therefore \frac{q'L}{k} = T_h - T_c$

$\therefore T_c = T_h - \frac{q'L}{k} = 300 - \frac{(8600000)(0.005)}{215}$

$\therefore T_c = 100$

Case 2: Conduction through 2 slabs

Consider the heat transfer across two slabs.



where:

- T_h: Hot temperature on one side of the slab (K)
- T_1: Temperature at the end of slab 1 (K)
- T_c: Cooler temperature at the end of the slabs (K)
- $\dot{q}$: Heat Flow (W)
- L_1: Length of slab 1 (m)
- L_2: Length of slab 2 (m)
- x: Direction in which heat flows (m)
- k_1: Thermal conductivity of slab 1 (W/(m.K))
- k_2: Thermal conductivity of the slab 2 (W/(m.K))

Assumptions:

- Area through slab is constant
- $k_1 \neq k_2$
- Steady state conditions; therefore Energy in one face of the slab = Energy out other face of the slab

From single slab calculation:

$$\dot{q} = \frac{kA(T_h - T_c)}{L} \qquad \textbf{1.9}$$

Therefore:

Slab 1: $$\dot{q} = \frac{k_1 A(T_h - T_1)}{L_1} \qquad \textbf{1.10}$$

Slab 2: $$\dot{q} = \frac{k_2 A(T_1 - T_c)}{L_2} \qquad \textbf{1.11}$$

Re-arranging, this gives:

$$T_h - T_1 = \frac{\dot{q} L_1}{k_1 A} \qquad \textbf{1.12}$$

$$T_1 - T_c = \frac{\dot{q} L_2}{k_2 A} \qquad \textbf{1.13}$$

Adding these equations, we are left with an expression without T_1 and are able to calculate $\dot{q}$:

$$T_h - T_1 + T_1 - T_c = \frac{\dot{q}L_1}{k_1 A} + \frac{\dot{q}L_2}{k_2 A} \tag{1.14}$$

$$T_h \cancel{- T_1 + T_1} - T_c = \frac{\dot{q}L_1}{k_1 A} + \frac{\dot{q}L_2}{k_2 A} \tag{1.15}$$

$$T_h - T_c = \frac{\dot{q}}{A}\left(\frac{L_1}{k_1} + \frac{L_2}{k_2}\right) \tag{1.16}$$

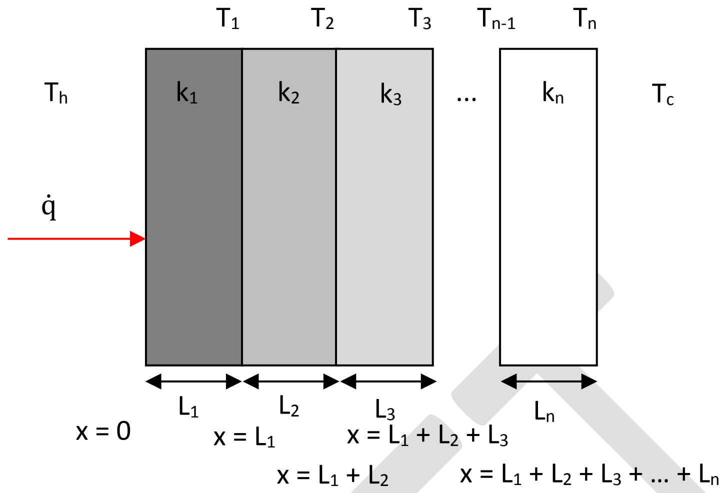
$$\frac{\dot{q}}{A} = \frac{T_h - T_c}{\left(\frac{L_1}{k_1} + \frac{L_2}{k_2}\right)} \tag{1.17}$$

In order to determine T_1, either of the original heat transfer equations can then be used.

> *<u>Example:</u>*
> *The wall of a furnace is constructed from an inner steel layer of 0.5 cm (k = 40 W/m.K) and a brick outer layer of 10 cm (k = 2.5 W/mk). The inner surface temperature is 900K and the outside surface temperature is 460K. What is the temperature between the steel and the brick?*
>
> *<u>Solution:</u>*
> *Assumptions:*
> - *Steady State (energy in = energy out)*
> - *Linear heat transfer (ignore the effects of edges)*
>
> *From Fourier's Law:* $q = -kA\frac{dT}{dx}$
> *For 2 slabs this simplifies to:* $\frac{q}{A} = \frac{T_{in} - T_{out}}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}$
>
> *Therefore:*
> $$\frac{q}{A} = \frac{(900 - 460)}{\frac{0.005}{40} + \frac{0.1}{2.5}} = \frac{440}{(0.000125 + 0.04)} = 10965.732 \ldots\ W/m^2$$
>
> *If we let T_1 be the temperature between the steel and the brick and:*
> $$T_1 = T_{in} - \frac{Q.L_1}{A.k_1} = 900 - (10965)(0.000125) = 898.629 \ldots K$$
>
> *The temperature between the brick and steel is approximately 625°C*

Case 3: Conduction through n-slabs

Consider the heat transfer across multiple slabs.



where:

T_h: Hot temperature on one side of the slab (K)
T_i: Temperature at the end of slab i (K)
T_c: Cooler temperature at the end of the slabs (K)
L_i: Length of slab i (m)
x: Direction in which heat flows (m)
k_i: Thermal conductivity of slab i (W/(m.K))
n: Number of slabs

Assumptions:

- Area through slab is constant
- $k_i \neq k_{i+1}$
- Steady state conditions; therefore Energy in one face of the slab = Energy out other face of the slab

As before we can solve for the heat flow through each slab:

Slab 1: $\dot{q} = \frac{k_1 A(T_h - T_1)}{L_1}$ **1.18**

Slab 2: $\dot{q} = \frac{k_2 A(T_1 - T_2)}{L_2}$ **1.19**

Slab 3: $\dot{q} = \frac{k_3 A(T_2 - T_3)}{L_3}$ **1.20**

Slab n: $\dot{q} = \frac{k_n A(T_{n-1} - T_c)}{L_n}$ **1.21**

Rearranging as before:

$T_h - T_1 = \frac{\dot{q} L_1}{k_1 A}$ **1.22**

$$T_1 - T_2 = \frac{\dot{q}L_2}{k_2A} \quad \textbf{1.23}$$

$$T_2 - T_3 = \frac{\dot{q}L_3}{k_3A} \quad \textbf{1.24}$$

$$T_{n-1} - T_c = \frac{\dot{q}L_n}{k_nA} \quad \textbf{1.25}$$

Adding these equations, we are left with an expression without T_i and are able to calculate $\dot{q}$:

$$T_h - T_c = \frac{\dot{q}}{A}\left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \cdots + \frac{L_n}{k_n}\right) \quad \textbf{1.26}$$

This is more commonly written as:

$$T_h - T_c = \dot{q}\left(\frac{L_1}{k_1A} + \frac{L_2}{k_2A} + \frac{L_3}{k_3A} + \cdots + \frac{L_n}{k_nA}\right) \quad \textbf{1.27}$$

Which can also be expressed as:

$$\dot{q} = \frac{\Delta T}{\sum_{i=1}^{n}\frac{L_i}{k_iA}} \quad \textbf{1.28}$$

Example:

On a cold winters day you decide to add on a few layers of clothes to stay warm. What heat flux do you experience for each successive layer added as below? Assume your skin is at 36°C and the temperature outside is at 4°C:

1) *T-Shirt (k = 0.05 W/mK; Thickness = 0.75 mm)*
2) *Jersey 1 (k = 0.06 W/mK; Thickness = 2 mm)*
3) *Jersey 2 (k = 0.05 W/mK; Thickness = 2 mm)*
4) *Jacket (k = 0.005 W/mK; Thickness = 3 mm)*
5) *Overcoat (k = 0.02 W/mK; Thickness = 4.5 mm)*

It is estimated that the body released between 90-140 W of heat. Further, the body has an average surface area of between 1.6 and 1.9m². Given this information comment on the answer above

Solution:

$$\dot{q} = \frac{\Delta T}{\sum_{i=1}^{n}\frac{L_i}{k_iA}} \; OR \; q' = \frac{\Delta T}{\sum_{i=1}^{n}\frac{L_i}{k_i}}$$

1) $q' = \frac{\Delta T}{\frac{L_1}{k_1}} = \frac{32}{\frac{0.75\times10^{-3}}{0.05}} = 2133.3\ W/m^2$
2) $q' = \frac{\Delta T}{\frac{L_1}{k_1}+\frac{L_2}{k_2}} = \frac{32}{\frac{0.75\times10^{-3}}{0.05}+\frac{2\times10^{-3}}{0.06}} = 662.1\ W/m^2$
3) $q' = \frac{\Delta T}{\frac{L_1}{k_1}+\frac{L_2}{k_2}+\frac{L_3}{k_3}} = \frac{32}{\frac{0.75\times10^{-3}}{0.05}+\frac{2\times10^{-3}}{0.06}+\frac{2\times10^{-3}}{0.05}} = 362.2\ W/m^2$
4) $q' = \frac{\Delta T}{\frac{L_1}{k_1}+\frac{L_2}{k_2}+\frac{L_3}{k_3}+\frac{L_4}{k_4}} = \frac{32}{\frac{0.75\times10^{-3}}{0.05}+\frac{2\times10^{-3}}{0.06}+\frac{2\times10^{-3}}{0.05}+\frac{3\times10^{-3}}{0.005}} = 46.5\ W/m^2$
5) $q' = \frac{\Delta T}{\frac{L_1}{k_1}+\frac{L_2}{k_2}+\frac{L_3}{k_3}+\frac{L_4}{k_4}+\frac{L_5}{k_5}} = \frac{32}{\frac{0.75\times10^{-3}}{0.05}+\frac{2\times10^{-3}}{0.06}+\frac{2\times10^{-3}}{0.05}+\frac{3\times10^{-3}}{0.005}+\frac{4.5\times10^{-3}}{0.02}} = 35\ W/m^2$

From the average body data, it can be shown that the body releases between 47 and 87 W/m². Therefore is would be a reasonable assumption that the average person would be most comfortable with four layers as calculated.

<u>Temperature profile through n-slabs</u>

From the calculation of heat transfer in one slab it was show that:

$$\dot{q} = \frac{kA(T_h - T_c)}{L} \qquad \textbf{1.8}$$

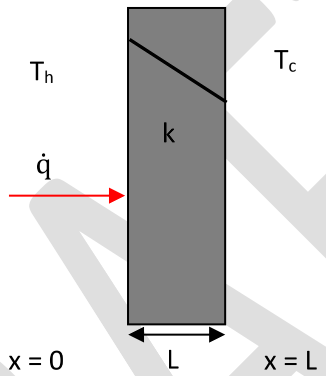
Rearranging:

$$\dot{q}L = kA(T_h - T_c) \qquad \textbf{1.29}$$

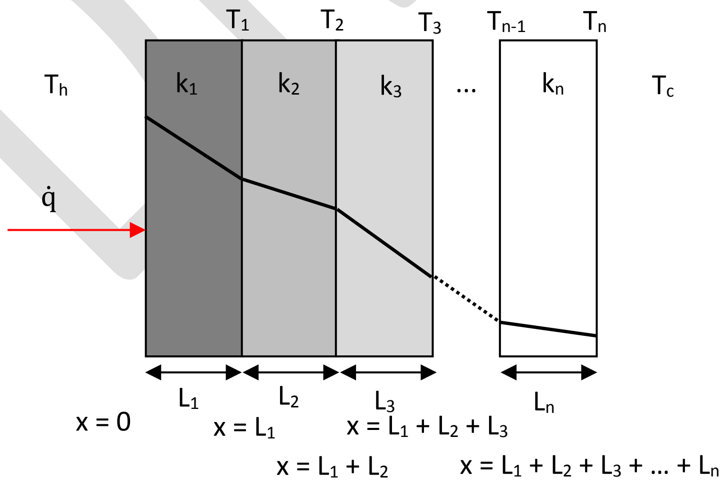
$$\frac{\dot{q}L}{kA} = T_h - T_c \qquad \textbf{1.30}$$

$$T_c = \frac{-\dot{q}L}{kA} + T_h \qquad \textbf{1.31}$$

When $y = mx + c$, plot of T *vs.* L gives a straight line temperature profile (for a constant k, as steady state). An insulator has a LOW thermal conductivity, therefore (-q̇/k.A) is LARGE, and the slope is great, *i.e.* not much temperature change. NOTE: Only applicable for a constant heat transfer constant.



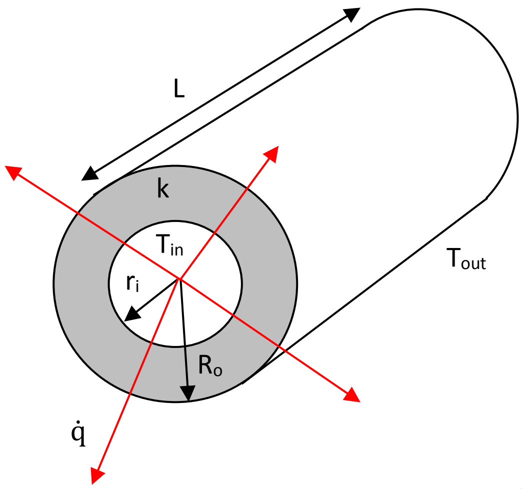
In the same way this can be extended for n-slabs in series. The temperature profile in each slab is linear (provided k is constant), with a different linear slope depending on the value of k.



Conduction through a Cylinder

<u>Case 1: Conduction through a single cylinder</u>

Consider the heat transfer across a single cylinder.



where:

T_{in}: Temperature on the inside of the cylinder (K)
T_{out}: Temperature on the outside of the cylinder (K)
$\dot{q}$: Heat Flow (W)
r_i: Inner radius of the cylinder (m)
r_o: Outer radius of the cylinder (m)
k: Thermal conductivity of the slab (W/(m.K))

Assumptions:

- Cylinder thickness is constant throughout
- k is constant
- Steady state conditions; therefore Energy in one face of the slab = Energy out other face of the slab
- No axial heat flow
- Area through which heat flows is NOT constant!

From Fourier's Law:

$$\dot{q} = -k.A\frac{dT}{dx} \qquad \textbf{1.3}$$

But we do not have an x-direction in a cylinder. Therefore convert Fourier's Law into cylindrical equivalent:

$$\dot{q} = -k.A\frac{dT}{dr} \qquad \textbf{1.32}$$

The area is that which is perpendicular to the direction of heat transfer. Therefore:
A = 2π.r.L (circumference of a cylinder x length) **1.33**

And Fourier's Law becomes:

$$\dot{q} = -k.(2\pi rL)\frac{dT}{dr} \qquad \textbf{1.34}$$

Assuming k is constant and solving the separable differential equation:

$$\frac{\dot{q}}{-k2\pi L}\int_{r_i}^{r_o}\frac{dr}{r} = \int_{T_{in}}^{T_{out}} dT \qquad \textbf{1.35}$$

Integrating:

$$\frac{\dot{q}}{-k2\pi L}\ln\left(\frac{r_o}{r_i}\right) = T_{out} - T_{in} \qquad \textbf{1.36}$$

Or:

$$T_{in} - T_{out} = \frac{\dot{q}}{k2\pi L}\ln\left(\frac{r_o}{r_i}\right) \qquad \textbf{1.37}$$

Example:

Water enters a pipe at a temperature of 60°C. Assuming the pipe is made of stainless steel (k = 30W/m²) and that the pipe has an inner and outer diameter of 20 and 25 cm respectively, what is the outer temperature of the pipe if heat is lost at a rate of 15kJ/m.s? Assume the temperature is constant down the length of the pipe.

Solution:

$$T_{in} - T_{out} = \frac{\dot{q}}{k2\pi L}ln\left(\frac{r_o}{r_i}\right)$$

$$T_{out} = T_{in} - \frac{\dot{q}}{k2\pi L}ln\left(\frac{r_o}{r_i}\right) = 60 - \frac{15000}{(30)2\pi}ln\left(\frac{25}{20}\right)$$

$$T_{out} = 42.2°C$$

Conduction through an Irregular Shape

When the heat flows through a material which is not a uniform shape, the area term can no longer be taken out of the integral term as was done previously.

From the Fourier form as before:

$$\int_{x=0}^{x=L} \frac{\dot{q}}{-kA} dx = \int_{T_h}^{T_c} dT \qquad \textbf{1.5}$$

Now:

$$\frac{\dot{q}}{-k} \int_{x=0}^{x=L} \frac{1}{A} dx = \int_{T_h}^{T_c} dT \qquad \textbf{1.38}$$

Typically, area will be given as a function of x (*e.g.* A = x +3; or A = $2x^2 - 4$ or others) and needs to be integrated as appropriate.

> *Example:*
>
> *In a particular building in a hot climate (45°C), the inside of a building is cooled to 15°C. The walls of the building include an irregular shaped steel girder which forms a structural member of the wall. Assuming that no heat is lost or gained to or from the insulation through the sides of the girder, calculate the heat flow from the room from a single girder.*
>
> *Given: Thickness of the wall, 20 cm; k = 45 $W.m^{-1}.K^{-1}$; A = $-50.x^2 + 3$*
>
> *Solution:*
>
> $$\dot{q} = -k.A\frac{dT}{dx}$$
>
> $$\dot{q} = -k.(-50x^2 + 3)\frac{dT}{dx}$$
>
> $$\frac{\dot{q}}{-k(-50x^2+3)} dx = dT$$
>
> $$\int_{x=0}^{x=L} \frac{\dot{q}}{-k(-50x^2+3)} dx = \int_{T_c}^{T_h} dT$$
>
> $$\frac{\dot{q}}{k} \int_{x=0}^{x=L} \frac{1}{50x^2 - 3} dx = \int_{T_c}^{T_h} dT$$
>
> $$\frac{\dot{q}}{k}\left[\frac{2}{\sqrt{4\times 50\times 3}} tan^{-1}\frac{2\times 3x}{\sqrt{4\times 50\times 3}}\right]_0^L = T_h - T_c$$
>
> $$\frac{\dot{q}}{45}\left[\frac{2}{\sqrt{600}} tan^{-1}\frac{6x}{\sqrt{600}}\right]_0^{0.2} = 45 - 15 = 30$$
>
> $$\dot{q} = \frac{(30 \times 45)}{\left(\frac{2}{\sqrt{600}} tan^{-1}\frac{6(0.2)}{\sqrt{600}}\right) - \left(\frac{2}{\sqrt{600}} tan^{-1}\frac{6(0)}{\sqrt{600}}\right)}$$
>
> $$\dot{q} = \frac{1350}{\left(0.081\, tan^{-1}\frac{1.2}{24.5}\right) - 0}$$
>
> $$\dot{q} = \frac{1350}{1.665} = 810.6\, W$$

Conduction when Thermal Conductivity is Not Constant

Typically, thermal conductivity is not constant, but a function of temperature. As such:

From the Fourier form as before:

$$\int_{x=0}^{x=L} \frac{\dot{q}}{-kA} dx = \int_{T_h}^{T_c} dT \qquad \textbf{1.5}$$

Now:

$$-\frac{\dot{q}}{A} \int_{x=0}^{x=L} \frac{1}{A} dx = \int_{T_h}^{T_c} k dT \qquad \textbf{1.39}$$

Typically, k will be given as a function of x (*e.g.* k = x +30; or A = $2x^2 - 4$ or others) and needs to be integrated as appropriate.

Thermal Resistance

From the slab calculations, it was shown that:

$$T_h - T_c = \dot{q}\left(\frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \cdots + \frac{L_n}{k_n A}\right) \qquad \textbf{1.40}$$

Or:

$$\dot{q} = \frac{T_h - T_c}{\left(\frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \cdots + \frac{L_n}{k_n A}\right)} \qquad \textbf{1.41}$$

$$\dot{q} = \frac{\Delta T}{\sum_{i=1}^{n} \frac{L_i}{k_i A}} \qquad \textbf{1.42}$$

This can be simplified to:

$$\dot{q} = \frac{\Delta T}{R} \qquad \textbf{1.43}$$

where:

ΔT: Temperature difference (K)

R: Thermal resistance (K/W)

For slab calculations, $R = \sum_{i=1}^{n} \frac{L_i}{k_i A}$, however, R can represent the resistance for any range of shapes.

Example:

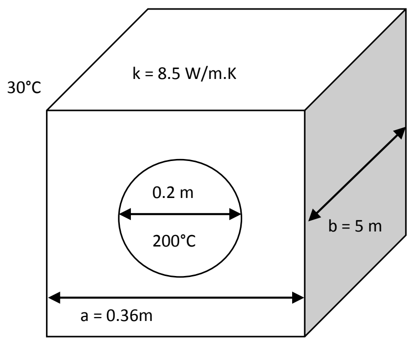
A circular pipe of 20 cm is enclosed centrally in a square section insulator of side 36 cm. The thermal conductivity of the material is given as 8.5W/mK. The inside surface is at 200°C; while the outside is at 30°C. Determine the heat flow for a length of 5 m.

Additional information:

$$R = \frac{1}{2\pi l}\left[\frac{1}{k} ln \frac{1.08a}{2r}\right]$$

where:

a = side length of length of square section
l = length
r = radius of inner portion



Solution:

$$\dot{q} = \frac{\Delta T}{R}$$

$$R = \frac{1}{2\pi \times 5}\left[\frac{1}{8.5} ln \frac{1.08 \times 0.36}{2.\frac{0.2}{2}}\right] = 0.002489362 \ ℃/W$$

$$\therefore \dot{q} = \frac{\Delta T}{R} = \frac{200 - 30}{0.002489 \ldots} = 68290.59 = 68\,000 \ W$$

Example:

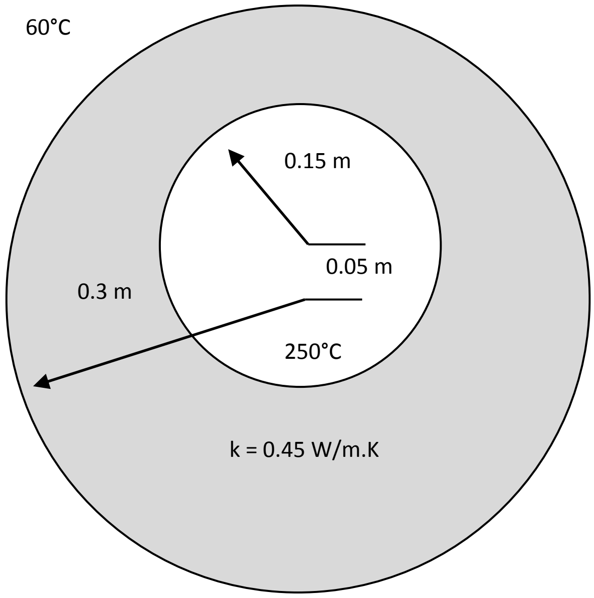
A pipe of 30 cm OD is insulated by a material of thermal conductivity 0.45 W/mK. Due to space restrictions, the inner pipe is placed slightly off-centre (5cm), resulting in a portion of insulation thicker than the rest of the pipe (see diagram). The inner surface is at 250°C and the outer surface is at 60°C. Determine the heat loss for a 5 m length of pipe.

Additional information:

$$R = \frac{1}{2\pi.k.l} ln \frac{\sqrt{[(r_2+r_1)^2 - e^2]} + \sqrt{[(r_2-r_1)^2 - e^2]}}{\sqrt{[(r_2+r_1)^2 - e^2]} - \sqrt{[(r_2-r_1)^2 - e^2]}}$$

where:

e = eccentricity, m



Solution:

$$\dot{q} = \frac{\Delta T}{R}$$

$$R = \frac{1}{2\pi \times 0.45 \times 5} ln \frac{\sqrt{[(0.15+0.3)^2 - 0.05^2]} + \sqrt{[(0.15-0.3)^2 - 0.05^2]}}{\sqrt{[(0.15+0.3)^2 - 0.05^2]} - \sqrt{[(0.15-0.3)^2 - 0.05^2]}}$$

$$R = \frac{1}{2\pi \times 0.45 \times 5} ln \frac{0.4472 + 0.14142}{0.4472 - 0.14142} = 0.046325\ ℃/W$$

$$\therefore \dot{q} = \frac{250-60}{0.046325} = 4101\ W$$

1.3 Convective Heat Transfer

Newton's Law of Cooling

Convection (film) heat transfer occurs through the bulk movement of fluid within a fluid or between a solid and a fluid.

Conduction is governed by Newton's Law of Cooling:

$$\dot{q} = h.A.\Delta T \qquad \mathbf{1.44}$$

where:

h = Heat Transfer Coefficient (W/m^2.K)

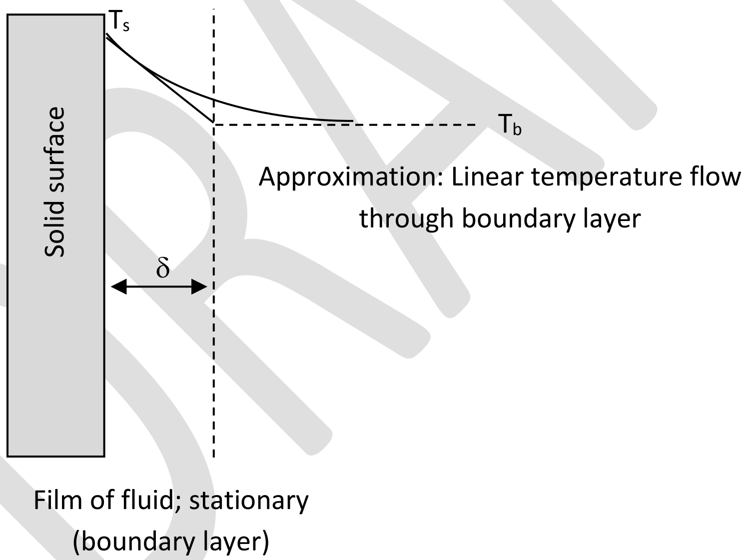
A = Area (m^2)

ΔT = Temperature difference (K)

Convection (and Conduction) Through a Wall

Case 1: Convection from a single slab:

Consider the convective heat from a single slab.



where:

δ = thickness of stationary boundary layer

T_s = Surface temperature

T_b = Bulk temperature

If we assume that there is no movement in a boundary layer between the slab and bulk fluid (δ – Greek delta), we can approximate the heat transfer to Fourier's Law:

$$\dot{q} = \frac{kA(T_s - T_b)}{\delta} \qquad \mathbf{1.45}$$

Defining a heat transfer coefficient as h:

$$h = \frac{k}{\delta} \quad \textbf{1.46}$$

And equation [the one above] becomes:

$$\dot{q} = hA(T_s - T_b) \quad \textbf{1.47}$$

where:

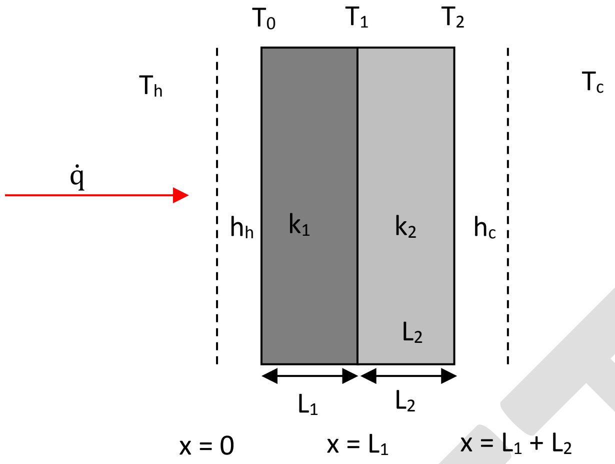
h = Heat Transfer Coefficient (W/m^2.K)

Note:

It is harder to measure wall temperatures compared to the bulk fluid temperatures. Therefore, the equations below are developed to use the bulk fluid temperatures and not intermediate wall temperatures.

Case 2: Convection on either side of 2 slabs:

Consider the heat across two slabs, taking into account both conduction and convection.



where:

- T_h: Hot temperature on one side of the slab (K)
- T_1: Temperature at the end of slab 1 (K)
- T_c: Cooler temperature at the end of the slabs (K)
- $\dot{q}$: Heat Flow (W)
- L_1: Length of slab 1 (m)
- L_2: Length of slab 2 (m)
- x: Direction in which heat flows (m)
- k_1: Thermal conductivity of slab 1 (W/(m.K))
- k_2: Thermal conductivity of the slab 2 (W/(m.K))
- h_h: Heat Transfer coefficient on the hot side (W/m^2.K)
- h_c: Heat Transfer coefficient on the cold side (W/m^2.K)

Assumptions:

- The slabs have thermal conductivities (k); the fluids have heat transfer coefficients (h)
- Area through the slabs is constant
- $k_1 \neq k_2$ (k_1 and k_2 are constant)
- Steady state conditions; therefore Energy in one face of the slab = Energy out other face of the slab

Heat transfer equations from inside to outside:

For inside convective film: $\dot{q} = h_h A(T_h - T_0)$ **1.48**

Slab 1 (conduction): $\dot{q} = \frac{k_1 A(T_0 - T_1)}{L_1}$ **1.49**

Slab 2 (conduction): $\dot{q} = \frac{k_2 A(T_1 - T_2)}{L_2}$ **1.50**

For outside film: $\dot{q} = h_c A(T_2 - T_c)$ **1.51**

Rearranging for each in terms of temperature:

$$T_h - T_0 = \frac{\dot{q}}{h_h A} \quad \textbf{1.52}$$

$$T_0 - T_1 = \frac{\dot{q} L_1}{k_1 A} \quad \textbf{1.53}$$

$$T_1 - T_2 = \frac{\dot{q} L_2}{k_2 A} \quad \textbf{1.54}$$

$$T_2 - T_c = \frac{\dot{q}}{h_c A} \quad \textbf{1.55}$$

Then adding the last four equations, we are left with an expression without T_c and are able to calculate $\dot{q}$:

$$T_h - T_0 + T_0 - T_1 + T_1 - T_2 + T_2 - T_c = \frac{\dot{q}}{h_h A} + \frac{\dot{q} L_1}{k_1 A} + \frac{\dot{q} L_2}{k_2 A} + \frac{\dot{q}}{h_c A} \quad \textbf{1.56}$$

$$T_h \cancel{- T_0 + T_0 - T_1 + T_1 - T_2 + T_2} - T_c = \frac{\dot{q}}{h_h A} + \frac{\dot{q} L_1}{k_1 A} + \frac{\dot{q} L_2}{k_2 A} + \frac{\dot{q}}{h_c A} \quad \textbf{1.57}$$

$$T_h - T_c = \frac{\dot{q}}{A}\left(\frac{1}{h_h} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_c}\right) \quad \textbf{1.58}$$

$$\frac{\dot{q}}{A} = \frac{T_h - T_c}{\left(\frac{1}{h_h} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_c}\right)} \quad \textbf{1.59}$$

Given the area, heat transfer coefficients ad thermal conductivities, only need to know inner and outer temperatures (NOT any of the inside temperatures) to solve for q.

Reminder:

$$\dot{q} = \frac{\Delta T}{R} \quad \textbf{1.60}$$

For the above equation:

$$\dot{q} = \frac{\Delta T}{\left(\frac{1}{h_h A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_c A}\right)} \quad \textbf{1.61}$$

$$\dot{q} = \frac{\Delta T}{(R_1 + R_2 + R_3 + R_4)} \quad \textbf{1.62}$$

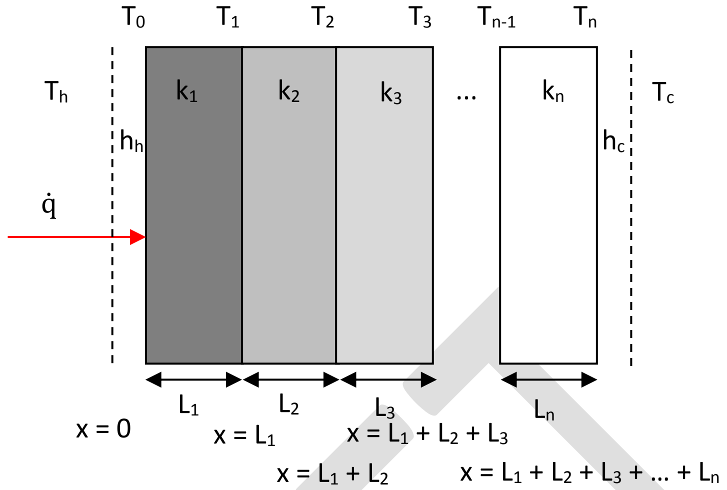
where:

R_i = individual thermal resistances for different layers

Compare this to U as defined later!!!

Case 3: Convection on either side of n-slabs:

Consider the heat transfer across multiple slabs.



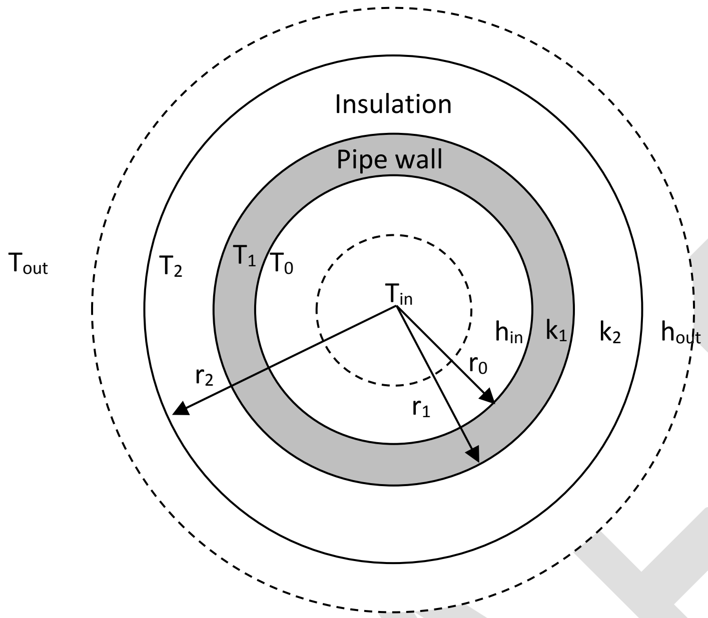
Using similar derivations as above:

$$\frac{\dot{q}}{A} = \frac{T_h - T_c}{\left(\frac{1}{h_h} + \sum_{i=1}^{n} \frac{L_i}{k_i} + \frac{1}{h_c}\right)} \qquad \textbf{1.63}$$

Convection (and Conduction) Through a Cylinder

<u>Case 1: Convection (and conduction) through concentric cylinders</u>

Consider the heat transfer across multiple layers of a cylinder.



where:

- T_{in}: Temperature on the inside of the cylinder (K)
- T_{out}: Temperature on the outside of the cylinder (K)
- $\dot{q}$: Heat Flow (W)
- r_0: Inner radius of the cylinder (m)
- r_1: Outer radius of the 1st cylinder (pipe) (m)
- r_2: Outer radius of the 2nd cylinder (insulation) (m)
- k_1: Thermal conductivity of cylinder 1 (W/(m.K))
- k_2: Thermal conductivity of cylinder 2 (W/(m.K))
- h_{in}: Heat Transfer coefficient on the inside (inside) ($W/m^2.K$)
- h_{out}: Heat Transfer coefficient on the outside (outside) ($W/m^2.K$)

<u>Assumptions:</u>

- Cylinder thickness is constant throughout
- Steady state conditions; therefore Energy in one face of the slab = Energy out other face of the slab
- Area through which heat flows is NOT constant!
- $k_1 \neq k_2$
- No axial heat transfer

<u>Convection on inner surface:</u>

$$\dot{q} = h_{in}A_0(T_{in} - T_0) \qquad \textbf{1.64}$$

$$A_0 = 2\pi r_0 L \qquad \textbf{1.65}$$

Area is the area in contact with the film (we don't know how thick the film is so can't use any other area)

$$\therefore \dot{q} = h_{in} 2\pi r_0 L (T_{in} - T_0) \tag{1.66}$$

$$T_{in} - T_0 = \frac{\dot{q}}{h_i 2\pi r_0 L} \tag{1.67}$$

<u>Conduction across pipe:</u>

$$\dot{q} = -k_1 . (2\pi r L) \frac{dT}{dr} \tag{1.68}$$

Assuming k is constant and solving the separable differential equation:

$$\therefore \frac{\dot{q}}{-k_1 2\pi L} \int_{r_0}^{r_1} \frac{dr}{r} = \int_{T_0}^{T_1} dT \tag{1.69}$$

Integrating:

$$T_0 - T_1 = \frac{\dot{q}}{k_1 2\pi L} \ln\left(\frac{r_1}{r_0}\right) \tag{1.70}$$

<u>Conduction across insulation:</u>

As above for conduction through pipe: $T_1 - T_2 = \frac{\dot{q}}{k_2 2\pi L} \ln\left(\frac{r_2}{r_1}\right)$ **1.71**

<u>Convection on outer surface:</u>

As for convection on inner surface:

$$\dot{q} = h_{out} 2\pi r_2 L (T_2 - T_{out}) \tag{1.72}$$

Therefore:

$$T_2 - T_{out} = \frac{\dot{q}}{h_o 2\pi r_2 L} \tag{1.73}$$

Consolidating for each layer:

$$T_{in} - T_0 = \frac{\dot{q}}{h_{in} 2\pi r_0 L} \tag{1.74}$$

$$T_0 - T_1 = \frac{\dot{q}}{k_1 2\pi L} \ln\left(\frac{r_1}{r_0}\right) \tag{1.75}$$

$$T_1 - T_2 = \frac{\dot{q}}{k_2 2\pi L} \ln\left(\frac{r_2}{r_1}\right) \tag{1.76}$$

$$T_2 - T_{out} = \frac{\dot{q}}{h_{out} 2\pi r_2 L} \tag{1.77}$$

Adding equations these four equations:

$$T_{in} - T_0 + T_0 - T_1 + T_1 - T_2 + T_2 - T_{out} = \frac{\dot{q}}{h_{in} 2\pi r_0 L} + \frac{\dot{q}}{k_1 2\pi L} \ln\left(\frac{r_1}{r_0}\right) + \frac{\dot{q}}{k_2 2\pi L} \ln\left(\frac{r_2}{r_1}\right) + \frac{\dot{q}}{h_{out} 2\pi r_2 L} \tag{1.78}$$

$$T_{in} \cancel{- T_0 + T_0 - T_1 + T_1 - T_2 + T_2} - T_{out} = \dot{q} \left(\frac{1}{h_{in} 2\pi r_0 L} + \frac{1}{k_1 2\pi L} \ln\left(\frac{r_1}{r_0}\right) + \frac{1}{k_2 2\pi L} \ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_{out} 2\pi r_2 L} \right) \tag{1.79}$$

$$T_{in} - T_{out} = \dot{q} \left(\frac{1}{h_{in} 2\pi r_0 L} + \frac{1}{k_1 2\pi L} \ln\left(\frac{r_1}{r_0}\right) + \frac{1}{k_2 2\pi L} \ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_{out} 2\pi r_2 L} \right) \tag{1.80}$$

$$\dot{q} = \frac{T_{in} - T_{out}}{\left(\frac{1}{h_{in} 2\pi r_0 L} + \frac{1}{k_1 2\pi L} \ln\left(\frac{r_1}{r_0}\right) + \frac{1}{k_2 2\pi L} \ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_{out} 2\pi r_2 L} \right)} \tag{1.81}$$

Log Mean Radius and Log Mean Area

We define the log mean radius as follows:

$$r_{LM} = \frac{r_2 - r_1}{\ln\frac{r_2}{r_1}} \qquad \textbf{1.82}$$

where:

r_{LM} = Log mean radius
r_2 = outer radius
r_1 = inner radius

Note:

$$\frac{r_2 - r_1}{\ln\frac{r_2}{r_1}} \neq \frac{r_2 - r_1}{2} \qquad \textbf{1.83}$$

From this we can define the log mean area:

$$A_{LM} = 2\pi r_{LM} L \qquad \textbf{1.84}$$

where:

A_{LM} = Log mean area

From the heat flow through two cylinders, (conduction and convection, it was shown that:

$$\dot{q} = \frac{T_{in} - T_{out}}{\left(\frac{1}{h_{in} 2\pi r_0 L} + \frac{1}{k_1 2\pi L}\ln\left(\frac{r_1}{r_0}\right) + \frac{1}{k_2 2\pi L}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_{out} 2\pi r_2 L}\right)} \qquad \textbf{1.85}$$

Which we can re-write as:

$$\dot{q} = \frac{T_{in} - T_{out}}{\left(\frac{1}{h_{in} 2\pi r_0 L} + \frac{1}{k_1 2\pi L}\ln\left(\frac{r_1}{r_0}\right)\cdot\frac{(r_1 - r_0)}{(r_1 - r_0)} + \frac{1}{k_2 2\pi L}\ln\left(\frac{r_2}{r_1}\right)\cdot\frac{(r_2 - r_1)}{(r_2 - r_1)} + \frac{1}{h_{out} 2\pi r_2 L}\right)} \qquad \textbf{1.86}$$

From the definition of R_{LM}, and converting wherever possible to surface area instead of radius, this becomes:

$$\dot{q} = \frac{T_{in} - T_{out}}{\left(\frac{1}{h_{in} A_0} + \frac{1}{k_1 2\pi L}\cdot\frac{(r_1 - r_0)}{r_{LM,10}} + \frac{1}{k_2 2\pi L}\cdot\frac{(r_2 - r_1)}{r_{LM,21}} + \frac{1}{h_{out} A_2}\right)} \qquad \textbf{1.87}$$

$$\dot{q} = \frac{T_{in} - T_{out}}{\left(\frac{1}{h_{in} A_0} + \frac{(r_1 - r_0)}{k_1 A_{LM,10}} + \frac{(r_2 - r_1)}{k_2 A_{LM,21}} + \frac{1}{h_{out} A_2}\right)} \qquad \textbf{1.88}$$

For n concentric circles:

$$\dot{q} = \frac{T_{in} - T_{out}}{\left(\frac{1}{h_{in} A_0} + \sum_{i=1}^{n} \frac{\Delta R_i}{k_i A_{LM,i}} + \frac{1}{h_{out} A_n}\right)} \qquad \textbf{1.89}$$

1.4 Overall Heat Transfer Coefficient

Previously, heat transfer was expressed in terms of thermal resistance as:

$$\dot{q} = \frac{\Delta T}{R} \tag{1.90}$$

where:

ΔT: Temperature difference (K)

R: Thermal resistance (K/W)

For multiple slabs in series the thermal resistance was shown to be:

$$R = \sum_{i=1}^{n} \frac{L_i}{k_i A} \tag{1.91}$$

Which can be re-written to include the convection as:

$$R = \frac{1}{h_h A} + \sum_{i=1}^{n} \frac{L_i}{k_i A} + \frac{1}{h_c A} \tag{1.92}$$

And for cylindrical systems:

$$R = \frac{1}{h_{in} A_0} + \sum_{i=1}^{n} \frac{\Delta R_i}{k_i A_{LM,i}} + \frac{1}{h_{out} A_n} \tag{1.93}$$

A new term is now introduced for the Overall Heat Transfer Coefficient (U):

$$U = \frac{1}{A.R} \tag{1.94}$$

$$\therefore \dot{q} = UA(T_{in} - T_{out}) \tag{1.95}$$

Note: Thermal resistance (R) includes the term for area (A), while the overall heat transfer coefficient (A) does not. Further, R and U are inverse relationships.

Defining the overall heat transfer coefficient for cylinders:

$$\dot{q} = UA(T_{in} - T_{out}) \tag{1.96}$$

We can measure heat and temperatures independently; UA can be calculated as a combined measured value.

$$UA = \frac{1}{\left(\frac{1}{h_{in}} + \sum_{i=1}^{n} \frac{\Delta r_i}{k_i} + \frac{1}{h_{out}}\right)} \tag{1.97}$$

We can then define different U values:

$$UA = U_0 A_0 = U_i A_i \tag{1.98}$$

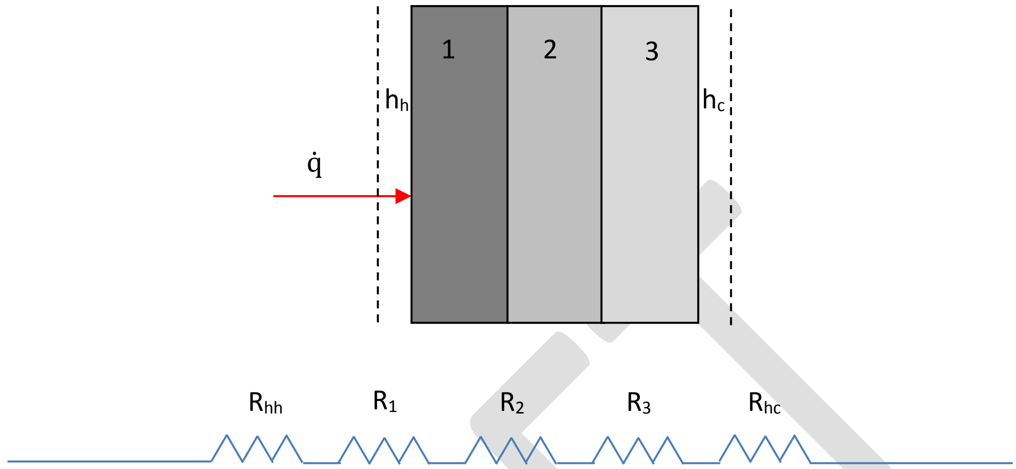
$$U_0 = \frac{UA}{A_0} \quad \text{For outer cylinder} \tag{1.99}$$

$$U_i = \frac{UA}{A_i} \quad \text{For inner cylinder} \tag{1.100}$$

1.5 Heat Transfer through Composite Walls

Heat transfer through series configuration

It is possible to relate thermal resistance to the resistance in an electrical circuit. In this way the resistance through solid slabs can be represented as below.



Where:

R = Thermal resistance through each layer (including convection and conduction)

Reminder:

$$\dot{q} = \frac{\Delta T}{R} \qquad \textbf{1.90}$$

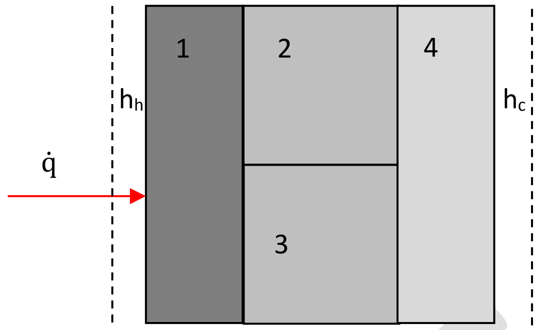
Therefore for the slab above:

$$\dot{q} = \frac{\Delta T}{R_{hh} + R_1 + R_2 + R_3 + R_{hc}} \qquad \textbf{1.101}$$

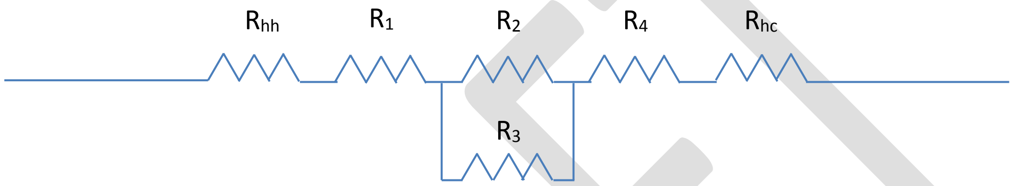
$$\dot{q} = \frac{\Delta T}{h_{hh}.A + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + h_{hc}.A} \qquad \textbf{1.102}$$

Heat transfer through parallel configuration

In a similar way, heat transfer through a parallel configuration can be equated to an electrical circuit.



Scenario 1: Assuming the surfaces normal to the direction of heat flow are isothermal (*i.e.* the entire length of slabs 1 and 4 have the same temperature), this gives:



Where:

R = Thermal resistance through each layer (including convection and conduction)

$$\dot{q} = \frac{\Delta T}{R} \quad \textbf{1.90}$$

In a similar manner to calculating electrical resistance, the resistance through the parallel section can be written as:

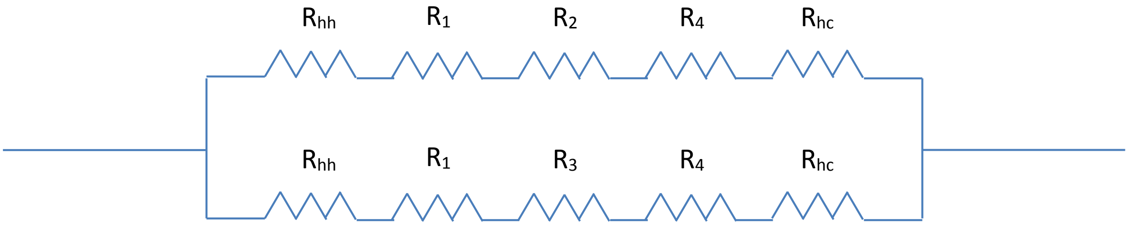
$$R_{2/3} = \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} \quad \textbf{1.103}$$

$$R_{2/3} = \left(\frac{k_2 A}{L_2} + \frac{k_3 A}{L_3}\right)^{-1} \quad \textbf{1.104}$$

Therefore:

$$\dot{q} = \frac{\Delta T}{h_{hh.}A + \frac{L_1}{k_1 A} + \left(\frac{k_2 A}{L_2} + \frac{k_3 A}{L_3}\right)^{-1} + \frac{L_4}{k_4 A} + h_{hc.}A} \quad \textbf{1.105}$$

Scenario 2: However, if we assume that the surfaces parallel to heat flow are adiabatic (*i.e.* no heat flows from slab 2 to 3), a better approximation is given by:



Now:

$$\dot{q} = \frac{\Delta T}{R} \qquad \textbf{1.90}$$

Adding all the sections which are in series, inverting to account for addition of multiple parallel sections and again inverting the sum of this:

$$\dot{q} = \frac{\Delta T}{\left(\frac{1}{h_{hh}.A+\frac{L_1}{k_1A}+\frac{L_2}{k_2A}+\frac{L_4}{k_4A}+h_{hc}.A}+\frac{1}{h_{hh}.A+\frac{L_1}{k_1A}+\frac{L_3}{k_3A}+\frac{L_4}{k_4A}+h_{hc}.A}\right)^{-1}} \qquad \textbf{1.106}$$

1.6 Heat Transfer from Extended Surfaces

Introduction

To cool something down, we may have a cooling fin. A hot object may have a long flat (or thin) piece protruding from it to increase surface area and also the conductive and convective heat transfer, thereby cooling the object down faster.

Fins may come in various forms:

- Straight Fins
  - Rectangular
  - Triangular
  - Parabolic
- Annular Fins
- Pin Fins
  - Rectangular
  - Triangular
  - Parabolic

Fins of Uniform Cross-Sectional Area

These could be rectangular or pins.

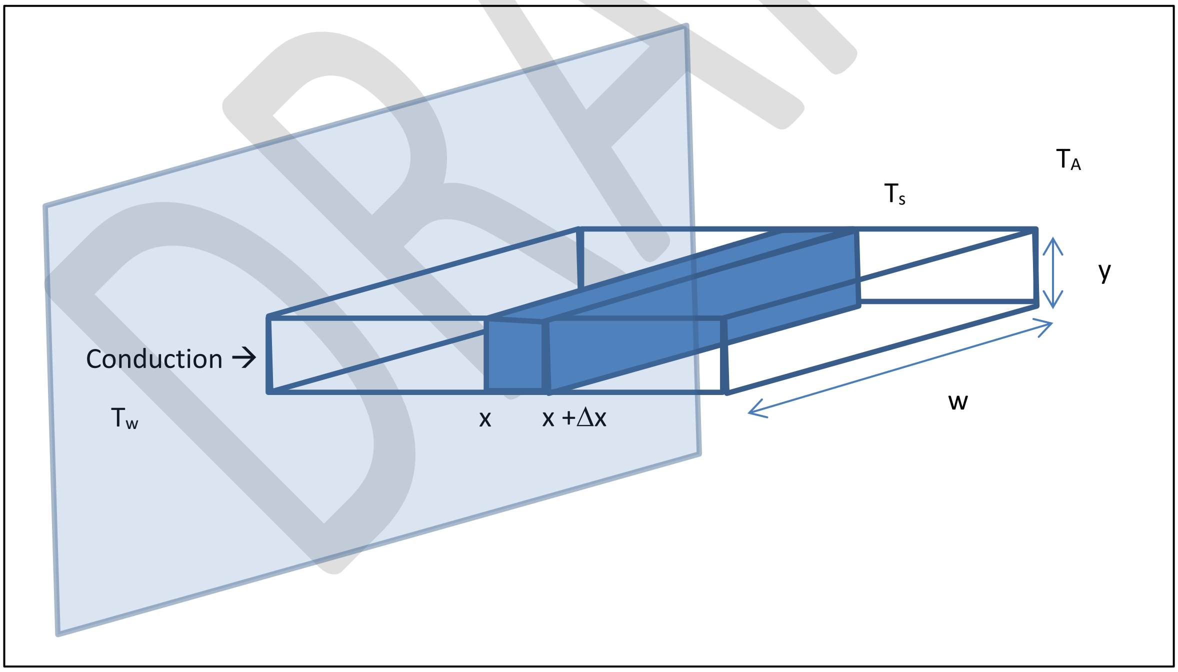


Figure 1.1: Graphical representation of a rectangular fin protruding from a wall

Simplifying assumption: Assume thin slice of fin that has a temperature profile in *x*-direction only, NOT in *y*-direction.

By an energy balance:

Energy in (left) = energy out (right) + energy out (top + bottom) **1.107**

$$\dot{q}|_x = \dot{q}|_{x+\Delta x} + h(2w\Delta x)(T_s - T_A) \qquad \textbf{1.108}$$

(Multiplied by 2 because there are 2 surfaces)

where:

- $\dot{q}$: Conductive heat transfer
- h: convective heat transfer coefficient
- Δx: element through which heat flows
- T_s: surface temperature
- T_A: Ambient/air temperature

Replacing $\dot{q}$ terms with conductive heat flow terms:

$$-kA\frac{dT}{dx}\Big|_x = -kA\frac{dT}{dx}\Big|_{x+\Delta x} + h(2w\Delta x)(T_s - T_A) \qquad \textbf{1.109}$$

$$-k(w.y)\frac{dT}{dx}\Big|_x = -k(w.y)\frac{dT}{dx}\Big|_{x+\Delta x} + h(2w\Delta x)(T_s - T_A) \qquad \textbf{1.110}$$

Rearrange and divide by Δx

$$\frac{-k(w.y)\frac{dT}{dx}\Big|_{x+\Delta x} + k(w.y)\frac{dT}{dx}\Big|_x}{\Delta x} = -h(2w)(T_s - T_A) \qquad \textbf{1.111}$$

Taking limits as $\Delta x \rightarrow 0$

$$\lim_{x\to 0}\frac{-k(w.y)\frac{dT}{dx}\Big|_{x+\Delta x} + k(w.y)\frac{dT}{dx}\Big|_x}{\Delta x} = -h(2w)(T_s - T_A) \qquad \textbf{1.112}$$

$$\frac{d}{dx}\left[-k(w.y)\frac{dT}{dx}\right] = -h(2w)(T_s - T_A) \qquad \textbf{1.113}$$

Assuming k is constant:

$$-k(w.y)\frac{d^2T}{dx^2} = -h(2w)(T_s - T_A) \qquad \textbf{1.114}$$

Simplifying:

$$ky\frac{d^2T}{dx^2} = 2h(T_s - T_A) \qquad \textbf{1.115}$$

And since there is no temperature profile in the y-direction, T_s = T at all x
Therefore T_s = T

And:

$$ky\frac{d^2T}{dx^2} = 2h(T - T_A) \qquad \textbf{1.116}$$

$$\frac{d^2T}{dx^2} = \frac{2h}{ky}(T - T_A) \qquad \textbf{1.117}$$

Let:

$$\theta = T - T_A \qquad \textbf{1.118}$$

AND

$$m = \sqrt{2h/ky} \qquad \textbf{1.119}$$

$$\frac{d^2\theta}{dx^2} = m^2\theta \qquad \textbf{1.120}$$

$$D^2 - m^2\theta = 0 \qquad \textbf{1.121}$$

$$(D - m)(D + m)\theta = 0 \qquad \textbf{1.122}$$

$$\theta = C_1e^{mx} + C_2e^{-mx} \qquad \textbf{1.123}$$

We have two constants, therefore need two boundary conditions:

BC1: At x = 0, $\theta = \theta_1$ — Known temp at wall

BC2: At x = L, $d\theta/dx = 0$ — Temp has stationary point at x = L

<u>From BC1:</u>

$\theta_1 = C_1 + C_2$ **[1]**

<u>From BC2:</u>

$$\frac{d\theta}{dx} = C_1me^{mx} - C_2me^{-mx}$$

$0 = C_1me^{mL} - C_2me^{-mL}$

$C_2 = C_1e^{2mL}$ **[2]**

$$C_1 = \frac{\theta_1}{1 + e^{2ml}}$$

$$C_2 = \frac{\theta_1 . e^{2ml}}{1 + e^{2ml}}$$

$$\theta = \frac{\theta_1 . e^{mx}}{1 + e^{2ml}} + \frac{\theta_1 . e^{2ml} . e^{-mx}}{1 + e^{2ml}}$$

$$\theta = \frac{\theta_1}{1 + e^{2ml}}\left[e^{mx} + e^{2ml} . e^{-mx}\right] = \theta_1 \frac{e^{ml}\left(e^{-ml} . e^{mx} + e^{ml} . e^{-mx}\right)}{e^{ml}\left(e^{-ml} + e^{ml}\right)}$$

$$\frac{\theta}{\theta_1} = \frac{e^{-ml} . e^{mx} + e^{ml} . e^{-mx}}{e^{-ml} + e^{ml}}$$

$$\frac{\theta}{\theta_1} = \frac{\cosh m.(L - x)}{\cosh mL}$$

Re-introducing the temperature and h, k and y terms:

Reminder:

$\theta = T - T_A$

$m = \sqrt{2h/ky}$

$$\frac{T - T_A}{T_W - T_A} = \frac{\cosh\sqrt{2h/ky}.(L - x)}{\cosh\sqrt{2h/ky}. L}$$

where:

Tw = Wall temperature

$$\therefore T = (T_w - T_A)\left(\frac{\cosh\sqrt{2h/ky}.(L-x)}{\cosh\sqrt{2h/ky}.L}\right) + T_A$$

1.7 Heat Exchangers

Introduction

In order to transfer heat efficiently, both for energy and cost savings, industrial processes require heat transfer equipment. Hot fluids may also need to be cooled before being discarded to the environment – either for environmental and/or personal safety reasons.

Various heat transfer equipment exists. These include:

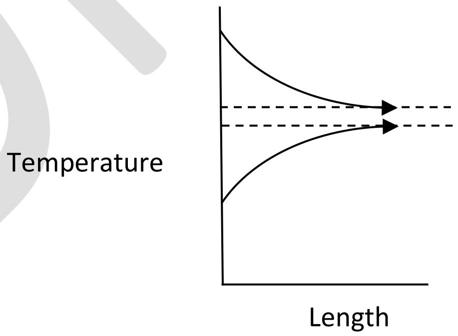
- Heat Exchanger
- Fin Fan Cooler
- Cooling Towers
- Plate Heat Exchangers
- Radiators
- OTHERS.....

A heat exchanger is a piece of equipment built for efficient heat transfer from one medium to another.

A shell and tube heat exchanger is a class of heat exchanger most commonly used in oil refineries and other large chemical processes. It consists of a shell with a bundle of tubes inside. One fluid runs through the tubes, and another fluid flows over the tubes (through the shell) to transfer heat between the two fluids.

Co-current Heat Exchangers

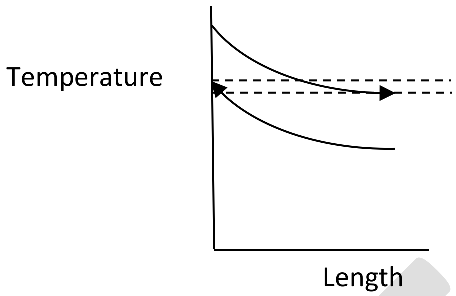
Two streams of fluid enter at the same end of the heat exchanger – one hot and one cold. The two fluids exchange energy with the hotter getting colder and the colder getting hotter. The temperature of the cold stream can never be greater than the hot stream and vice versa.



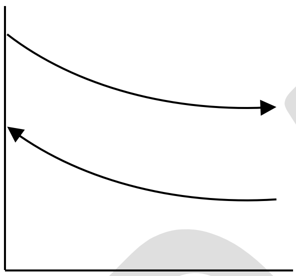
Counter Current Heat Exchangers

Two streams of fluid enter on opposite ends heat exchanger – one hot and one cold. The two fluids exchange energy with the hotter getting colder and the colder getting hotter. The temperature of the cold stream exiting can be hotter than the temperature of the hot stream exiting.

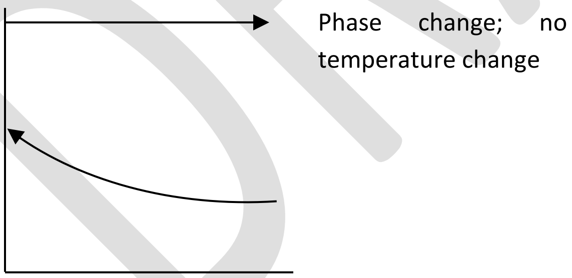
Typically, the maximum temperature difference at any point can be no lower than 10°C. Note in the counter current setup that the outlet temperature difference can be less than this.



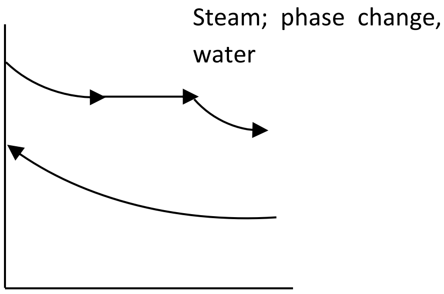
If there is a phase change, deal with each phase separately
The ΔT_{LM} calculation can only be done on a fluid which does not have a phase change OR if there is a phase change ONLY.



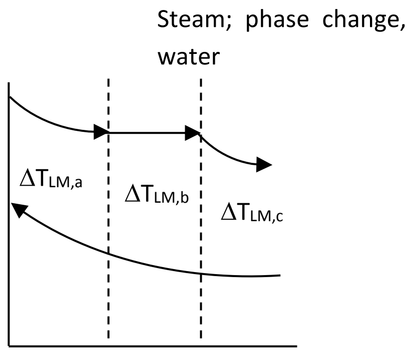
Can use ΔT_{LM} calculation



Can use ΔT_{LM} calculation



CAN NOT use ΔT_{LM} calculation



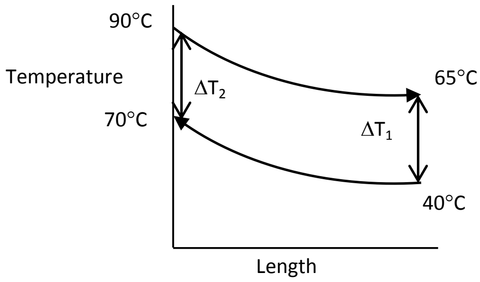
BUT: If we break the problem into 3, can use ΔT_{LM} calculation

<u>Example:</u>

Calculate the log mean temperature given the following information:

Temperature hot in:	*90 ℃*
Temperature hot out:	*65 ℃*
Temperature cold in:	*40 ℃*
Temperature cold out:	*70 ℃*

<u>Solution:</u>



ΔT_1 = 25 ℃; $T_{hot,in} - T_{cold,in}$ = 65 – 40 ℃

ΔT_2 = 20 ℃; $T_{hot,out} - T_{cold,out}$ = 90 – 70 ℃

$$\Delta T_{LM} = \left(\frac{\Delta T_1 - \Delta T_2}{ln\frac{\Delta T_1}{\Delta T_2}}\right) = \left(\frac{20 - 25}{ln\left(\frac{20}{25}\right)}\right) = 22.4°\text{C}$$

This is not the same as average temperature:

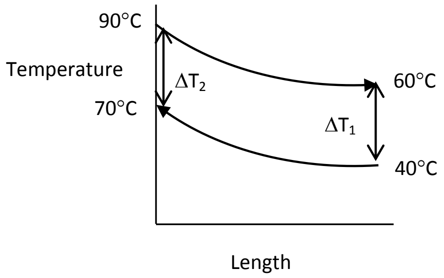
$$\Delta T_{AVE} = \left(\frac{20 + 25}{2}\right) = 22.5°\text{C}$$

Example:

Calculate the log mean temperature given the following information:

Temperature hot in:	*90 ℃*	*Temperature cold in:*	*40 ℃*
Temperature hot out:	*60 ℃*	*Temperature cold out:*	*70 ℃*

Solution:



ΔT_1 = 20 ℃; $T_{hot,out} - T_{cold,in}$ = 60 – 40 ℃
ΔT_2 = 20 ℃; $T_{hot,out} - T_{cold,out}$ = 90 – 70 ℃

$$\Delta T_{LM} = \left(\frac{\Delta T_1 - \Delta T_2}{ln\frac{\Delta T_1}{\Delta T_2}}\right) = \left(\frac{20-20}{ln\left(\frac{20}{20}\right)}\right) = undefined$$

Since ΔT_{LM} is undefined, can use ΔT_{AVE}. Also: since ΔT is constant across the length of heat exchanger (and also equal to ΔT_{AVE}), ΔT_{AVE} can be used.

$$\Delta T_{AVE} = \left(\frac{20+20}{2}\right) = 20℃$$

1.8 Heat Generation

Heat generation in terms of heat transfer:

Energy can be generated in a solid element by:

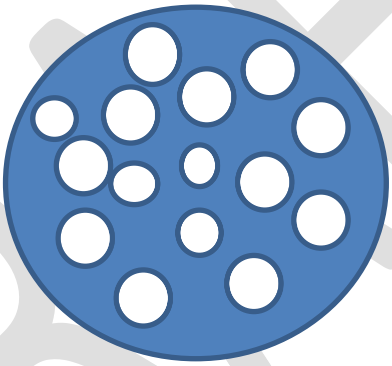
- Nuclear reaction;
- Catalyst and exothermic reactions; and
- Electrical current

Nuclear Reactions (Bird, Stewart and Lightfoot, p296)

Given in terms of G (per volume of radioactive material)

Catalysts (Bird, Stewart and Lightfoot, p300)

Metals are expensive. In order to maximize the surface area to volume ratio of a catalyst, we typically use metal oxide supports which have a large surface area, *e.g.* activated carbon. Reactants adsorb an react in the pores; products desorb and are released.



Electric Current (Bird, Stewart and Lightfoot, p292)

<u>Example:</u>

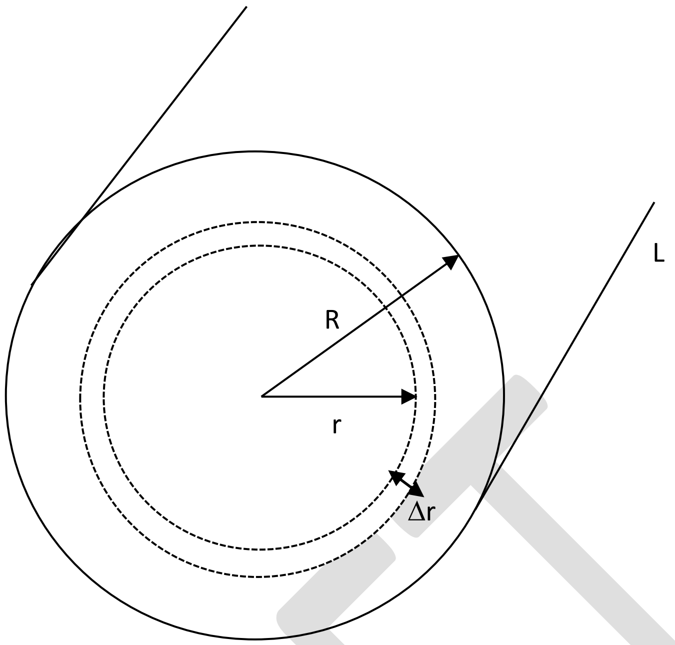
Consider an electric wire of circular cross section with radius R and electrical conductivity k_e $\text{ohm}^{-1}\text{cm}^{-1}$. Through this wire an electric current is passed with a current density of I amp/cm^2. This process is an irreversible process, converting some electrical energy into heat (thermal energy). The rate of heat production per unit volume is given by the expression:

$$S_e = \frac{I^2}{k_e}$$

The quantity S_e is the heat source resulting from electrical dissipation. It is assumed that the temperature rise in the wire is not so large that the temperature dependence of either the electrical or thermal conductivity need be considered. The surface of the wire is maintained at temperature T_0.

Find the radial temperature profile within the wire.

Solution:



For the system we take a cylindrical shell of thickness Δr and length L.

The contributors to the energy balance across this shell are:

Rate of heat across the cylinder at r	$(2\pi rL)(q_r\|_r) = (2\pi rLq_r)\,\|_r$
Rate of heat out across cylindrical surface at r + Δr	$(2\pi(r+\Delta r)L)(q_r\|_{r+\Delta r}) = (2\pi rLq_r)\,\|_{r+\Delta r}$
Rate of thermal production by electrical dissipation	$(2\pi r\Delta rL)S_e$

By an energy balance: Energy in + Energy generated = Energy out

$$(2\pi rLq_r)\,|_r + (2\pi r\Delta rL)S_e = (2\pi rLq_r)\,|_{r+\Delta r}$$

Re-arranging

$$(2\pi rLq_r)\,|_{r+\Delta r} - (2\pi rLq_r)\,|_r = (2\pi r\Delta rL)S_e$$

÷ $2\pi L\Delta r$

$$\frac{(rq_r)\,|_{r+\Delta r} - (rq_r)\,|_r}{\Delta r} = S_e r$$

Taking limits as $\Delta r \rightarrow 0$

$$\lim_{\Delta r\to 0}\frac{(rq_r)\,|_{r+\Delta r} - (rq_r)\,|_r}{\Delta r} = S_e r$$

$$\frac{d}{dr}(rq_r) = S_e r$$

This is a first order DE and can be integrated to:

$$q_r = \frac{S_e r}{2} + \frac{C_1}{r}$$

The integration constant C_1 must be zero, since at r = 0, q_r is not infinite

$$q_r = \frac{S_e r}{2}$$

Now, from Fourier's Law for heat transfer:

$$q_r = -k.(dT/dr)$$

Equating:

$$-k.\left(\frac{dT}{dr}\right) = \frac{S_e r}{2}$$

Assuming k is constant, integrating:

$$T = \frac{S_e r^2}{4k} + C_2$$

At r = R; T = T_0

$$C_2 = \frac{S_e R^2}{4k} + T_0$$

$$T - T_0 = \frac{S_e R^2}{4k}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Maximum temperature is at r = 0

$$T_{max} - T_0 = \frac{S_e R^2}{4k}$$

$$T_{max} = \frac{S_e R^2}{4k} + T_0$$

Average temperature:

$$\langle T \rangle - T_0 = \frac{\int_0^{2\pi}\int_0^{R}(T(r) - T_0) r dr. d\theta}{\int_0^{2\pi}\int_0^{R} r dr. d\theta} = \frac{S_e R^2}{8k}$$

1.9 Radiative Heat Transfer

Black Body

A perfect idealized physical body which absorbs all incident electromagnetic radiation and is also the best possible emitter of thermal radiation.

$$\dot{q} = A\sigma T^4 \qquad \text{Stefan Boltzmann Law} \qquad \textbf{1.124}$$

where:

- $\dot{q}$: Heat Flux (W)
- A: Cross Sectional Area (m^2)
- σ: Stefan Boltzmann Constant = 5.67×10^{-8} $W.m^{-2}K^{-4}$

  $= \dfrac{2\pi^5 k_B^4}{15h^3c^2}$

  - k_B = Boltzmann constant – 1.38×10^{-23} J/K;
  - h = Plank constant – 6.626×10^{-34} J.s
  - c = speed of light m/s
- T = Temperature (K)

> *Example:*
>
> *What is the radiative heat transfer per square meter for a body at the following temperatures?*
>
> *a) 100°C*
> *b) 400°C*
> *c) 800°C*
>
> *Solution:*
>
> $$\dot{q}/A = A\sigma T^4$$
> $$\dot{q}/A = \sigma T^4$$
> $$\dot{q}/A = 5.76 \times 10^{-8}.T^4$$
>
> *a) $\dot{q}/A$ = 1.12 kW/m²*
> *b) $\dot{q}/A$ = 11.8 kW/m²*
> *c) $\dot{q}/A$ = 76.4 kW/m²*

Given that the Stefan Boltzmann constant is very small, radiative heat transfer only plays a part at high temperatures (approx. greater than 400°C), while conduction and convection describe heat transfer at lower temperatures.

Additional Reading

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