CHAPTER 3

The Simplex Method and Sensitivity Analysis

Set 3.1a

1

$(x_1, x_2) = (3, 1)$

$M1: S_1 = 24 - (6\times 3 + 4\times 1) = 2$ tons/day

$M2: S_2 = 6 - (1\times 3 + 2\times 1) = 1$ ton/day

2

$S_1 = x_1 + x_2 - 800$
$= 500 + 600 - 800 = 300$ lb

3

$10x_1 - 3x_2 \geq -5 \equiv -10x_1 + 3x_2 \leq 5$

Thus, $-10x_1 + 3x_2 + S_1 = 5$ ①

Also, $10x_1 - 3x_2 \geq -5 \equiv 10x_1 - 3x_2 - S_2 = -5$

Thus, $-10x_1 + 3x_2 + S_2 = 5$ ②

① and ② are the same

4

x_{ij} = number of units of product i manufactured on machine j

LP model

Maximize $z = 10(x_{11} + x_{12}) + 15(x_{21} + x_{22})$

subject to

$$|(x_{11} + x_{21}) - (x_{12} + x_{22})| \leq 5$$
$$x_{11} + x_{21} \leq 200$$
$$x_{12} + x_{22} \leq 250$$
$$x_{ij} \geq 0 \text{ for all } i \,\&\, j$$

Equation form:

$$|(x_{11} + x_{21}) - (x_{12} + x_{22})| \leq 5$$

to

$$x_{11} + x_{21} - x_{12} - x_{22} \leq 5$$
$$x_{11} + x_{21} - x_{12} - x_{22} \geq -5$$

Maximize $z = 10x_{11} + 10x_{12} + 15x_{21} + 15x_{22}$

Subject to

$$\begin{aligned}
x_{11} + x_{21} - x_{12} - x_{22} + S_1 &= 5\\
-x_{11} - x_{21} + x_{12} + x_{22} + S_2 &= 5\\
x_{11} + x_{21} + S_3 &= 200\\
x_{12} + x_{22} + S_4 &= 250
\end{aligned}$$

$x_{ij} \geq 0$ for all i and j
$S_i \geq 0$ for all i

continued...

5

$y = \max\{|x_1 - x_2 + 3x_3|, |-x_1 + 3x_2 - x_3|\}$

Hence

$$|x_1 - x_2 + 3x_3| \leq y$$
$$|-x_1 + 3x_2 - x_3| \leq y$$

LP model:

minimize $z = y$

Subject to

$$\begin{aligned}
x_1 - x_2 + 3x_3 &\leq y\\
x_1 - x_2 + 3x_3 &\geq -y\\
-x_1 + 3x_2 - x_3 &\leq y\\
-x_1 + 3x_2 - x_3 &\geq -y
\end{aligned}$$

$$x_1, x_2, x_3, y \geq 0$$

Equation form:

Minimize $z = y$

Subject to

$$\begin{aligned}
-y + x_1 - x_2 + 3x_3 + S_1 &= 0\\
-y - x_1 + x_2 - 3x_3 + S_2 &= 0\\
-y - x_1 + 3x_2 - x_3 + S_3 &= 0\\
-y + x_1 - 3x_2 + x_3 + S_4 &= 0
\end{aligned}$$

$$x_1, x_2, x_3, y, S_1, S_2, S_3, S_4 \geq 0$$

6

$$\sum_{j=1}^{n} a_{ij}x_j = b_i \Leftrightarrow \begin{cases} \sum_{j=1}^{n} a_{ij}x_j \leq b_i & ① \\ \sum_{j=1}^{n} a_{ij}x_j \geq b_i & ② \end{cases}$$

From ②, for $i = 1, 2, \ldots, m$, we have

$$\sum_{j=1}^{n} a_{ij}x_j \geq b_i \Leftrightarrow \sum_{i=1}^{m}\left(\sum_{j=1}^{n} a_{ij}x_j\right) \geq \sum_{i=1}^{m} b_i$$
$$\Leftrightarrow \sum_{j=1}^{n}\left(\sum_{i=1}^{m} a_{ij}\right)x_j \geq \sum_{i=1}^{m} b_i$$

Thus, ① and ② are equivalent to

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i, \quad i = 1, 2, \ldots, m$$
$$\sum_{j=1}^{n}\left(\sum_{i=1}^{m} a_{ij}\right)x_j \geq \sum_{i=1}^{m} b_i$$

1

x_1 = Nbr. $\frac{1}{4}$-lb / day

x_2 = Nbr. cheeseburgers/day

Maximize $z = .2x_1 + .15x_2 - .25x_3^+$

s.t.

$.25x_1 + .2x_2 + x_3^- - x_3^+ = 200$

$x_1 + x_2 \le 900$

Solution: $z = \$173.35$

$x_1 = 900,\ x_2 = 0,\ x_3^+ = 25$ lb

2

(a) x_j = # units of product j per day, $j = 1, 2$

x_3^+ = unused minutes of machine time / day

x_3^- = machine overtime per day in minutes

Maximize $z = 6x_1 + 7.5x_2 - .5x_3^-$

Subject to

$10x_1 + 12x_2 + x_3^+ - x_3^- = 2500$

$150 \le x_1 \le 200$

$x_2 \le 45$

$x_1, x_2 \ge 0$

$x_3^+, x_3^- \ge 0$

TORA optimum solution:

x_1 = 200 units/day

x_2 = 45 units/day

x_3^- = overtime minutes = 40 minutes/day

$z = \$1517.50$

(b) Overtime at \$1.50/min yields $x_3^- = 0$, which means no overtime is needed

3

x_j = # of units of products 1, 2, and 3

Maximize $z = 2x_1 + 5x_2 + 3x_3 - 15x_4^+ - 10x_5^+$

Subject to

$2x_1 + x_2 + 2x_3 + x_4^- - x_4^+ = 80$

$x_1 + x_2 + 2x_3 + x_5^- - x_5^+ = 65$

all variables ≥ 0

Solution: $z = \$325$

$x_2 = 65$ units, $x_4^- = 15$

All other variables = 0

continued...

4

Formulation 1:

Maximize $z = -2x_1 + 3x_2^+ - 3x_2^- - 2x_3^+ + 2x_3^-$

Subject to

$4x_1 - x_2^+ + x_2^- - 5x_3^+ + 5x_3^- = 10$

$2x_1 + 3x_2^+ - 3x_2^- + 2x_3^+ - 2x_3^- = 12$

all variables ≥ 0

Optimum solution:

$x_1 = 0$

$\left.\begin{array}{l} x_2^+ = 6.15 \\ x_2^- = 0 \end{array}\right\} \Rightarrow x_2 = 6.15$

$\left.\begin{array}{l} x_3^+ = 0 \\ x_3^- = 3.23 \end{array}\right\} \Rightarrow x_3 = -3.23$

$z = 24.92$

Formulation 2:

Maximize $z = -2x_1 + 3x_2^+ - 2x_3^+ - w$

Subject to

$4x_1 - x_2^+ - 5x_3^+ + 6w = 10$

$2x_1 + 3x_2^+ + 2x_3^+ - 5w = 12$

all variables ≥ 0

Optimum solution:

$x_1 = 0$

$\left.\begin{array}{l} x_2^+ = 9.38 \\ w = 3.23 \end{array}\right\} \Rightarrow x_2 = 9.38 - 3.23 = 6.15$

$\left.\begin{array}{l} x_3^+ = 0 \\ w = 3.23 \end{array}\right\} \Rightarrow x_3 = 0 - 3.23 = -3.23$

$z = 24.92$

continued...

Set 3.2a

1

(a)

Equation form:

Maximize $z = 2x_1 + 3x_2$

subject to

$$x_1 + 3x_2 + x_3 = 6$$
$$3x_1 + 2x_2 + x_4 = 6$$
$$x_1, x_2, x_3, x_4 \geq 0$$

(b) Basic (x_1, x_2) (Point B):

$$x_1 + 3x_2 = 6$$
$$3x_1 + 2x_2 = 6$$

Solution: $(x_1, x_2) = (\frac{6}{7}, \frac{12}{7})$, $z = 6\frac{6}{7}$

Basic (x_1, x_3) (Point E):

$$x_1 + x_3 = 6$$
$$3x_1 = 6$$

Solution: $(x_1, x_3) = (2, 4)$, $z = 4$

Basic (x_1, x_4) (Point C):

$$x_1 = 6$$
$$3x_1 + x_4 = 6$$

Solution: $(x_1, x_4) = (6, -12)$
Unique but infeasible

Basic (x_2, x_3) (Point A):

$$3x_2 + x_3 = 6$$
$$2x_2 = 6$$

Solution: $(x_2, x_3) = (3, -3)$
Unique but infeasible

Basic (x_2, x_4) (Point D):

$$3x_2 = 6$$
$$2x_2 + x_4 = 6$$

Solution: $(x_2, x_4) = (2, 2)$, $z = 6$

Basic (x_3, x_4) (Point F):

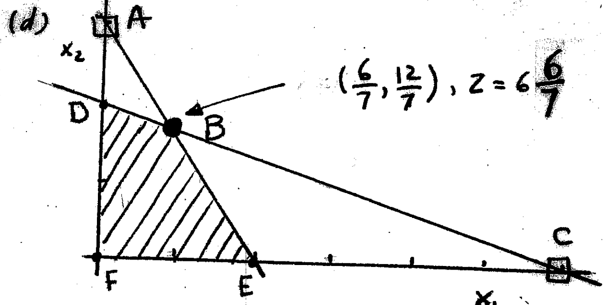
$$x_3 = 6$$
$$x_4 = 6$$

Solution: $(x_3, x_4) = (6, 6)$, $z = 0$

(c) Optimum solution occurs at B:
$(x_1, x_2) = (\frac{6}{7}, \frac{12}{7})$ with $z = 6\frac{6}{7}$

continued...

(d)



(e) From the graph in (d), we have

A: $x_2 = 3$, $x_3 = -3$

C: $x_1 = 6$, $x_4 = -12$

2

(a) Maximize $z = 2x_1 - 4x_2 + 5x_3 - 6x_4$

Subject to

$$x_1 + 4x_2 - 2x_3 + 8x_4 + x_5 = 2$$
$$-x_1 + 2x_2 + 3x_3 + 4x_4 + x_6 = 1$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Combination	Solution	Status	z
x_1, x_2	0, 1/2	Feasible	-2
x_1, x_3	8, 3	Feasible	$\boxed{31}$
x_1, x_4	0, 1/4	Feasible	-3/2
x_1, x_5	-1, 3	Infeasible	—
x_1, x_6	2, 3	Feasible	4
x_2, x_3	1/2, 0	Feasible	-2
x_2, x_4	1/2, 0	Feasible	-2
x_2, x_5	1/2, 0	Feasible	-2
x_2, x_6	1/2, 0	Feasible	-2
x_3, x_4	0, 1/4	Feasible	-3/2
x_3, x_5	1/3, 8/3	Feasible	5/3
x_3, x_6	-1, 4	Infeasible	—
x_4, x_5	1/4, 0	Feasible	-3/2
x_4, x_6	1/4, 0	Feasible	-3/2
x_5, x_6	2, 1	Feasible	0

Optimum Solution:

$x_1 = 8$, $x_2 = 0$, $x_3 = 3$, $x_4 = 0$

$z = 31$

continued...

(b) Minimize $z = x_1 + 2x_2 - 3x_3 - 2x_4$

subject to

$$x_1 + 2x_2 - 3x_3 + x_4 = 4$$
$$x_1 + 2x_2 + x_3 + 2x_4 = 4$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Combination	Solution	Status	Z
x_1, x_2	infinity of solutions		—
x_1, x_3	4, 0	Feasible	4
x_1, x_4	4, 0	Feasible	4
x_2, x_3	2, 0	Feasible	4
x_2, x_4	2, 0	Feasible	4
x_3, x_4	$-\frac{4}{7}, \frac{16}{7}$	Infeasible	—

Alternative optima:

x_1	x_2	x_3	x_4	z
4	0	0	0	4
0	2	0	0	4

3

maximize $z = x_1 + x_2$

subject to

$$x_1 + 2x_2 + x_3 = 6$$
$$2x_1 + x_2 - x_4 = 16$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Combination	Solution	Status
x_1, x_2	26/3, −4/3	Infeasible
x_1, x_3	8, −2	Infeasible
x_1, x_4	6, −4	Infeasible
x_2, x_3	16, −26	Infeasible
x_2, x_4	3, −13	Infeasible
x_3, x_4	6, −16	Infeasible

continued...

4

Maximize $z = 2x_1 + 3x_2^- - 3x_2^+ + 5x_3$

Subject to

$$-6x_1 + 7x_2^- - 7x_2^+ - 9x_3 - x_4 = 4$$
$$x_1 + x_2^- - x_2^+ + 4x_3 = 10$$
$$x_1, x_2^-, x_2^+, x_3, x_4 \geq 0$$

(x_2, x_3):

$$7x_2^- - 7x_2^+ = 4$$
$$x_2^- - x_2^+ = 10$$

Since $(7x_2^- - 7x_2^+)$ and $(x_2^- - x_2^+)$ are dependent, it is impossible for x_2^- and x_2^+ to be basic simultaneously. This means that at least x_2^- and x_2^+ must be nonbasic at zero level; thus making it impossible for x_2^- and x_2^+ to assume positive values simultaneously in any basic solution.

5

maximize $z = x_1 + 3x_2$

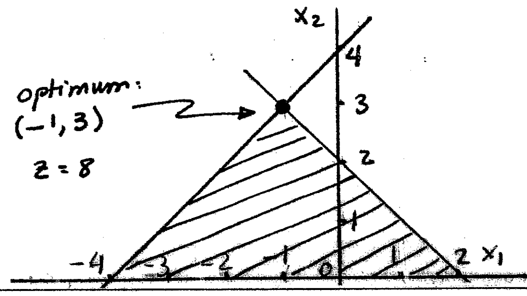
subject to

$$x_1 + x_2 + x_3 = 2$$
$$-x_1 + x_2 + x_4 = 4$$
$$x_1 \text{ unrestricted}$$
$$x_2, x_3 \geq 0$$

Combination	Solution	Status	Z
x_1, x_2	−1, 3	Feasible	$\boxed{8}$
x_1, x_3	−4, 6	Feasible	−4
x_1, x_4	2, 6	Feasible	2
x_2, x_3	4, −2	Infeasible	—
x_2, x_4	2, 2	Feasible	6
x_3, x_4	2, 4	Feasible	0

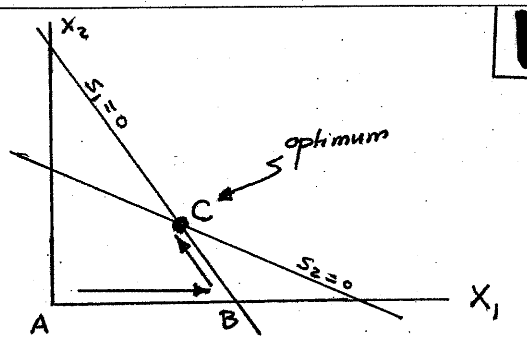
Optimum: $x_1 = -1$, $x_2 = 3$, $z = 8$

(c)



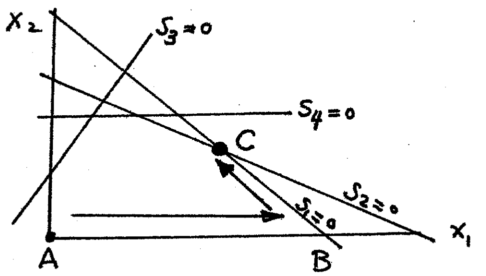
Set 3.3a

1



Extreme point	Basic	Nonbasic
A	S_1, S_2	x_1, x_2
B	x_1, S_2	x_2, S_1
C	x_1, x_2	S_1, S_2

2



Extreme point	Basic	Nonbasic
A	S_1, S_2, S_3, S_4	x_1, x_2
B	x_1, S_2, S_3, S_4	S_1, x_2
C	x_1, x_2, S_3, S_4	S_1, S_2

3

(a) (A, B) adjacent, hence can be on a simplex path. Remaining pairs cannot be on a simplex path because they are not adjacent.

(b) (i) Yes, because connects adjacent extreme points

(ii) No, because C and I are not adjacent.

(iii) No, because the path returns to a previous extreme point.

4

Extreme Point	Basic	Nonbasic
A	S_1, S_2, S_3, S_4	x_1, x_2, x_3
B	S_1, x_1, S_3, S_4	S_2, x_2, S_3
C	x_2, S_2, S_3, S_4	S_1, x_1, x_3
D	S_1, S_2, x_3, S_4	x_1, x_2, S_3
E	x_1, x_2, S_3, S_4	S_1, S_2, x_3
F	x_2, S_2, x_3, S_4	x_1, S_1, S_3
G	S_1, x_1, x_3, S_4	S_2, x_2, S_3
H	S_1, x_1, x_2, x_3	S_2, S_3, S_4
I	x_1, x_2, x_3, S_3	S_1, S_2, S_4
J	x_1, S_2, x_2, x_3	S_1, S_3, S_4

5

(a) x_3 enters at value 1

$z = 0 + 3 \times 1 = 3$

(b) x_1 enters at value 1

$z = 0 + 5 \times 1 = 5$

(c) x_2 enters at value 1

$z = 0 + 7 \times 1 = 7$

(d) Tie broken arbitrarily between x_1, x_2, and x_3. Entering value = 1

$z = 0 + 1 \times 1 = 1$

1

Basic	z	x_1	x_2	s_1	s_2	s_3	s_4	Sol
z	1	-5	-4	0	0	0	0	0
s_1	0	6	4	1	0	0	0	24
s_2	0	1	2	0	1	0	0	6
s_3	0	-1	1	0	0	1	0	1
s_4	0	0	1	0	0	0	1	2
z	1	0	6	0	5	0	0	30
s_1	0	0	-8	1	-6	0	0	-12
x_1	0	1	2	0	1	0	0	6
s_3	0	0	3	0	1	1	0	7
s_4	0	0	1	0	0	0	1	2

2

(a)

Basic	x1	x2	x3	x4	sx5	sx6	sx7	Solution
z	-2.00	-1.00	3.00	-5.00	0.00	0.00	0.00	0.00
1)sx5	1.00	2.00	2.00	4.00	1.00	0.00	0.00	40.00
2)sx6	2.00	-1.00	1.00	2.00	0.00	1.00	0.00	8.00
3)sx7	4.00	-2.00	1.00	-1.00	0.00	0.00	1.00	10.00
z	3.00	-3.50	5.50	0.00	0.00	2.50	0.00	20.00
1)sx5	-3.00	4.00	0.00	0.00	1.00	-2.00	0.00	24.00
2)x4	1.00	-0.50	0.50	1.00	0.00	0.50	0.00	4.00
3)sx7	5.00	-2.50	1.50	0.00	0.00	0.50	1.00	14.00
z	0.38	0.00	5.50	0.00	0.88	0.75	0.00	41.00
1)x2	-0.75	1.00	0.00	0.00	0.25	-0.50	0.00	6.00
2)x4	0.62	0.00	0.50	1.00	0.12	0.25	0.00	7.00
3)sx7	3.12	0.00	1.50	0.00	0.62	-0.75	1.00	29.00

(b)

Basic	x1	x2	x3	x4	sx5	sx6	sx7	Solution
z	-8.00	-6.00	-3.00	2.00	0.00	0.00	0.00	0.00
1)sx5	1.00	2.00	2.00	4.00	1.00	0.00	0.00	40.00
2)sx6	2.00	-1.00	1.00	2.00	0.00	1.00	0.00	8.00
3)sx7	4.00	-2.00	1.00	-1.00	0.00	0.00	1.00	10.00
z	0.00	-10.00	-1.00	0.00	0.00	0.00	2.00	20.00
1)sx5	0.00	2.50	1.75	4.25	1.00	0.00	-0.25	37.00
2)sx6	0.00	0.00	0.50	2.50	0.00	1.00	-0.50	3.00
3)x1	1.00	-0.50	0.25	-0.25	0.00	0.00	0.25	2.50
z	0.00	0.00	6.00	17.00	4.00	0.00	1.00	170.00
1)x2	0.00	1.00	0.70	1.70	0.40	0.00	-0.10	15.00
2)sx6	0.00	0.00	0.50	2.50	0.00	1.00	-0.50	3.00
3)x1	1.00	0.00	0.60	0.60	0.20	0.00	0.20	10.00

(c)

Basic	x1	x2	x3	x4	sx5	sx6	sx7	Solution
z	-3.00	1.00	-3.00	-4.00	0.00	0.00	0.00	0.00
1)sx5	1.00	2.00	2.00	4.00	1.00	0.00	0.00	40.00
2)sx6	2.00	-1.00	1.00	2.00	0.00	1.00	0.00	8.00
3)sx7	4.00	-2.00	1.00	-1.00	0.00	0.00	1.00	10.00
z	1.00	-1.00	-1.00	0.00	0.00	2.00	0.00	16.00
1)sx5	-3.00	4.00	0.00	0.00	1.00	-2.00	0.00	24.00
2)x4	1.00	-0.50	0.50	1.00	0.00	0.50	0.00	4.00
3)sx7	5.00	-2.50	1.50	0.00	0.00	0.50	1.00	14.00
z	0.25	0.00	-1.00	0.00	0.25	1.50	0.00	22.00
1)x2	-0.75	1.00	0.00	0.00	0.25	-0.50	0.00	6.00
2)x4	0.62	0.00	0.50	1.00	0.12	0.25	0.00	7.00
3)sx7	3.12	0.00	1.50	0.00	0.62	-0.75	1.00	29.00
z	1.50	0.00	0.00	2.00	0.50	2.00	0.00	36.00
1)x2	-0.75	1.00	0.00	0.00	0.25	-0.50	0.00	6.00
2)x3	1.25	0.00	1.00	2.00	0.25	0.50	0.00	14.00
3)sx7	1.25	0.00	0.00	-3.00	0.25	-1.50	1.00	8.00

continued...

(d)

Basic	x1	x2	x3	x4	sx5	sx6	sx7	Solution
z	-5.00	4.00	-6.00	8.00	0.00	0.00	0.00	0.00
1)sx5	1.00	2.00	2.00	4.00	1.00	0.00	0.00	40.00
2)sx6	2.00	-1.00	1.00	2.00	0.00	1.00	0.00	8.00
3)sx7	4.00	-2.00	1.00	-1.00	0.00	0.00	1.00	10.00
z	-13.00	8.00	-10.00	0.00	0.00	-4.00	0.00	-32.00
1)sx5	-3.00	4.00	0.00	0.00	1.00	-2.00	0.00	24.00
2)x4	1.00	-0.50	0.50	1.00	0.00	0.50	0.00	4.00
3)sx7	5.00	-2.50	1.50	0.00	0.00	0.50	1.00	14.00
z	-7.00	0.00	-10.00	0.00	-2.00	0.00	0.00	-80.00
1)x2	-0.75	1.00	0.00	0.00	0.25	-0.50	0.00	6.00
2)x4	0.62	0.00	0.50	1.00	0.12	0.25	0.00	7.00
3)sx7	3.12	0.00	1.50	0.00	0.62	-0.75	1.00	29.00

3

	Ratios			
Basic	x_1	x_2	x_3	x_4
x_5	4/1	4/2	--	4/5
x_6	8/5	--	--	8/6
x_7	3/2	3/3	--	3/3
x_8	--	--	0/1	--
Value	1.5	1	0	0.8
Leaving var	x_7	x_7	x_8	x_5

4

(a) Nonbasic x_1 will improve solution.

Basic	x_1-ratios
x_2	4/5 $\Rightarrow$ x_2 leaves, $x_1 = \frac{4}{5}$
x_3	8/6
x_4	3/3

$x_1 = \frac{4}{5} = .8$, $x_3 = 8 - 6 \times .8 = 3.6$, $x_4 = 3 - 3 \times .8 = .6$

$x_2 = 0$, $z = .8 \times 1 = .8$

(b) x_1 remains nonbasic at zero. Current solution, $x_2 = 4$, $x_3 = 8$, $x_4 = 3$, $z = 0$ is optimum

5

Basic solutions consist of one variable each. Thus,

$x_1 = 90/1 = 90$, $z = 5 \times 90 = 450$

$x_2 = 90/3 = 30$, $z = -6 \times 30 = -180$

$x_3 = 90/5 = 18$, $z = 3 \times 18 = 54$

$x_4 = 90/6 = 15$, $z = -5 \times 15 = -75$

$x_5 = 90/3 = 30$, $z = 12 \times 30 = 360$

Optimum solution:

$x_1 = 90$, $x_2 = x_3 = x_4 = x_5 = 0$, $z = 450$

6

(a) Basic: $(x_8, x_3, x_1) = (12, 6, 0)$, $z = 620$

Nonbasic: $(x_2, x_4, x_5, x_6, x_7) = (0,0,0,0,0)$

(b) x_2, x_5, x_6 will improve solution.

x_2 enters: $x_2 = \min\left(\frac{12}{3}, \frac{6}{1}, -\right) = 4$. Thus, x_8 leaves, $\Delta z = 4 \times 5 = 20$

continued...

Set 3.3b

x_5 enters: $x_5 = \min(-, \frac{6}{1}, \frac{0}{6}) = 0$. Thus, $\Delta Z = 1 \times 0 = 0$ (x_1 leaves)

x_6 enters: $x_6 = \min(-, -, -)$. Thus, no leaving variable and x_6 can be increased to ∞. $\Delta Z = +\infty$

(c) x_4 can improve solution.

x_4 enters: $x_4 = \min(-, \frac{6}{3}, -) = 2$. Thus, x_3 leaves. $\Delta Z = -4 \times 2 = -8$

(d) As shown in (b), x_5 cannot change Z because it enters the solution a level zero. x_7 cannot change Z either because its objective equation coefficient $= 0$. $\Delta Z = 0 \times \min(\frac{12}{5}, \frac{6}{3}, -) = 0$

7

(a) Maximize $Z = 3x_1 + 6x_2$:

x_2 is the first entering variable. Resulting path is $A \rightarrow G \rightarrow F \rightarrow E$.

(b) Maximize $Z = 4x_1 + x_2$:

Entering variable $x_1 = \left(\begin{matrix}\text{min intercept with} \\ x_1\text{-axis}\end{matrix}\right)$

$x_1 = \min(2, 3, 5) = 2$ at B

$\Delta Z = 4 \times 2 = 8$

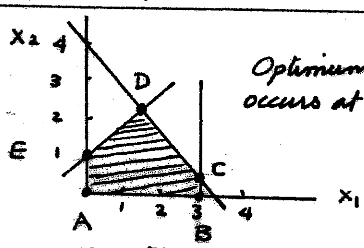
(c) Maximize $Z = x_1 + 4x_2$:

Entering variable $x_2 = \left(\begin{matrix}\text{min intercept} \\ \text{with } x_2\text{-axis}\end{matrix}\right)$

$x_2 = \min(1, 2, 4) = 1$

$\Delta Z = 4 \times 1 = 4$

8



Optimum occurs at D

(a) x_1 will enter first and the iterations will follow the path $A \rightarrow B \rightarrow C \rightarrow D$

(b) x_2 enters first and the iterations will follow the path $A \rightarrow E \rightarrow D$

(c) The most-negative criterion requires more iterations (4 vs. 3). This criterion is only a heuristic, and although it does not guarantee the smallest number of iterations, computational experience demonstrates that, on the average, the most-negative criterion is more efficient.

continued...

(d) Iterations are identical, with the exception of the objective row, which should appear with an opposite sign

9

Optimum tableau:

Basic	x_1	x_2	s_1	s_2	s_3	s_4	
z	0	0	$\frac{3}{4}$	$\frac{1}{2}$	0	0	21
x_1	1	0	¼	-1/2	0	0	3
x_2	0	1	-1/8	¾	0	0	3/2
s_3	0	0	3/8	-5/4	1	0	5/2
s_4	0	0	1/8	-3/4	0	1	1/2

If s_1 enters, its value $= \min\left\{\frac{3}{1/4}, -, \frac{5/2}{3/8}, \frac{1/2}{1/8}\right\} = 4$

New $Z = 21 - 3/4 \times 4 = 18$

If s_2 enters, its value $= \min\left\{-, \frac{3/2}{3/4}, -, -\right\} = 2$

New $Z = 21 - 1/2 \times 2 = 20$. The second best Z is associated with s_2 entering the basic solution

10

Not easily extendable because the third best solution may not be an adjacent corner point of the current optimum point.

11

x_1 = number of purses per day

x_2 = number of bags per day

x_3 = number of backpacks per day

Maximize $Z = 24x_1 + 22x_2 + 45x_3$

subject to

$$
\begin{aligned}
2x_1 + x_2 + 3x_3 &\le 42 \\
2x_1 + x_2 + 2x_3 &\le 40 \\
x_1 + .5x_2 + x_3 &\le 45 \\
x_1, x_2, x_3 &\ge 0
\end{aligned}
$$

TORA's optimum solution:

$x_1 = 0$, $x_2 = 36$, $x_3 = 2$, $Z = \$882$

Status of resources:

Resource	slack	Status
Leather	0	scarce
Sewing	0	scarce
Finishing	25	abundant

12

From TORA Iterations module, click [All Iterations], then go to the optimal iteration and click any of the associated nonbasic variables (X4, SX6, SX7, SX8). Now, click [Next Iteration] to produce the new iteration in which the selected variable becomes basic. The associated value of Z will deteriorate.

13

To determine the next-best solution, follow the procedure in Problem 1. First, let X4 enter the basic solution and record the associated value of Z. Next, click [View/Modify Input Data] and re-solve the problem to produce the same optimum tableau that was used before X4 was entered into the basic solution. Now, enter SX6 into the basic solution and record the associated value of Z. Repeat the procedure for SX7 and SX8. You will get the following results:

Entering variable	Z
X4	2.63
SX6	1.00
SX7	6.40
SX8	1.90

The next-best solution is associated with entering SX7 into the basic solution. Associated values of the variables are

$x_1 = 1.6$

$x_2 = 0$

$x_3 = 1.6$

$x_4 = 0$

$Z = 6.40$

1

Iteration	Basic	x_1	x_2	x_3	R_1	R_2	x_4	Solution
0 (starting)	z	$-4+7M$	$-1+4M$	$-M$	0	0	0	$9M$
x_1 enters	R_1	3	1	0	1	0	0	3
R_1 leaves	R_2	4	3	-1	0	1	0	6
	x_4	1	2	0	0	0	1	4
1	z	0	$\frac{1+5M}{3}$	$-M$	$\frac{4-7M}{3}$	0	0	$4+2M$
x_2 enters	x_1	1	1/3	0	1/3	0	0	1
R_2 leaves	R_2	0	5/3	-1	$-4/3$	1	0	2
	x_4	0	5/3	0	$-1/3$	0	1	3
2	z	0	0	1/5	$8/5-M$	$-1/5-M$	0	18/5
x_3 enters	x_1	1	0	1/5	3/5	$-1/5$	0	3/5
x_4 leaves	x_2	0	1	$-3/5$	$-4/5$	3/5	0	6/5
	x_4	0	0	1	1	-1	1	1
3	z	0	0	0	$7/5-M$	$-M$	$-1/5$	17/5
(optimum)	x_1	1	0	0	2/5	0	$-1/5$	2/5
	x_2	0	1	0	$-1/5$	0	3/5	9/5
	x_3	0	0	1	1	-1	1	1

2

$M=1$:

Optimum solution: $x_1=0, x_2=2, xR4=1$

$z=3$

Solution is infeasible because $xR4$ is positive. The reason $M=1$ produces an infeasible solution is that it does not play the role of a penalty relative to the objective coefficients of the real variables, x_1 and x_2. Using $M=1$ makes $xR4$ more attractive than x_1 from the standpoint of minimization.

$M=10$:

Optimum solution: $x_1=.4$, $x_2=1.8$, $z=3.4$

The solution is feasible because it does not include artificials at positive level. $M=10$ is relatively much larger than the objective coefficients of x_1 and x_2, and hence properly plays the role of a penalty.

$M=1000$:

It produces the optimum solution as with $M=10$. The conclusion is that it suffices to select M reasonably larger than the objective coefficients of the real variables. Actually, $M=1000$ is an "overkill" in this case, and selecting such huge values could result in adverse roundoff error.

3

(a) Minimize $z = 4x_1 + x_2 + M(R_1+R_2+R_3)$

subject to

$$3x_1 + x_2 + R_1 = 3$$
$$4x_1 + 3x_2 - S_2 + R_2 = 6$$
$$x_1 + 2x_2 - S_3 + R_3 = 4$$
$$x_1, x_2, S_2, S_3, R_1, R_2, R_3 \geq 0$$

Basic	x_1	x_2	S_2	S_3	R_1	R_2	R_3	
z	-4	-1			$-M$	$-M$	$-M$	0
R_1	3	1			1			3
R_2	4	3	-1			1		6
R_3	1	2		-1			1	4
z	$-4+8M$	$-1+6M$	$-M$	$-M$	0	0	0	$10M$
R_1	3	1			1			3
R_2	4	3	-1			1		6
R_3	1	2		-1			1	4

(b) Minimize $z = 4x_1 + x_2 + M R_1$

subject to

$$3x_1 + x_2 + R_1 = 3$$
$$4x_1 + 3x_2 + S_2 = 6$$
$$x_1 + 2x_2 + S_3 = 4$$

Basic	x_1	x_2	R_1	S_2	S_3	
z	-4	-1	$-M$			0
R_1	3	1	1			3
S_2	4	3		1		6
R_3	1	2			1	4
z	$-4+3M$	$-1+M$	0	0	0	$3M$
R_1	3	1	1			3
S_2	4	3		1		6
R_3	1	2			1	4

(c) Minimize $z = 4x_1 + x_2 + M(R_1+R_2)$

subject to

$$3x_1 + x_2 + R_1 = 3$$
$$4x_1 + 3x_2 + R_2 = 6$$
$$x_1 + 2x_3 + S_3 = 4$$

Basic	x_1	x_2	R_1	R_2	S_3	
z	-4	-1	$-M$	$-M$	0	0
R_1	3	1	1			3
R_2	4	3		1		6
S_3	1	2			1	4
z	$-4+7M$	$-1+4M$	0	0	0	$9M$
R_1	3	1	1			3
R_2	4	3		1		6
S_3	1	2			1	4

continued…

(d) Maximize $Z = 4X_1 + X_2 - M(R_1 + R_2)$

subject to

$$\begin{aligned} 3X_1 + X_2 + R_1 &= 3 \\ 4X_1 + 3X_2 - S_2 + R_2 &= 6 \\ X_1 + 2X_2 + S_3 &= 4 \end{aligned}$$

Basic	x_1	x_2	S_2	R_1	R_2	S_3	
Z	-4	-1	0	M	M	0	0
R_1	3	1		1			3
R_2	4	3	-1		1		6
S_3	1	2				1	4
Z	-4-7M	-1-4M	M	0	0	0	-9M
R_1	3	1		1			3
R_2	4	3	-1		1		6
S_3	1	2				1	4

4

(a) Maximize $z = 5x_1 + 6x_2 - MR_1$

subject to

$$\begin{aligned} -2x_1 + 3x_2 + R_1 &= 3 \quad (1) \\ x_1 + 2x_2 + S_3 &= 5 \quad (3) \\ 6x_1 + 7x_2 + S_4 &= 3 \quad (4) \end{aligned}$$

$$Z - (5-2M)x_1 - (6+3M)x_2 = -3M$$

(b) Maximize $Z = 2x_1 - 7x_2 - M(R_1 + R_2 + R_5)$

subject to

$$\begin{aligned} -2x_1 + 3x_2 + R_1 &= 3 \quad (1) \\ 4x_1 + 5x_2 - S_2 + R_2 &= 10 \quad (2) \\ 6x_1 + 7x_2 + S_4 &= 3 \quad (4) \\ 4x_1 + 8x_2 - S_5 + R_5 &= 5 \quad (5) \end{aligned}$$

$$Z - (2+6M)X_1 - (-7+16M)X_2 + MS_2 + MS_5 = -18M$$

(c) Minimize $z = 3x_1 + 6x_2 + MR_5$

subject to

$$\begin{aligned} x_1 + 2x_2 + S_1 &= 5 \quad (3) \\ 6x_1 + 7x_2 + S_2 &= 3 \quad (4) \\ 4x_1 + 8x_2 - S_5 + R_5 &= 5 \quad (5) \end{aligned}$$

$$Z - (3-4M)X_1 - (6-8M)X_2 - MS_5 = 5M$$

(d) Minimize $Z = 4X_1 + 6X_2 + M(R_1 + R_2 + R_5)$

subject to

$$\begin{aligned} -2X_1 + 3X_2 + R_1 &= 3 \quad (1) \\ 4X_1 + 5X_2 - S_2 + R_2 &= 10 \quad (2) \\ 4X_1 + 8X_2 - S_5 + R_5 &= 5 \quad (5) \end{aligned}$$

$$Z - (4-6M) - (6-16M)X_2 - MS_2 - MS_5 = 18M$$

(e) Minimize $z = 3x_1 + 2x_2 + M(R_1 + R_5)$

subject to

$$\begin{aligned} -2x_1 + 3x_2 + R_1 &= 3 \quad (1) \\ 4x_1 + 8x_2 - S_5 + R_5 &= 5 \quad (5) \end{aligned}$$

$$Z - (3-2M)X_1 - (2-11M)X_2 - MS_5 = 8M$$

continued...

5

(a)

	Basic	x_1	x_2	x_3	S_2	R_1	R_2	
0	z	-2 -3M	-3 +4M	5 -2M	M	0	0	-17M
	R_1	1	1	1	0	1	0	7
	R_2	$\boxed{2}$	-5	1	-1	0	1	10
I	z	0	-8 $-\frac{7M}{2}$	6 $-\frac{M}{2}$	-1 $-\frac{M}{2}$	0	1 $+\frac{3M}{2}$	10 -2M
	R_1	0	$\boxed{7/2}$	1/2	1/2	1	-1/2	2
	x_1	1	-5/2	1/2	-1/2	0	1/2	5
II	z	0	0	50/7	1/7	16/7 +M	-1/7 +M	$\frac{102}{7}$
	x_2	0	1	1/7	1/7	2/7	-1/7	4/7
	x_1	1	0	6/7	-1/7	5/7	1/7	45/7

(b)

	Basic	x_1	x_2	x_3	S_2	R_1	R_2	Sol'n
0	z	-2 +3M	-3 -4M	5 +2M	-M	0	0	17M
	R_1	1	1	1	0	1	0	7
	R_2	2	-5	1	-1	0	1	10
I	z	0	-8 +7M/2	6 +M/2	-1 +M/2	0	1 -3M/2	10 + 2M
	R_1	0	7/2	1/2	1/2	1	-1/2	2
	x_1	1	-5/2	1/2	-1/2	0	1/2	5
II	z	0	0	50/7	1/7	16/7 -M	-1/7 -M	$\frac{102}{7}$
	x_2	0	1	1/7	1/7	2/7	-1/7	4/7
	x_1	1	0	6/7	-1/7	5/7	1/7	45/7
III	z	0	-50	0	-7	-12 -M	7 -M	-14
	x_3	0	7	1	1	2	-1	4
	x_1	1	-6	0	-1	-1	1	3

continued...

(c)

	Basic	x_1	x_2	x_3	S_2	R_1	R_2	Sol'n
	z	-1	-2	-1	0	M	M	—
0	R_1	1	1	1	0	1	0	7
	R_2	2	-5	1	-1	0	1	10
	z	-1 -3M	-2 +4M	-1 -2M	M	0	0	-17M
I	R_1	1	1	1	0	1	0	7
	R_2	2	-5	1	-1	0	1	10
	z	0	-9/2 -7M/2	-1/2 - M/2	-1/2 -M/2	0	1/2 +3M/2	5 -2M
II	R_1	0	7/2	1/2	1/2	1	-1/2	2
	x_1	1	-5/2	1/2	-1/2	0	1/2	5
	z	0	0	1/7	1/7	9/7 +M	-1/7 +M	53/7
III	x_2	0	1	1/7	1/7	2/7	-1/7	4/7
	x_1	1	0	6/7	-1/7	5/7	1/7	45/7

(d)

	Basic	x_1	x_2	x_3	S_2	R_1	R_2	Sol'n
	z	-4	8	-3	0	-M	-M	0
0	R_1	1	1	1	0	1	0	7
	R_2	2	-5	1	-1	0	1	10
	z	-4 +3M	8 -4M	-3 +2M	-M	0	0	17M
I	R_1	1	1	1	0	1	0	7
	R_2	2	-5	1	-1	0	1	10
	z	0	-2 +7M/2	-1 +M/2	-2 +M/2	0	2 -3M/2	20 +2M
II	R_1	0	7/2	1/2	1/2	1	-1/2	2
	x_1	1	-5/2	1/2	-1/2	0	1/2	5
	z	0	0	-5/7	-12/7	4/7 -M	12/7 -M	148/7
III	x_2	0	1	1/7	1/7	2/7	-1/7	4/7
	x_1	1	0	6/7	-1/7	-5/7	1/7	45/7

6

In the first iteration, we must substitute out the starting solution variables, x_3 and x_4, in the z-equation, exactly as we do with the artificial variables

	Basic	x_1	x_2	x_3	x_4	Solution
	z	-2	-4	-4	3	—
0	x_3	1	1	1	0	4
	x_4	1	4	0	1	8
	z	-1	-12	0	0	-8
I	x_3	1	1	1	0	4
	x_4	1	$\boxed{4}$	0	1	8
	z	2	0	0	3	16
II	x_3	3/4	0	1	-1/4	2
	x_2	1/4	1	0	1/4	2

7

After adding surplus S_1 and S_2, substitute out x_3 in the z-equation

	Basic	x_1	x_2	S_1	S_2	x_3	x_4	Solution
	z	-3	-2	0	0	-3	0	—
0	x_3	1	4	-1	0	1	0	7
	x_4	2	1	0	-1	0	1	10
	z	0	10	-3	0	0	0	21
I	x_3	1	4	-1	0	1	0	7
	x_4	2	1	0	-1	0	1	10
	z	-5/2	0	-1/2	0	-5/2	0	7/2
II	x_2	1/4	1	-1/4	0	1/4	0	7/4
	x_4	7/4	0	1/4	-1	-1/4	1	33/4

8

Both x_3 and R (the starting solution variables) must be substituted out in the z-equation

	Basic	x_1	x_2	x_3	R	Solution
	z	-1	-5	-3	M	—
0	x_3	1	2	1	0	3
	R	2	-1	0	1	4
	z	2-2M	1+M	0	0	9-4M
I	x_3	1	2	1	0	3
	R	$\boxed{2}$	-1	0	1	4
	z	0	2	0	-1+M	5
II	x_3	0	5/2	1	-1/2	1
	x_1	1	-1/2	0	1/2	2

9

Maximize $Z = 2x_1 + 5x_2 - MR_1$

subject to

$$3x_1 + 2x_2 - S_1 + R_1 = 6$$
$$2x_1 + x_2 + S_2 = 2$$
$$x_1, x_2, S_1, R_1, S_2 \geq 0$$

Basic	x_1	x_2	S_1	R_1	S_2	
z	-2	-5	0	M	0	–
R_1	3	2	-1	1	0	6
S_2	2	1	0	0	1	2
z	-2-3M	-5-2M	M	0	0	-6M
R_1	3	2	-1	1	0	6
S_2	2	1	0	0	1	2
z	0	-4-M/2	M	0	1+3M/2	-2 +3M
R_1	0	1/2	-1	1	-3/2	3
x_1	1	1/2	0	0	1/2	1
z	8+M	0	M	0	5+2M	10 -2M
R_1	-1	0	-1	1	-2	2
x_2	2	1	0	0	1	2

The z-row shows that the solution is optimal (all nonbasic coefficients in the z-row are ≥ 0). However, the solution is infeasible because the artificial variable R_1 assumes a positive value. Having a positive value for the artificial variable R_1 is the same as regarding the constraint $3x_1 + 2x_2 \geq 6$ as $3x_1 + 2x_2 \leq 6$, which violates the constraints of the original model.

Set 3.4b

1

In Phase I, we always minimize the sum of the artificial variables because the sum represents a measure of infeasibility in the problem

2

(a) Minimize $r = R_1$

(b) Minimize $r = R_1 + R_2 + R_5$

(c) Minimize $r = R_5$

(d) Minimize $r = R_1 + R_2 + R_5$

(e) Minimize $r = R_1 + R_5$

3

(a) Phase I:

Basic	x_1	x_2	x_3	s_2	R_1	R_2	
r	0	0	0	0	-1	-1	0
R_1	1	1	1	0	1	0	7
R_2	2	-5	1	-1	0	1	10
r	3	-4	2	-1	0	0	17
R_1	1	1	1	0	1	0	7
R_2	2	-5	1	-1	0	1	10
r	0	7/2	1/2	1/2	0	-3/2	2
R_1	0	7/2	1/2	1/2	1	-1/2	2
x_1	1	-5/2	1/2	-1/2	0	1/2	5
r	0	0	0	0	-1	-1	0
x_2	0	1	1/7	1/7	2/7	-1/7	4/7
x_1	1	0	6/7	-1/7	5/7	1/7	45/7

Phase II

Basic	x_1	x_2	x_3	s	Sol'n
z	-2	-3	5	0	0
x_2	0	1	1/7	1/7	4/7
x_1	1	0	6/7	-1/7	45/7
z	0	0	50/7	1/7	102/7
x_2	0	1	1/7	1/7	4/7
x_1	1	0	6/7	-1/7	45/7

(b) Phase I is the same as in (a)

Phase II

Basic	x_1	x_2	x_3	s_2	Sol'n
z	-2	-3	5	0	0
x_2	0	1	1/7	1/7	4/7
x_1	1	0	6/7	-1/7	45/7
z	0	0	50/7	1/7	102/7
x_2	0	1	1/7	1/7	4/7
x_1	1	0	6/7	-1/7	45/7
z	0	-50	0	-7	-14
x_3	0	7	1	1	4
x_1	1	-6	0	-1	3

continued...

(c) Phase I is the same as in (a)

Phase II:

Basic	x_1	x_2	x_3	s_2	Sol'n
z	-1	-2	-1	0	0
x_2	0	1	1/7	1/7	4/7
x_1	1	0	6/7	-1/7	45/7
z	0	0	1/7	1/7	53/7
x_2	0	1	1/7	1/7	4/7
x_1	1	0	6/7	-1/7	45/7

(d) Phase I is the same as in (a)

Phase II:

Basic	x_1	x_2	x_3	x_4	Sol'n
z	-4	8	-3	0	0
x_2	0	1	1/7	1/7	4/7
x_1	1	0	6/7	-1/7	45/7
z	0	0	-5/7	-12/7	$21\frac{1}{7}$
x_2	0	1	1/7	1/7	4/7
x_1	1	0	6/7	-1/7	45/7

4

Minimize $r = R_1$

subject to

$$3x_1 + 2x_2 - S_1 + R_1 = 6$$
$$2x_1 + x_2 + S_2 = 2$$
$$x_1, x_2, S_1, R_1, S_2 \geq 0$$

Solution of Phase I by TORA yields $r = 2$, which indicates that the problem has no feasible space

5

Minimize $Z = R_2$

subject to

$$2x_1 + x_2 + x_3 + S_1 = 2$$
$$3x_1 + 4x_2 + 2x_3 - S_2 + R_2 = 8$$
$$x_1, x_2, x_3, S_1, S_2, R_2 \geq 0$$

Phase I Optimal solution:

Basic	x_1	x_2	x_3	s_2	s_1	R_2	Sol'n
r	-5	0	-2	-1	-4	0	0
x_2	2	1	1	0	1	0	2
R_2	-5	0	-2	-1	-4	1	0

$R_2 = 0$ is basic in the Phase I Solution

continued...

6

(b)

Phase I (continued): R2 leaves, x1 enters (also x3, s2, and s1 are candidates for the entering variable).

	x1	x2	x3	s2	s1	R2	Sol.
r	-5	0	-2	-1	-4	0	0
x2	2	1	1	0	1	0	2
R2	-5	0	-2	-1	-4	1	0
r	0	0	0	0	0	-1	
x2	0	1	1/5	-2/5	-3/5	2/5	2
x1	1	0	2/5	1/5	4/5	-1/5	0

Drop R2-column.

Phase II:

	x1	x2	x3	s2	s1	Sol.
z	-2	-2	-4	0	0	0
x2	0	1	1/5	-2/5	-3/5	2
x1	1	0	2/5	1/5	4/5	0
z	0	0	-14/5	-2/5	2/5	4
x2	0	1	1/5	-2/5	-3/5	2
x1	1	0	2/5	1/5	4/5	0
z	7	0	0	1	6	4
x2	-1/2	1	0	-1/2	-1	2
x3	5/2	0	1	1/2	2	0

Optimum solution:

$x_1 = 0, x_2 = 2, x_3 = 0, z = 4$

Phase I:

	x1	x2	x3	R1	R2	R3	Sol
r	-10	0	-4	-8	0	0	0
x2	2	1	1	1	0	0	2
R2	-5	0	-2	-3	1	0	0
R3	-5	0	-2	-4	0	1	0
r	0	0	1	-2	-2	0	0
x2	0	1	1/5	-1/5	2/5	0	2
x1	1	0	2/5	3/5	-1/5	0	0
R3	0	0	0	-1	-1	1	0

Remove R1- and R2 columns, which gives

	x1	x2	x3	R3	Sol
r	0	0	1	0	0
x2	0	1	1/5	0	2
x1	1	0	2/5	0	0
R3	0	0	0	1	0

The R3-row is R3 = 0, which is redundant. Hence the R3-row and R3-column can be dropped from the tableau with no consequences.

Phase II:

	x1	x2	x3	Sol
z	-3	-2	-3	0
x2	0	1	1/5	2
x1	1	0	2/5	0
z	0	0	-7/5	4
x2	0	1	1/5	2
x1	1	0	2/5	0
z	7/2	0	0	4
x2	-1/2	1	0	2
x1	5/2	0	1	0

Optimum solution:

$x_1 = 0, x_2 = 2, x_3 = 0, z = 4$

7

If x_1, x_3, x_4, or x_5 assume a positive value, the value of the objective function at the end of Phase I must necessarily become positive. This follows because these variables have nonzero z-row coefficients in the optimal Phase I tableau. A positive objective value at the end of Phase I means that Phase I solution is infeasible. Since Phase II uses the same constraints as in Phase I, it follows that Phase II must have $x_1 = x_3 = x_4 = x_5 = 0$ as well.

Phase II:

Basic	x_2	R	Sol'n
z	-2	0	0
x_2	1	0	2
R	0	1	0
z	0	0	4
x_2	1	0	2
R	0	1	0

Optimum solution:

$x_1 = 0 \quad x_2 = 2 \quad x_3 = x_4 = x_5 = 0$

$z = 4$

8

$$\begin{aligned}
-5x_1 + 6x_2 - 2x_3 + x_4 \quad\quad &= -5 \\
x_1 - 3x_2 - 5x_3 \quad + x_5 \quad &= -8 \\
2x_1 + 5x_2 - 4x_3 \quad\quad + x_6 &= 9
\end{aligned}$$

x_1	x_2	x_3	x_4	x_5	x_6	R	
0	0	0	0	0	0	-1	
-5	6	-2	1	0	0	-1	-5
1	-3	-5	0	1	0	$\boxed{-1}$	-8
2	5	-4	0	0	1	0	9
-1	3	5	0	-1	0	0	8
-6	9	3	1	-1	0	0	3
-1	3	5	0	-1	0	1	8
2	5	-4	0	0	1	0	9

Phase I problem:

minimize $r = R$

subject to

$$\begin{aligned}
-6x_1 + 9x_2 + 3x_3 + x_4 - x_5 \quad &= 3 \\
-x_1 + 3x_2 + 5x_3 \quad - x_5 + R &= 8 \\
2x_1 + 5x_2 - 4x_3 \quad\quad + x_6 &= 9
\end{aligned}$$

all variables ≥ 0

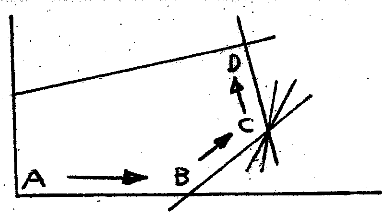
The logic of the procedure is as follows:

In the R-column, enter -1 for any constraint with negative RHS and 0 for all other constraints.

Next, use the R-column as a pivot column and select the pivot element as the one corresponding to the most negative RHS. This procedure will always require one artificial variable regardless of the number of constraints.

1

(a)

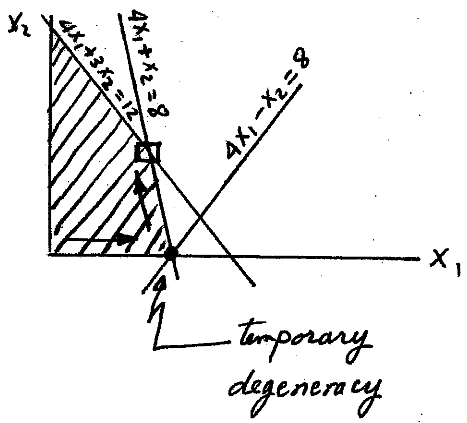


(b) A: 1, B: 1, C: $\binom{3}{2} = 3$, D: 1

2

(a) From TORA, iterations 2 and 3 are degenerate. Degeneracy is removed in iteration 4.

(b)



3

(a) Four iterations

(b) Three iterations: In iteration 2, degeneracy is removed because basic sx5 = 0 corresponds to a negative constraint coefficient in the entering variable column (x2).

(c) In part (a), solution encounters 2 degenerate basic solution at the same corner point. In part (b), only one basic solution was encountered.

1

	Basic	x_1	x_2	x_3	s_1	s_2	s_3	Solution
0	z	-1	-2	-3	0	0	0	0
	s_1	1	2	$\boxed{3}$	1	0	0	10
	s_2	1	1	0	0	1	0	5
	s_3	1	0	0	0	0	1	1
I	z	0	$\boxed{0}$	0	1	0	0	10
	x_3	1/3	2/3	1	1/3	0	0	10/3
	s_2	1	$\boxed{1}$	0	0	1	0	5
	s_3	1	0	0	0	0	1	1
II	z	$\boxed{0}$	0	0	1	0	0	10
	x_3	-1/3	0	1	1/3	-2/3	0	0
	x_2	1	1	0	0	1	0	5
	s_3	$\boxed{1}$	0	0	0	0	1	1
III	z	0	0	0	1	$\boxed{0}$	0	10
	x_3	0	0	1	1/3	-2/3	1/3	1/3
	x_2	0	1	0	0	$\boxed{1}$	-1	4
	x_1	1	0	0	0	0	1	1
IV	z	0	0	0	1	0	0	10
	x_3	0	2/3	1	1/3	0	-1/3	3
	s_2	0	1	0	0	1	-1	4
	x_1	1	0	0	0	0	1	1

Three alternative basic optima:

$$(x_1, x_2, x_3) = \begin{cases} (0, 0, 10/3) \\ (0, 5, 0) \\ (1, 4, 1/3) \end{cases}$$

The associated <u>nonbasic</u> alternative optima are

$$\tilde{x}_1 = \lambda_3$$
$$\tilde{x}_2 = 5\lambda_2 + 4\lambda_3$$
$$\tilde{x}_3 = 10/3\,\lambda_1 + 1/3\,\lambda_3$$

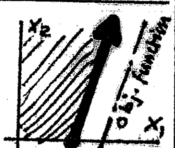
where

$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$
$$0 \le \lambda_i \le 1,\ i = 1, 2, 3$$

2

Basic	x_1	x_2	x_3	s_1	s_2	
z	-2	1	3	0	0	0
s_1	1	-1	$\boxed{5}$	1	0	10
s_2	2	-1	3	0	1	40
z	-7/5	2/5	0	3/5	0	6
x_3	$\boxed{1/5}$	-1/5	1	1/5	0	2
s_2	7/5	-2/5	0	-3/5	1	34
z	0	-1	7	2	0	20
x_1	1	-1	5	1	0	10
s_2	0	$\boxed{1}$	-7	-2	1	20
z	0	0	$\boxed{0}$	$\boxed{0}$	1	40
x_1	1	0	-2	-1	0	30
x_2	0	1	-7	-2	1	20

x_3 and s_1 can yield alternative optima. However, because all their constraint coefficients are negative (in general, ≤ 0), none can yield an alternative (corner point) basic solution.



3

	Basic	x_1	x_2	x_3	s_1	s_2	s_3	soln
0	z	-3	-1	0	0	0	0	0
	s_1	1	2	0	1	0	0	5
	s_2	$\boxed{1}$	1	-1	0	1	0	2
	s_3	7	3	-5	0	0	1	20
I	z	0	2	-3	0	3	0	6
	s_1	0	1	$\boxed{1}$	1	-1	0	3
	x_1	1	1	-1	0	1	0	2
	s_3	0	-4	2	0	-7	1	6
II	z	0	5	0	3	$\boxed{0}$	0	15
	x_3	0	1	1	1	-1	0	3
	x_1	1	2	0	1	0	0	5
	s_3	0	-6	0	-2	-5	1	0

The optimum solution is degenerate because s_3 is basic and equal to zero. Also, it has alternative <u>nonbasic</u> solutions because s_2 has a zero coefficient in the z-row and all its constraint coefficients are ≤ 0.

1

Basic	x_1	x_2	s_1	s_2	Solution
z	-2	-1	0	0	0
s_1	1	-1	1	0	10
s_2	2	0	0	1	40
z	0	-3	2	0	20
x_1	1	-1	1	0	10
s_2	0	2	-2	1	20
z	0	0	-1	3/2	50
x_1	1	0	0	1/2	20
x_2	0	1	-1	1/2	10

unbounded ↑

2

(a)

x_2
-10
-3
0
-1

$\Rightarrow$ Solution space unbounded in the direction of x_2

(b) Objective value is unbounded because each unit increase in x_2 increases z by 10

3

If, at any iteration, all the constraint coefficients of a variable are ≤ 0, then the solution space is unbounded in the direction of that variable.

A more "foolproof" way of accomplishing this task is to solve a sequence of LPs in which the objective function is

Maximize $z = x_j$, $j = 1, 2, \cdots, n$

Subject to the constraints of the problem. For the unbounded variables, $z = \infty$.

1

x_1 = number of units of T1
x_2 = number of units of T2
x_3 = number of units of T3

Constraints:

$$3x_1 + 5x_2 + 6x_3 \le 1000$$
$$5x_1 + 3x_2 + 4x_3 \le 1200$$
$$x_1 + x_2 + x_3 \ge 500$$
$$x_1, x_2, x_3 \ge 0$$

We can use Phase I to see whether the problem has a feasible solution; that is,

minimize $r = R_3$
subject to

$$3x_1 + 5x_2 + 6x_3 + S_1 = 1000$$
$$5x_1 + 3x_2 + 4x_3 + S_2 = 1200$$
$$x_1 + x_2 + x_3 - S_3 + R_3 = 500$$
$$x_1, x_2, x_3, S_1, S_2, S_3, R_3 \ge 0$$

Optimum solution from TORA:
$R_3 = r = 225$ units

This is interpreted as a deficiency of 225 units. The most that can be produced is 500 − 225 = 275 units

2

Basic	x_1	x_2	x_3	S_1	S_2	R_1	Sol^{n}
z	−3 −3M	−2 −4M	−3 −2M	M	0	0	−8M
S_1	2	$\boxed{1}$	1	0	1	0	2
R_1	3	4	2	−1	0	1	8
z	−1 +5M	0	−1 +2M	M	2 +4M	0	4
x_2	2	1	1	0	1	0	2
R_1	−5	0	−2	−1	−4	1	0

Because $R_1 = 0$ in the optimal tableau, the problem has a feasible solution. The optimum solution is

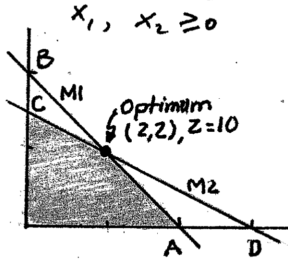
$x_1 = 0, \; x_2 = 2, \; z = 4$

Note that in the first iteration, R_1 could have been used as the leaving variable, in which case it would not be basic in the optimum iteration.

1

x_1 = Nbr. units of product A
x_2 = Nbr. units of product B

Maximize $z = 2x_1 + 3x_2$
s.t.
$2x_1 + 2x_2 \le 8$ (M1)
$3x_1 + 6x_2 \le 18$ (M2)
$x_1, x_2 \ge 0$



	M1	M2	Z
A=(4,0)		12	8
B=(0,4)		24	12
C=(0,3)	6		9
D=(6,0)	12		12

(a) M1 at C $= 2(0)+2(3) = 6$
M1 at D $= 2(6)+2(0) = 12$
Z at C $= 2(0)+3(3) = 9$
Z at D $= 2(6)+3(0) = 12$
Dual price $= \frac{12-9}{12-6} = \$.50$/unit
Allowable range $= (6 \le M1 \le 12)$
M2 at A $= 3(4)+6(0) = 12$
M2 at B $= 3(0)+6(4) = 24$
Z at A $= 2(4)+3(0) = 8$
Z at B $= 2(0)+3(4) = 12$
Dual price $= \frac{12-8}{24-12} = \$.33$/unit
Range: $12 \le M2 \le 24$

(b) Dual price = \$.50/unit valid in the range $6 \le M1 \le 12$
Increase in revenue $= .5 \times 4 = \$2.00$
Increase in cost $= .3 \times 4 = \$1.20$
Cost < Revenue – purchase recommended

(c) Dual price = \$.33/unit valid in the range $12 \le M2 \le 24$
Purchase price/unit < \$.33

(d) Dual price = \$.33/unit valid in the range $12 \le M2 \le 24$. M2 is increased from 18 to 23 units
Increase in revenue
$= 5 \times .33 = \$1.65$

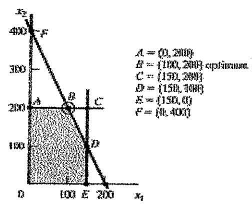
New optimum revenue $= 10 + 1.65 = \$11.65$

2

x_1 = daily number of type 1 hat
x_2 = daily number of type 2 hat

Maximize $z = 8x_1 + 5x_2$
$2x_1 + x_2 \le 400$
$x_1 \le 150$
$x_2 \le 200$
$x_1, x_2 \ge 0$

(a) Optimum occurs at B:
$x_1 = 100$ type 1 hats
$x_2 = 200$ type 2 hats
$z = \$1800$



(b) A = (0, 200), C = (150, 200)

	capacity	z
A	$2\times0 + 1\times200 = 200$	$8\times0 + 5\times200 = 1000$
C	$2\times150 + 1\times200 = 500$	$8\times150 + 5\times200 = 2200$

worth/capacity unit $= \frac{2200-1000}{500-200}$
= \$4 per type 2 hat
Range: (200, 500)

(c) Dual price = 0 in the range $(100, \infty)$
Thus, change from $x_1 \le 150$ to $x_1 \le 120$ has no effect on optimum z.

(d) Let d = demand limit for type 2 hat

	d	z
D(150,100)	100	$8(150)+5(100) = \$1700$
F(0,400)	400	$8(0)+5(400) = \$2000$

Dual price $= \frac{2000-1700}{400-100} = \$1.00$
Range (100, 400)
Maximum increase in demand limit for type 2 hat $= 400 - 200 = 200$ hats

Set 3.6b

1

(a) $\frac{3}{6} \le \frac{C_A}{C_B} \le \frac{2}{2}$, or

$.5 \le \frac{C_A}{C_B} \le 1$ or $1 \le \frac{C_B}{C_A} \le 2$

(b) Maximize $z = 2x_A + 3x_B$

$C_B = 3$: $3 \times .5 \le C_A \le 3 \times 1$

$1.5 \le C_A \le 3$

$C_A = 2$: $2 \times .5 \le C_B \le 2 \times 2$

$1 \le C_B \le 4$

(c) $\frac{C_A}{C_B} = \frac{5}{4} = 1.25$, which falls outside the range $.5 \le \frac{C_A}{C_B} \le 1$. Optimum solution changes and must be computed anew. New solution: $x_A = 4$, $x_B = 0$, $z = \$20$.

(d) Case 1: $z = 5x_A + 3x_B$

$C_A = 5$ falls outside the range $(1.5, 3)$, hence the optimum changes. New Optimum is $x_A = 4$, $x_B = 0$, $z = \$20$.

Case 2: $z = 2x_A + 4x_B$

$C_B = 4$ falls in the range $(1, 4)$, hence optimum is unchanged at $x_A = x_B = 2$, $z = 2(2) + 4(2) = \$12$

2

(a) $\frac{1}{2} \le \frac{C_1}{C_2} \le \frac{6}{4}$, or

$.5 \le \frac{C_1}{C_2} \le 1.5$ or $\frac{2}{3} \le \frac{C_2}{C_1} \le 2$

(b) Given $C_1 = 5$, then

$5(\frac{2}{3}) \le C_2 \le 5(2)$, or $\frac{10}{3} \le C_2 \le 10$

(c) $\frac{C_1}{C_2} = \frac{5}{3} = 1.67$, which falls outside the range $.5 \le \frac{C_1}{C_2} \le 1.5$. Hence the solution changes

3

(a) $\frac{0}{1} \le \frac{C_1}{C_2} \le \frac{2}{1}$, or

$0 \le \frac{C_1}{C_2} \le 2$

(b) $\frac{C_1}{C_2} = 1$, which falls in the range $0 \le \frac{C_1}{C_2} \le 2$. Hence, the solution is unchanged.

1

Feasibility conditions:

$X_2 = 100 + \frac{1}{2}D_1 - \frac{1}{4}D_2$
$X_3 = 230 + \frac{1}{2}D_2$
$X_6 = 20 - 2D_1 + D_2 + D_3$

(a) $D_1 = 438 - 430 = 8$ min
$D_2 = 500 - 460 = 40$
$D_3 = 410 - 420 = -10$

$X_2 = 100 + \frac{1}{2}(8) - \frac{1}{4}(40) = 94 > 0$
$X_3 = 230 + \frac{1}{2}(40) = 250 > 0$
$X_6 = 20 - 2(8) + 40 - 10 = 34 > 0$

Dual prices:
Resource 1 = \$1/min, $-200 \le D_1 \le 10$
2 = \$2/min, $-20 \le D_2 \le 400$
3 = \$0/min, $-20 \le D_3 < \infty$

New profit $= 1350 + D_1 + 2D_2 + 0D_3$
$= 1350 + 8 + 2 \times 40 =$ \$1438

(b) $D_1 = 460 - 430 = 30$ min
$D_2 = 440 - 460 = -20$
$D_3 = 380 - 420 = -40$
$X_2 = 100 + \frac{1}{2}(30) - \frac{1}{4}(-20) = 120 > 0$
$X_3 = 230 + \frac{1}{2}(-20) = 220 > 0$
$X_6 = 20 - 2(30) - 20 - 40 = -100 < 0$

2

(a) Overtime cost $\frac{50}{60} =$ \$.83/min
Revenue (dual price) for operation 1 is \$1/min.
Cost < Revenue $\Rightarrow$ advantageous

(b) Dual price for operation 2 = \$2/min valid in the range $-20 \le D_2 \le 400$
$D_2 = 120$ minutes
Revenue increase $= 120 \times 2 =$ \$240
Cost increase $= 2($55) =$ \$110
Revenue > cost $\Rightarrow$ accept.

(c) No, resource 3 is already abundant. This is the reason its dual price = 0

(d) Dual price for operation 1 is \$1/min, valid in the range $-200 \le D_1 \le 10$

continued...

$D_1 = 440 - 430 = 10$ min
Cost $= \frac{10}{60} \times 40 =$ \$6.67
New revenue $= 1350 + 1 \times 10 =$ \$1360
Net revenue $= 1360 -$ \$6.67 = \$1353.33

(e) Dual price = \$2/min, $-20 \le D_2 \le 400$
$D_2 = -$ min
Decrease in cost $= \frac{15}{60} \times$ \$30 = \$7.50
Lost revenue $= 15 \times$ \$2.00 = \$30.00
Lost revenue > Decrease in cost
Not recommended.

3

X_j = units of product $i = 1, 2, 3$

Maximize $z = 20X_1 + 50X_2 + 35X_3$
s.t.
$-.5X_1 + .5X_2 + .5X_3 \le 0$
$X_1 \le 75$
$2X_1 + 4X_2 + 3X_3 \le 240$
$X_1, X_2, X_3 \ge 0$

(a) Solution: $z =$ \$2800
$X_1 = X_2 = 40$, $X_3 = 0$

	x_1	x_2	x_3	S_1	S_2	S_3	
z	0	0	10/3	20/3	0	35/3	2800
x_2	0	0	5/6	2/3	0	1/6	40
S_2	1	0	1/6	4/3	1	-1/6	35
x_1	0	1	-1/6	-4/3	0	1/6	40

(b) $z + 10/3X_3 + 20/3S_1 + 0S_2 + 35/3S_3 = 2800$
Dual-price for raw material = \$35/3 /lb

$$\left.\begin{aligned} x_2 &= 40 + D_3/6 \\ S_2 &= 35 - D_3/6 \\ x_1 &= 40 + D_3/6 \end{aligned}\right\} \Rightarrow -240 \le D_3 \le 210$$

$D_3 = 120$ lb falls in the range $(-240, 210)$
New solution:
$x_1 = 40 + \frac{120}{6} = 60$ units
$x_2 = 40 + \frac{120}{6} = 60$ units
$x_3 = 0$
New revenue $= 2800 + (35/3)(120)$
$=$ \$4200

continued...

(c) Dual price = 0, $-35 \le D_2 < \infty$

$\pm 10\%$ of 75 = ± 7.5 or

Change has no effect on the solution

4

x_j = units of product j, $j = 1, 2, 3$,

Maximize $z = 4.5x_1 + 5x_2 + 4x_3$

s.t.

$$10x_1 + 5x_2 + 6x_3 \le 600$$
$$6x_1 + 8x_2 + 9x_3 \le 600$$
$$8x_1 + 10x_2 + 12x_3 \le 600$$
$$x_1, x_2, x_3 \ge 0$$

(a) Solution: $z = \$325$

$x_1 = 50, x_2 = 20, x_3 = 0$

(b) Optimum tableau

	x_1	x_2	x_3	s_1	s_2	s_3	
z	0	0	2	.083	0	.458	325
x_1	1	0	0	.167	0	-.083	50
s_2	0	0	-.6	.067	1	-.833	140
x_2	0	1	1.2	-.133	0	.167	20

$z + 2x_3 + .083s_1 + 0s_2 + .458s_3 = 325$

Dual prices:

Process 1: \$.083/min

2: \$0/min

3: \$.458/min

Process 3 > Process 1

(c) Process 1: $60 \times .083 = \$4.98$

2: 0

3: $60 \times .458 = \$27.48$

5

x_1 = Nbr. of practical courses

x_2 = Nbr. of humanistic courses

Maximize $z = 1500x_1 + 1000x_2$

$$x_1 + x_2 + s_1 = 30 \quad (1)$$
$$x_1 - s_2 = 10 \quad (2)$$
$$x_2 - s_3 = 10 \quad (3)$$
$$x_1, x_2, s_1, s_2, s_3 \ge 0$$

(a) Solution:

$z = \$40,000$

x_1 = 20 courses

x_2 = 10 courses

continued...

(b) From Tora,

$z + 1500s_1 + 0s_2 + 500s_3 = 40,000$

s_1 is a slack, s_2 and s_3 are surplus

Dual prices:

constraint 1: \$1500/course

Constraint 2: \$0/min limit course

Constraint 3: -\$500/min limit course

Dual price for constraint 1 equals the revenue per practical course. Hence, an additional course must necessarily be of the practical type.

(c) From TORA,

$$\left.\begin{array}{l} s_2 = 10 + D_1 \ge 0 \\ x_1 = 20 + D_1 \ge 0 \end{array}\right\} -10 \le D_1 < \infty$$
$$x_2 = 10$$

Thus, the dual price of \$1500 for constraint 1 is valid for any number of courses $\ge 30 - 10 = 20$.

(d) Dual price = -\$500. To determine the range where it applies, we have from TORA

$$\left.\begin{array}{l} s_1 = 10 - D_3 \ge 0 \\ x_1 = 20 - D_3 \ge 0 \\ x_2 = 10 + D_3 \ge 0 \end{array}\right\} -10 \le D_3 \le 10$$

A unit increase in lower limit on humanistic course offering (i.e. from 10 to 11) decreases revenue by \$500

6

x_1 = Radio minutes

x_2 = TV minutes

x_3 = Newspaper ads

Maximize $z = x_1 + 50x_2 + 5x_3$

s.t.

$$15x_1 + 300x_2 + 50x_3 \le 10000 \quad (1)$$
$$x_3 \ge 5 \quad (2)$$
$$x_1 \le 400 \quad (3)$$
$$-x_1 + 2x_2 \le 0 \quad (4)$$
$$x_1, x_2, x_3 \ge 0$$

Solution: $z = 1561.36$

$x_1 = 59.09$ min, $x_2 = 29.55$ min, $x_3 = 5$ ads

continued...

(b) S_1, S_3, S_4 = Slacks associated with constraints 1, 3, and 4

S_2 = Surplus associated with constraint 2

From TORA's optimum tableau:

$$Z + 2.879 S_2 + .158 S_1 + 0 S_2 + 1.364 S_3 = 1561.36$$

$$59.091 + .006D_1 - .303D_2 \qquad - .909D_4 \geq 0$$

$$5 + D_2 \geq 0$$

$$340.909 - .006D_1 + .303D_2 + D_3 + .909D_4 \geq 0$$

$$29.545 + .003D_1 - .152D_2 + .045D_4 \geq 0$$

Constraint	Dual Price	RHS Range$^+$
1	.158	(250, 66250)
2	−2.879*	(0, 2000)
3	0	(59.09, ∞)
4	1.3636	(−375, 65)

* Negative because S_2 is a surplus variable

\+ These results are taken from TORA output. They differ from those computed from the given D_i - conditions because of roundoff error

Conclusions:

1. Increasing the lower limit on the number of newspaper ads is not advantageous because the associated dual price is negative (= −2.879)
2. Increasing the upper limit on radio minutes is not warranted because its dual price is zero (the current limit is already abundant).

(c) Dual price = .158/budget \$ valid in the range $250 \leq \$ \leq 66250$.

50% budget increase = \$5000, or budget will be increased to 15,000.

Increase in Z = .158 × 5000 = 790

7

(a) x_1 = Nbr. Shirts / week

x_2 = Nbr. blouses / week

Maximize $z = 8x_1 + 12x_2$

s.t.

$$20x_1 + 60x_2 \leq 25 \times 60 \times 40 = 60{,}000$$

$$70x_1 + 60x_2 \leq 35 \times 60 \times 40 = 84{,}000$$

$$12x_1 + 4x_2 \leq 5 \times 60 \times 40 = 12{,}000$$

$$x_1, x_2 \geq 0$$

continued...

Solution: $z = \$13920$ / week

x_1 = 480 shirts, x_2 = 840 blouses

(b) Let s_1, s_2, and s_3 be the slack variables associated with the cutting, sewing, and packaging constraints. From the optimum TORA tableau, we have

$$z + .12S_1 + .08S_2 + 0S_3 = 13920$$

Dept.	Worth/hr (Dual price)
Cutting	\$.12/ = \$7.20/hr
Sewing	\$.08/min = \$4.80/hr
Packaging	\$0/hr

(c) Breakeven wages are \$7.20/hr for cutting and \$4.80 for sewing

8

(a) x_1 = units of solution A

x_2 = units of solution B

Maximize $z = 8x_1 + 10x_2$

s.t.

$$.5x_1 + .5x_2 \leq 150 \qquad (1)$$

$$.6x_1 + .4x_2 \leq 145 \qquad (2)$$

$$30 \leq x_1 \leq 150 \qquad (3)$$

$$40 \leq x_2 \leq 200 \qquad (4)$$

Solution: $z = \$2800$

x_1 = 100 units, x_2 = 200 units

(b) Define

S_1, S_2, S_3, S_4 = slacks in constraints 1, 2, 3, 4

S_5, S_6 = surplus variables associated with the lower bounds of constraints 3 and 4.

From TORA's optimum tableau:

$$z + 16S_1 + 0S_2 + 0S_3 + 2S_4 + 0S_5 + 0S_6 = 2800$$

Conditions:

$$S_5 = 70 + 2D_1 - D_4 - D_5 \geq 0$$

$$S_2 = 5 - 1.2D_1 + D_2 + .2D_4 \geq 0$$

$$S_3 = 50 - 2D_1 + D_3 + D_4 \geq 0$$

$$x_1 = 100 + 2D_1 - D_4 \geq 0$$

$$x_2 = 200 + D_4 \geq 0$$

$$S_4 = 160 + D_4 - D_6 \geq 0$$

continued...

Constraint	Dual price	RHS-range
1	16	(115, 154.17)
2	0	(140, ∞)
3 (upper)	0	(100, ∞)
3 (lower)	0	(−∞, 100)
4 (upper)	2	(175, 270)
4 (lower)	0	(−∞, 200)

Increase in raw material 1 and in the upper bound on solution B is advantageous because their dual prices (16 and 2) are positive.

(c) Increase in revenue/unit = \$16
Increase in cost/unit = \$20
Not recommended!

(d) Dual price for raw material 2 is zero because it is abundant. No increase is warrented.

9

x_1 = Nbr. DiGi-1
x_2 = Nbr. DiGi-2
S_i = Idle minutes for station i, $i = 1, 2, 3$

The objective is to minimize $S_1 + S_2 + S_3$. To express the objective function in terms of x_1 and x_2, consider

$$6x_1 + 4x_2 + S_1 = .9 \times 480 = 432$$
$$5x_1 + 4x_2 + S_2 = .86 \times 480 = 412.8$$
$$4x_1 + 6x_2 + S_3 = .88 \times 480 = 422.4$$

Thus, $S_1 + S_2 + S_3 = 1267.2 - 15x_1 - 14x_2$

(a)
Maximize $z = 15x_1 + 14x_2$
s.t.

$$6x_1 + 4x_2 + S_1 = 432$$
$$5x_1 + 4x_2 + S_2 = 412.8$$
$$4x_1 + 6x_2 + S_3 = 422.4$$
$$x_1, x_2, S_1, S_2, S_3 \ge 0$$

z represents the total used time in the three stations in minutes.

Solution: $z = 1241.28$ minutes
$x_1 = 45.12$ units, $x_2 = 40.32$ units

Utilization $= \dfrac{1241.28}{1267.20} \times 100 = 97.95\%$

continued...

(b) From TORA,

$$z + 1.7S_1 + 0S_2 + 1.2S_3 = 1241.28$$

Conditions:

$$x_1 = .3D_1 - .2D_3 + 45.12 \ge 0$$
$$S_2 = -.7D_1 + D_2 - .2D_3 + 25.92 \ge 0$$
$$x_2 = -.2D_1 + .3D_3 + 40.32 \ge 0$$

Station	Dual Price	RHS Range
1	1.7	281.6, 469.03
2	0	386.88, ∞
3	1.2	288, 552

1% decrease in maintenance time is equivalent to $D_1 = D_2 = D_3 = 4.8$ minutes. This is equivalent to having

Station	Daily minutes
1	436.8
2	417.6
3	427.2

All three daily minutes fall within the allowable ranges. Thus

Station	Increase in utilized time/day
1	$4.8 \times 1.7 = 8.16$ minutes
2	$4.8 \times 0 = 0$
3	$4.8 \times 1.2 = 5.76$

(c) $D_1 = .9(600 - 480) = 108$ min
$D_2 = .86(600 - 480) = 103.2$
$D_3 = .88(600 - 480) = 105.6$

From the conditions in (b)

$$x_1 = .3 \times 108 - .2 \times 105.6 + 45.12 = 56.4$$
$$S_2 = -.7 \times 108 + 103.2 - .2 \times 105.6 + 25.92 = 32.4$$
$$x_2 = -.2 \times 108 + .3 \times 105.6 + 40.32 = 50.4$$

Solution is feasible. Hence dual prices remain applicable and the net utilization is increased by $1.7 \times 108 + 0 \times 103.2 + 1.2 \times 105.6 = 310.32$ minutes. Because station 2 has zero dual price, its capacity need not be increased. The associated cost thus equals $1.5(600 - 480) + 0 + 1.5(600 - 480) = \$360$.

The proposal can be improved by recommending that station 2 time remain unchanged.

10

x_1 = Nbr. purses / day
x_2 = Nbr. bags / day
x_3 = Nbr. backpacks/day

Maximize $z = 24x_1 + 22x_2 + 45x_3$
s.t.

$$2x_1 + x_2 + 3x_3 \le 42$$
$$2x_1 + x_2 + 2x_3 \le 40$$
$$x_1 + .5x_2 + x_3 \le 45$$
$$x_1, x_2, x_3 \ge 0$$

Solution: $z = \$882$, $x_1 = 0, x_2 = 2, x_3 = 36$

Letting s_1, s_2, s_3 be the slacks in constraints 1, 2, and 3, we get

$$z + 20x_1 + s_1 + 21s_2 + 0s_3 = 882$$

Conditions:

$$x_3 = 2 + D_1 - D_2 \ge 0$$
$$x_2 = 36 - 2D_1 + 3D_2 \ge 0$$
$$s_3 = 25 - .5D_2 + D_3 \ge 0$$

Resource	Dual price	RHS Ranges
Leather	1	(40, 60)
Sewing	21	(28, 42)
Finishing	0	(20, ∞)

(a) Available leather = 45 ft² falls in the RHS range. Solution remains feasible. $D_1 = 45 - 42 = 3$. New solution:

$x_1 = 0$
$x_2 = 36 - 2\times3 = 30$
$x_3 = 2 + 3 = 5$
$z = 882 + 1\times D_1 = 882 + 1\times3 = \$885$

(b) Available leather = 41 ft² falls in the RHS range and the solution remains feasible. $D_1 = 41 - 42 = -1$

$x_2 = 36 - (2\times-1) = 38$
$x_3 = 2 - 1 = 1$
$z = 882 + (1\times-1) = \$881$

(c) Sewing hours = 38 falls within the RHS range. $D_2 = 38 - 40 = -2$. Dual price = 21

$x_2 = 36 + 3\times-2 = 30$
$x_3 = 2 - (-2) = 4$
$z = 882 + (21\times-2) = \$840$

(d) Sewing hours = 46 hours falls outside the RHS range. Thus, the current optimum basic solution is infeasible. To obtain the new solution, either solve the problem anew or use the algorithms in chapter 4.

(e) Finishing hours = 15, which falls outside the RHS range. Hence, resolve the problem

(f) Sewing hours = 50, which falls in the RHS range. $D_3 = 50 - 45 = 5$. Solution remains unchanged because dual price is zero and D_3 does not appear in the expression for x_2 or x_3.

(g) Dual price = \$21/hr, which is higher than the cost of an additional worker per hour. Hiring is recommended.

continued....

11

x_1 = Nbr. model 1 units
x_2 = Nbr. model 2 units

Maximize $z = 3x_1 + 4x_2$
s.t.

$$2x_1 + 3x_2 \le 1200$$
$$2x_1 + x_2 \le 1000$$
$$4x_2 \le 800$$
$$x_1, x_2 \ge 0$$

Solution: $z = \$1750$
$x_1 = 450,\ x_2 = 100$

(a) $S_1 = 0 \Rightarrow$ Resistors scarce
$S_2 = 0 \Rightarrow$ capacitors scarce
$S_3 = 400 \Rightarrow$ chips abundant

(b) $z + \frac{5}{4}S_1 + \frac{1}{4}S_2 = 1750$

Resource	Dual price
Resistors	\$1.25/resistor
Capacitors	\$.25/capacitor
Chips	\$0/chip

(c) Conditions:

$$x_1 = 450 - \tfrac{1}{4}D_1 + \tfrac{3}{4}D_2 \ge 0$$
$$S_3 = 400 - 2D_1 + 2D_2 + D_3 \ge 0$$
$$x_2 = 100 + \tfrac{1}{2}D_1 - \tfrac{1}{2}D_2 \ge 0$$

Feasibility ranges:

$$\left.\begin{array}{l}450 - .25D_1 \ge 0\\ 400 - 2D_1 \ge 0\\ 100 + .5D_1 \ge 0\end{array}\right\} \Rightarrow -200 \le D_1 \le 200$$

continued...

Set 3.6c

$$\left.\begin{array}{l} 450 + .75D_2 \ge 0 \\ 400 + 2D_2 \ge 0 \\ 100 - .5D_2 \ge 0 \end{array}\right\} \Rightarrow -200 \le D_2 \le 200$$

$400 + D_3 \ge 0 \Rightarrow -400 \le D_3 < \infty$

(d) $D_1 = 1300 - 1200 = 100$ in the allowable range $-200 \le D_1 \le 200$.

$\Delta Z = 100 \times 1.25 = \$125$

$x_1 = 450 - .25 \times 100 = 425$

$x_2 = 100 + .5 \times 100 = 150$

New $Z = 1750 + \Delta Z = \$1875$

(e) $D_3 = 350 - 800 = -450$, which falls outside allowable range $-400 \le D_3$. Thus, basic solution and dual price change and the problem must be solved anew.

(f) $-200 \le D_2 \le 200$, dual price $= .25$. Thus,

$-200 \times .25 \le \Delta Z \le 200 \times .5$

$-50 \le \Delta Z \le 50$

$\$1700 \le Z \le \$1800$

$450 - .75 \times 200 \le x_1 \le 450 + .75 \times 200$

$100 - \frac{1}{2}(-200) \le x_2 \le 100 - \frac{1}{2}(+200)$

(g) Cost of purchasing 500 additional resistors $= 500 \times .40 = \$200$

$D_1 = 500$ resistors

Dual price of \$1.25 is valid in $-200 \le D_1 \le 200$. Thus, for the first 200 resistors alone, HiDec will get an additional revenue of $200 \times 1.25 = \$250$, which is more than the cost of all 500 resistors. Accept.

12

From Example 3.6-2, we have for the TOYCO model

$-200 \le D_1 \le 10$

$-20 \le D_2 \le 400$

$-20 \le D_3 < \infty$

(a) $D_1 = 8$, $D_2 = 40$, $D_3 = -10$

All D_i, $i = 1, 2, 3$ fall within the feasibility ranges. Thus

continued...

$r_1 = \frac{8}{10}$, $r_2 = \frac{40}{400}$, $r_3 = \frac{-10}{-20}$

$r_1 + r_2 + r_3 = .8 + .1 + .5 = 1.4 > 1$

Hence, no conclusion can be made about the feasibility of the new RHS (438, 500, 410). Problem 1(a) shows that these new values do produce a feasible solution.

(b) $D_1 = 30$, $D_2 = -20$, $D_3 = -40$.

Because D_1 and D_3 fall outside the given feasibility ranges, the 100% rule cannot be applied in this case

13

(a) From TORA,

$x_1 = 2 + \frac{2}{3}D_1 - \frac{1}{3}D_2 \ge 0$

$x_2 = 2 - \frac{1}{3}D_1 + \frac{2}{3}D_2 \ge 0$

Feasibility ranges:

$-3 \le D_1 \le 6$

$-3 \le D_2 \le 6$

(b) $D_1 = D_2 = \Delta > 0$. Thus

$$\left.\begin{array}{l} x_1 = 2 + \Delta/3 > 0 \\ x_2 = 2 + \Delta/3 > 0 \end{array}\right\} \text{for all } \Delta > 0$$

100% rule for $0 < \Delta \le 3$:

$r_1 = r_2 = \frac{\Delta}{6} \le \frac{3}{6} \Rightarrow r_1 + r_2 < 1$, which confirms feasibility for $0 < D < 3$

100% rule for $3 < \Delta \le 6$:

$r_1 = r_2 = \frac{\Delta}{6} \Rightarrow \frac{3}{6} < r_1, r_2 \le \frac{6}{6}$

$r_1 + r_2 > 1 \Rightarrow$ cannot confirm feasibility.

100% rule for $\Delta > 6$:

Δ is outside $-3 \le D_1, D_2 \le 6$. Thus, the rule is not applicable.

1

From Section 3.6.3, we have the following optimality conditions for the TOYCO model:

$x_1: 4 - \frac{1}{4}d_2 + \frac{3}{2}d_3 - d_1 \geq 0$

$x_4: 1 + \frac{1}{2}d_2 \geq 0$

$x_5: 2 - \frac{1}{4}d_2 + \frac{1}{2}d_3 \geq 0$

(i) $Z = 2x_1 + x_2 + 4x_3$

$d_1 = 2-3 = -1$, $d_2 = 1-2 = -1$, $d_3 = 4-5 = -1$

$x_1: 4 - \frac{1}{4}(-1) + \frac{3}{2}(-1) - (-1) = 3.75 > 0$

$x_4: 1 + \frac{1}{2}(-1) = .5 > 0$

$x_5: 2 - \frac{1}{4}(-1) + \frac{1}{2}(-1) = 1.75 > 0$

Conclusion: Solution is unchanged

(ii) $Z = 3x_1 + 6x_2 + x_3$

$d_1 = 3-3 = 0$, $d_2 = 6-2 = 4$, $d_3 = 1-5 = -4$

$x_1: 4 - \frac{1}{4}(4) + \frac{3}{2}(4) - (0) = -3 < 0$

Conclusion: solution changes

(iii) $Z = 8x_1 + 3x_2 + 9x_3$

$d_1 = 8-3 = 5$, $d_2 = 3-2 = 1$, $d_3 = 9-5 = 4$

$x_1: 4 - \frac{1}{4}(1) + \frac{3}{2}(4) - (5) = 4.75 > 0$

$x_4: 1 + \frac{1}{2}(1) = 1.5 > 0$

$x_5: 2 - \frac{1}{4}(1) + \frac{1}{2}(4) = 3.75 > 0$

Conclusion: Solution is unchanged

2

x_1 = Nbr cans of A1

x_2 = Nbr. cans of A2

x_3 = Nbr. cans of BK

Maximize $Z = 80x_1 + 70x_2 + 60x_3$

s.t. $x_1 + x_2 + x_3 \leq 500 \quad \Leftarrow S_1$

$x_1 \geq 100 \quad \Leftarrow S_2$

$4x_1 - 2x_2 - 2x_3 \leq 0 \quad \Leftarrow S_3$

$x_1, x_2, x_3 \geq 0$

TORA optimum tableau:

Basic	x_1	x_2	x_3	s_1	S_2	s_3	Solution
Z	0	0	10	73.33	0	1.67	36666.67
x_2	0	1	1	.67	0	-.17	333.33
x_1	1	0	0	.33	0	.17	166.67
S_2	0	0	0	.33	1	.17	66.67

continued...

(a) $Z = \$366.67$

$x_1 = 166.67$, $x_2 = 333.33$, $x_3 = 0$

(b) Reduced cost for X_3 = 10 cents. Price should be increased by more than 10 cents/can

(c) $d_1 = d_2 = d_3 = -5$ cents

From the optimum tableau, reduced costs:

$x_3: 10 + d_2 - d_3 = 10 - 5 - (-5) = 10 > 0$

$S_1: 73.33 + .67d_2 + .33d_1$
$= 73.33 + .67(-5) + .33(-5) = 68.33 > 0$

$S_3: 1.67 - .17d_2 + .17d_1 = 1.67 - .17(-5) + .17(-5)$
$= 1.67 > 0$

Conclusion: Solution is unchanged.

3

(a) Available carpenter hours in a 10-day period $= 4 \times 10 \times 8 = 320$

x_1 = Nbr. chairs assembled in 10 days

x_2 = Nbr. tables assembled in 10 days

Maximize $Z = 50x_1 + 135x_2$

s.t.

$.5x_1 + 2x_2 \leq 320$

$4 \leq \frac{x_1}{x_2} \leq 6 \Rightarrow \begin{cases} x_1 - 4x_2 \geq 0 \\ x_1 - 6x_2 \leq 0 \end{cases}$

$x_1, x_2 \geq 0$

Solution: $Z = \$27,840$, $x_1 = 384$, $x_2 = 64$

(b) Optimum tableau:

	x_1	x_2	s_1	S_2	s_3	solution
Z	0	0	87	0	6.5	27840
x_2	0	1	.2	0	-.1	64
x_1	1	0	1.2	0	.4	384
S_2	0	0	.4	1	.8	128

Optimality conditions:

$s_1: 87 + 1.2d_1 + .2d_2 \geq 0$

$s_3: 6.5 + .4d_1 - .1d_2 \geq 0$

For $d_1 = -5$, $d_2 = -13.5$:

$S_1: 87 + 1.2(-5) + .2(-13.5) = 78.3 > 0$

$S_3: 6.5 + .4(-5) - .1(-13.5) = 5.85 > 0$

Solution remains the same

(c) $d_1 = 25 - 50 = -25$, $d_2 = 120 - 135 = -15$

$S_1: 87 + 1.2(-25) + .2(-15) = 58.5 > 0$

$S_3: 6.5 + 4(-25) - .1(-15) = -2 < 0$

Solution changes

Set 3.6d

4

(a) x_1 = Amt. of personal loan (\$)
x_2 = Amt. of car loan (\$)

Maximize $z = .14(x_1 - .03x_1) + .12(x_2 - .02x_2) - .03x_1 - .02x_2 = .1058x_1 + .0976x_2$

s.t.

$$x_1 + x_2 \le 200{,}000$$

$$\frac{x_2}{x_1} \ge 2 \text{ or } 2x_1 - x_2 \le 0$$

$$x_1,\ x_2 \ge 0$$

Solution: $z = \$20{,}067$

$x_1 = \$66{,}667$, $x_2 = \$133{,}333$

Rate of return $= \frac{20{,}067}{200{,}000} \times 100 = 10.03\%$

(b) Optimum tableau:

	x_1	x_2	s_1	s_2	Solution
z	0	0	.1003	.0027	20066.67
x_2	0	1	.6667	-.3333	133333.33
x_1	1	0	.3333	.3333	66666.67

Optimality conditions:

s_1: $.1003 + .333d_1 + .6667d_2 \ge 0$

s_2: $.0027 + .3333d_1 - .3333d_2 \ge 0$

New x_1-objective coefficient $= .14(1-.04) - .04 = .0944$

New x_2-objective coefficient $= .12(1-.03) - .03 = .0864$

$d_1 = .0944 - .1058 = -.0114$

$d_2 = .0864 - .0976 = -.0112$

s_1: $.1003 + .3333(-.0114) + .6667(-.0112) = .08907 > 0$

s_2: $.0027 + .3333(-.0114) - .3333(-.0112) = .00267 > 0$

Solution does not change

5

(a) x_i = Nbr of units of motor i, $i = 1,2,3,4$

Maximize $z = 60x_1 + 40x_2 + 25x_3 + 30x_4$

s.t.

$$8x_1 + 5x_2 + 4x_3 + 6x_4 \le 8000$$

$$x_1 \le 500,\ x_2 \le 500,\ x_3 \le 800,\ x_4 \le 750$$

$$x_1, x_2, x_3, x_4 \ge 0$$

Solution: $z = \$59{,}375$, $x_1 = 500$, $x_2 = 500$, $x_3 = 375$, $x_4 = 0$

continued...

(b) Optimality conditions (from TORA):

x_4: $7.5 + 1.5d_3 - d_4 \ge 0$

s_1: $6.25 + .25d_3 \ge 0$

s_2: $10 - 2d_3 + d_1 \ge 0$

s_3: $8.75 - 1.25d_3 + d_2 \ge 0$

From s_3, $8.75 + d_2 \ge 0 \Rightarrow -8.75 \le d_2 < \infty$

Thus, price of type 2 motor can be reduced by at most \$8.75 without causing a solution change.

(c) $d_1 = -15$, $d_2 = -10$, $d_3 = -6.25$, $d_4 = -7.5$

Solution remains the same because

x_4: $7.5 + 1.5(-6.25) - (-7.5) = 5.625 > 0$

s_1: $6.25 + .25(-6.25) = 4.6875 > 0$

s_2: $10 - 2(-6.25) + (-15) = 7.5 > 0$

s_3: $8.75 - 1.25(-6.25) + (-10) = 6.5625 > 0$

(d) Reduced cost for $x_4 = 7.5$. Increase price of type 4 motor by more than \$7.50.

6

(a) x_1 = Cases of juice/day
x_2 = Cases of sauce/day
x_3 = Cases of paste/day

Maximize $z = 21x_1 + 9x_2 + 12x_3$

s.t.

$$(1\times 24)x_1 + (\tfrac{1}{2}\times 24)x_2 + (\tfrac{3}{4}\times 24)x_3 \le 60{,}000$$

$$x_1 \le 2000,\ x_2 \le 5000,\ x_3 \le 6000$$

$$x_1, x_2, x_3 \ge 0$$

Solution: $z = \$51{,}000$

$x_1 = 2000$, $x_2 = 1000$, $x_3 = 0$

(b) From TORA, optimality conditions given d_2:

x_3: $1.5 + 1.5d_2 \ge 0 \Rightarrow d_2 \ge -1$

s_1: $.75 + .083d_2 \ge 0 \Rightarrow d_2 \ge -9$

s_2: $3 - 2d_2 \ge 0 \Rightarrow d_2 \le 1.5$

Thus, $-1 \le d_2 \le 1.5$, or

$9 - 1 \le$ price/case of sauce $\le 9 + 1.5$

Solution mix remains the same if the price per case of sauce remains between \$8 and \$10.50.

7

(a) x_1 = Nbr. regular cabinets/day
x_2 = Nbr. deluxe cabinets/day

Maximize $z = 100x_1 + 140x_2$

s.t.

$$.5x_1 + x_2 \le 180$$
$$x_1 \le 200$$
$$x_2 \le 150$$
$$x_1, x_2 \ge 0$$

Solution: $z = \$31,200$
$x_1 = 200$ regular
$x_2 = 80$ deluxe

(b) From TORA, optimality conditions:

S_1: $140 + d_2 \ge 0$
S_2: $30 + d_1 - .5d_2 \ge 0$

$d_1 = 80 - 100 = -20$
$d_2 = 80 - 140 = -60$

S_1: $140 + (-60) = 80 > 0$
S_2: $30 + (-20) - .5(-60) = 40 > 0$

Solution remains the same

The 100% rule is nonconclusive. Yet Problem 1(iii) shows that the solution remains unchanged.

The two cases demonstrate that the 100% rule is too weak to be effective in decision making, and that it is more reliable to utilize the simultaneous optimality conditions given in Section 3.6.3.

(b) $-30 \le d_1 < \infty$, $-140 \le d_2 \le 60$

New $Z = 80x_1 + 80x_2$
Original $z = 100x_1 + 140x_2$

i	d_i	u_i	v_i	r_i
1	−20	−30	∞	$-20/-30 = 2/3$
2	−60	−140	60	$-60/-140 = 3/7$

$r_1 + r_2 = 2/3 + 3/7 = \frac{23}{21} > 1$

The 100% rule is nonconclusive. Yet, Problem 7(b) shows that the solution remains unchanged.

8

(a) For the original TOYCO model, TORA gives (also see Section 3.6.3)

$$-\infty < d_1 \le 4, \ -2 \le d_2 \le 8, \ -8/3 \le d_3 < \infty$$

(ii) Original $Z = 3x_1 + 2x_2 + 5x_3$
New $z = 3x_1 + 6x_2 + x_3$

i	d_i	u_i	v_i	r_i
1	0		4	$0/4 = 0$
2	4		8	$4/8 = 1/2$
3	−4	−8/3		$-4/-\frac{8}{3} = 3/2$

$r_1 + r_2 + r_3 = 0 + 1/2 + 3/2 = 2 > 1$

The 100% rule is nonconclusive in this case. The solution in Problem 1(ii) shows that the solution will change

(iii) Original $z = 3x_1 + 2x_2 + 5x_3$
New $z = 8x_1 + 3x_2 + 9x_3$

i	d_i	u_i	v_i	r_i
1	5		4	5/4
2	1		8	1/8
3	4		∞	$4/\infty = 0$

$r_1 + r_2 + r_3 = \frac{5}{4} + \frac{1}{8} = \frac{11}{8} > 1$

continued...

Set 3.6e

1

See file solver3.6e-1.xls in ch3Files

Dual prices for years 1, 2, 3, and 4 are 0, 0, 0, 2.89. Thus, for year 4, one (thousand) additional dollars increases Z by \$2.89 thousand. It is worthwhile to increase the funding for year 4.

2

See file tora3.6e-2.txt

Constraint	Dual Price	Range
1	5.36	$(0, \infty)$
2	-3.73	$(-\infty, 6000)$
3	-1.13	$(-\infty, 6800)$
4	-1.07	$(-\infty, 33642)$
5	-1.00	$(-\infty, 53628.73)$

(a) Constraint 1: $x_1 + x_2 + x_4 + y_1 \leq 10{,}000$

Dual price = \$5.36/invested \$

Rate of return = 536%

(b) Constraint 2: \$1000 spend on pleasure

$.5x_1 + .6x_2 - x_3 + .4x_4 + 1.065y_1 - y_2 = 1000$

Dual price = −3.73/pleasure \$

Range = $(-\infty, 6000)$

Spending \$1000 at end of year 1 reduces total return by \$3730.

3

See file tora3.6e-3.txt in ch3Files

Quarter	Dual price	Range
1	1.2488	.6647, 2.5806
2	1.2443	.6580, 2.6122
3	1.1945	−.2646, 1.1245
4	1.0200	−.2553, ∞
5	1.0000	−4.8366, ∞

(a) An additional \$ available at the start of quarter 1 is worth \$1.24888 at the end of 4 quarters. Similarly, an additional dollar at the start of periods 2, 3, and 4 is worth \$1.2443, \$1.1945, and \$1.02, respectively. The dual price for quarter 4 (= \$1.02) shows that all we can do with the money then is to invest it at 2% for the quarter.

We can use the dual price to determine

continued...

the rate of return for each quarter — namely,

quarter 1:

$1.2488 = 1.2243(1+i_1) \Rightarrow i_1 = .02$

quarter 2:

$1.2243 = 1.1945(1+i_2) \Rightarrow i_2 = .025$

quarter 3:

$1.1945 = 1.02(1+i_3) \Rightarrow i_3 = .171$

quarter 4:

$1.02 = 1.0(1+i_4) \Rightarrow i_4 = .02$

(b) The dual price associated with the upper bound on B_3 (UB-X10) is \$.149. It represents the networth per dollar borrowed in period 3. Also, an extra dollar in period 3 is worth \$1.1945 at the end of the horizon. However, if that dollar is borrowed, it must be repaid as \$1.025 in the next quarter. The repayment is equivalent to forgoing making 2% in interest. Thus, the networth of borrowing in period 3 is

$1.1945 - 1.025 \times 1.02 = .149$

This result is consistent with the dual price for the upper bound on B_3

4

Constraint	Current RHS	Min RHS	Max RHS	Dual Price
1 (=)	2.0000	0.0000	infinity	2.1756
2 (=)	2.0000	-0.1667	infinity	2.0173
3 (=)	2.5000	-0.3472	infinity	1.8647
4 (=)	2.5000	-0.5767	infinity	1.7296
5 (=)	3.0000	-0.8248	infinity	1.6044
6 (=)	3.5000	-1.1331	infinity	1.4356
7 (=)	3.5000	-6.1137	infinity	1.3355
8 (=)	4.0000	-11.4678	infinity	1.2423
9 (=)	4.0000	-20.6663	infinity	1.1556
10 (=)	5.0000	-32.5201	infinity	1.0750

The dual price provides the worth per additional \$ at the end of year 10.

Annual rate of return:

Period 1: $2.1756 = 2.0173(1+i_1) \Rightarrow i_1 = .0785$

Period 2: $2.0173 = 1.8647(1+i_2) \Rightarrow i_2 = .0818$

Period 3: $1.8647 = 1.7296(1+i_3) \Rightarrow i_3 = .0781$

Period 4: $1.7296 = 1.6044(1+i_4) \Rightarrow i_4 = .0780$

etc...

5

See file tora3.6e-5.txt in ch3Files

The dual price for constraint 1

$$x_{1A} + x_{1B} \le 100,000$$

is \$5.10. Thus, each invested \$ is worth \$5.10 at the end of the investment horizon. Range $(0, \infty)$

6

Dual price for the constraint

$$x_1 + x_2 + x_3 + x_4 \le 500$$

is \$2.35 per \$ invested, range $(0, \infty)$

The gambler should bet the largest amount possible.

7

See file tora3.6e-7.txt in ch3Files

For, $x_{W1} + x_{W2} + x_{W3} \ge 1500$, the dual price is \$11.4, range $(800, \infty)$

One extra wrench automatically requires the production of two chisels, thus leading to the following changes:

Cost of one wrench using subcont.	= \$3.00
Cost of 2 chisels using subcont.	= 2x \$4.20
total	= \$11.40

$x_{W1} \le 550$, dual price = -\$1, range $(-\infty, 1250)$. If regular time capacity for wrenches is increased by 1 unit, one less wrench will be produced by subcontractor, which saves \$3-\$2 = \$1.

Similar interpretations can be given for the remaining dual prices

8

See file tora3.6e-8.txt in ch3Files

Machine	Capacity	Dual price	Range
1	500	2	(253.33, 570)
2	380	12	(333.33, 750)

The company should pay less than \$2/hr for machine 1 and less than \$12/hr for machine 2.

9

See file tora3.6e-9.txt in ch3Files

(a) Constraint $2x_1 + 3x_2 + 5x_3 \le 4000$ corresponds to raw material A. Its dual price is \$10.27/lb. For a purchase price of \$12/lb, acquisition of additional raw material A is not recommended.

(b) Constraint $4x_1 + 2x_2 + 7x_3 \le 6000$ is associated with raw material B. Its dual price is \$0/lb. Resource B is already abundant. Thus, no additional purchase is recommended.

10

(a) See file tora3.6e-10.txt

Constraint	Dual price
1	0
2	0
3	-400
4	-750
5	0
6	0
7	0

Constraints 3 and 4 have negative dual price. These correspond respectively to the third specification for alloy A and the first specification for alloy B. Changes in these specifications affects profit adversely

(b) For the ore constraints, the dual prices are \$90, \$110, and \$30 per additional ton of ores 1, 2, and 3, respectively. These are the maximum prices the company should pay.