CHAPTER 2

Modeling with Linear Programming

Set 2.1a

1

(a) $x_2 - x_1 \geq 1$ or $-x_1 + x_2 \geq 1$

(b) $x_1 + 2x_2 \geq 3$ and $x_1 + 2x_2 \leq 6$

(c) $x_2 \geq x_1$ or $x_1 - x_2 \leq 0$

(d) $x_1 + x_2 \geq 3$

(e) $\frac{x_2}{x_1 + x_2} \leq .5$ or $.5x_1 - .5x_2 \geq 0$

2

(a) $(x_1, x_2) = (1, 4)$

$(x_1, x_2) \geq 0$

$6\times1 + 4\times4 = 22 < 24$

$1\times1 + 2\times4 = 9 \not\leq 6$ infeasible

(b) $(x_1, x_2) = (2, 2)$

$(x_1, x_2) \geq 0$

$$\left.\begin{array}{lll} 6\times2 + 4\times2 & = 20 & < 24 \\ 1\times2 + 2\times2 & = 6 & = 6 \\ -1\times2 + 1\times2 & = 0 & < 1 \\ 1\times2 & = 2 & = 2 \end{array}\right\} \text{feasible}$$

$Z = 5\times2 + 4\times2 = \$18$

(c) $(x_1, x_2) = (3, 1.5)$

$x_1, x_2 \geq 0$

$$\left.\begin{array}{lll} 6\times3 + 4\times1.5 & = 24 & = 24 \\ 1\times3 + 2\times1.5 & = 6 & = 6 \\ -1\times3 + 1\times1.5 & = -1.5 & < 1 \\ 1\times1.5 & = 1.5 & < 2 \end{array}\right\} \text{feasible}$$

$Z = 5\times3 + 4\times1.5 = \$21$

(d) $(x_1, x_2) = (2, 1)$

$x_1, x_2 \geq 0$

$$\left.\begin{array}{lll} 6\times2 + 4\times1 & = 16 & < 24 \\ 1\times2 + 2\times1 & = 4 & < 6 \\ -1\times2 + 1\times1 & = -1 & < 1 \\ 1\times1 & = 1 & < 2 \end{array}\right\} \text{feasible}$$

$Z = 5\times2 + 4\times1 = \$14$

(e) $(x_1, x_2) = (2, -1)$

$x_1 \geq 0$, $x_2 < 0$, infeasible

Conclusion: (c) gives the best feasible solution

3

$(x_1, x_2) = (2, 2)$

Let S_1 and S_2 be the unused daily amounts of M1 and M2.

For M1: $S_1 = 24 - (6x_1 + 4x_2) = 4$ tons/day

For M2: $S_2 = 6 - (x_1 + 2x_2)$

$= 6 - (2 + 2\times2) = 0$ tons/day

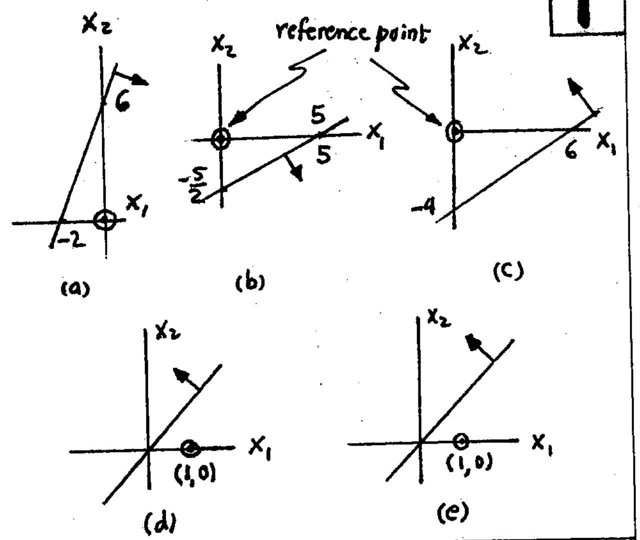
4

Quantity discount results in the following nonlinear objective function:

$$Z = \begin{cases} 5x_1 + 4x_2, & 0 \leq x_1 \leq 2 \\ 4.5x_1 + 4x_2, & x_1 > 2 \end{cases}$$

The situation cannot be treated as a linear program. Nonlinearity can be accounted for in this case using mixed integer programming (chapter 9).

1



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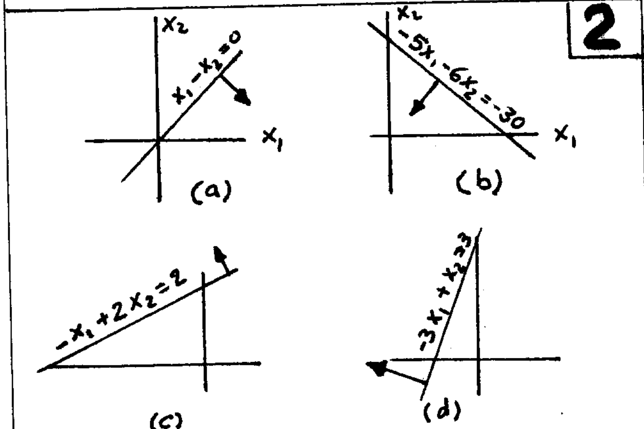
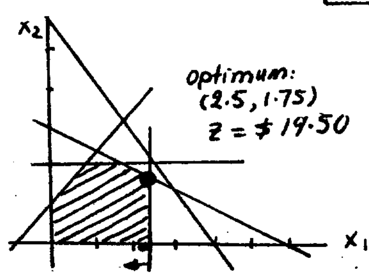


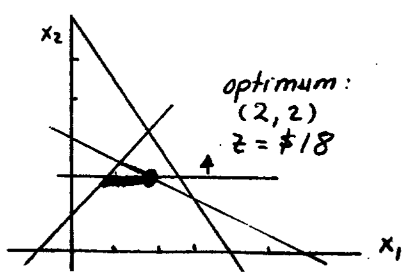
3

(a) $x_1 \le 2.5$



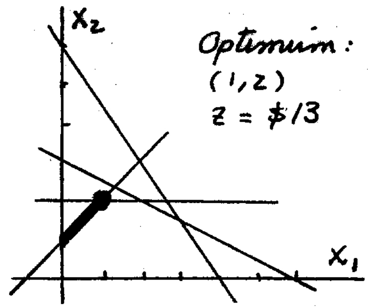
optimum:
(2.5, 1.75)
$z = \$19.50$

(b) $x_2 \ge 2$



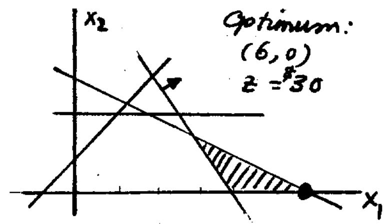
optimum:
(2, 2)
$z = \$18$

(c) $-x_1 + x_2 = 1$



Optimum:
(1, 2)
$z = \$13$

(d) $6x_1 + 4x_2 \ge 24$



Optimum:
(6, 0)
$z = \$30$

(e) No feasible space

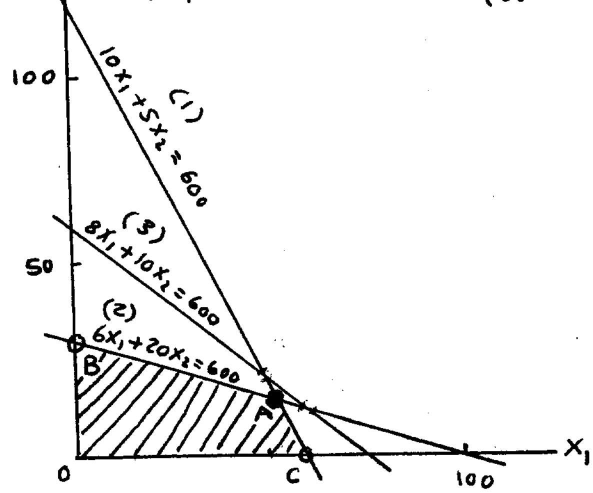
4

x_1 = daily units of product 1
x_2 = daily units of product 2

Maximize $z = 2x_1 + 3x_2$

s.t.

$$10x_1 + 5x_2 \le 600 \quad (1)$$
$$6x_1 + 20x_2 \le 600 \quad (2)$$
$$8x_1 + 10x_2 \le 600 \quad (3)$$



Optimum occurs at A:
$x_1 = 52.94$
$x_2 = 14.12$
$z = \$148.24$

continued...

Set 2.2a

5

x_1 = number of units of A
x_2 = number of units of B

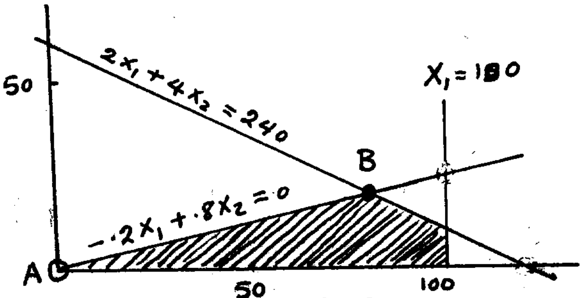
Maximize $Z = 20x_1 + 50x_2$

$\frac{x_1}{x_1+x_2} \geq .8$ or $-.2x_1 + .8x_2 \leq 0$

$x_1 \leq 100$

$2x_1 + 4x_2 \leq 240$

$x_1, x_2 \geq 0$



Optimal occurs at B:
$x_1 = 80$ units
$x_2 = 20$ units
$Z = \$2,600$

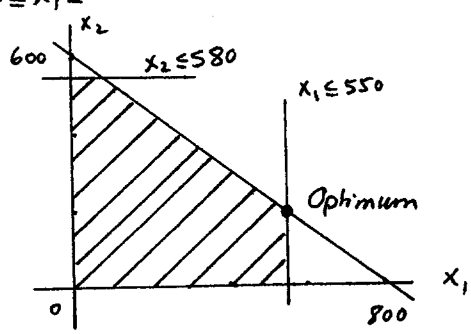
6

x_1 = number of sheets/day
x_2 = number of bars/day

Maximize $Z = 40x_1 + 35x_2$

s.t. $\frac{x_1}{800} + \frac{x_2}{600} \leq 1$

$0 \leq x_1 \leq 550$, $0 \leq x_2 \leq 580$



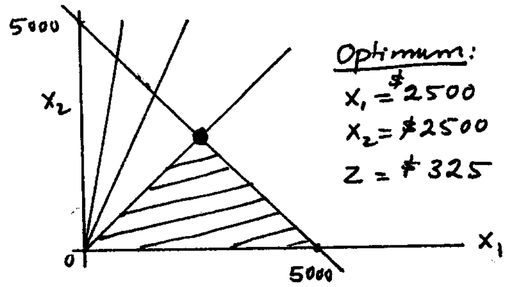
Optimum solution:
$x_1 = 550$ sheets
$x_2 = 187.13$ bars
$Z = \$28,549.40$

7

x_1 = \$ invested in A
x_2 = \$ invested in B

Maximize $Z = .05x_1 + .08x_2$

s.t.
$x_1 \geq .25(x_1 + x_2)$
$x_2 \leq .5(x_1 + x_2)$
$x_1 \geq .5x_2$
$x_1 + x_2 \leq 5000$
$x_1, x_2 \geq 0$



Optimum:
$x_1 = \$2500$
$x_2 = \$2500$
$Z = \$325$

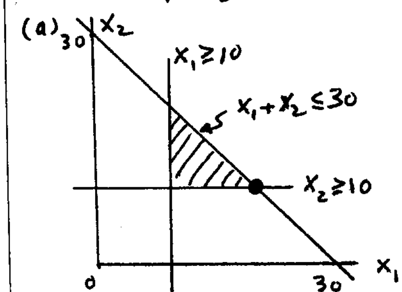
8

x_1 = number of practical courses
x_2 = number of humanistic courses

Maximize $Z = 1500x_1 + 1000x_2$

s.t.
$x_1 + x_2 \leq 30$
$x_1 \geq 10$
$x_2 \geq 10$
$x_1, x_2 \geq 0$

(a)



Optimum:
$x_1 = 20$
$x_2 = 10$
$Z = \$40,000$

(b) Change $x_1 + x_2 \leq 30$ to $x_1 + x_2 \leq 31$
Optimum $Z = \$41,500$
$\Delta Z = \$41,500 - 40,000 = \$1500$

Conclusion: Any additional course will be of the practical type.

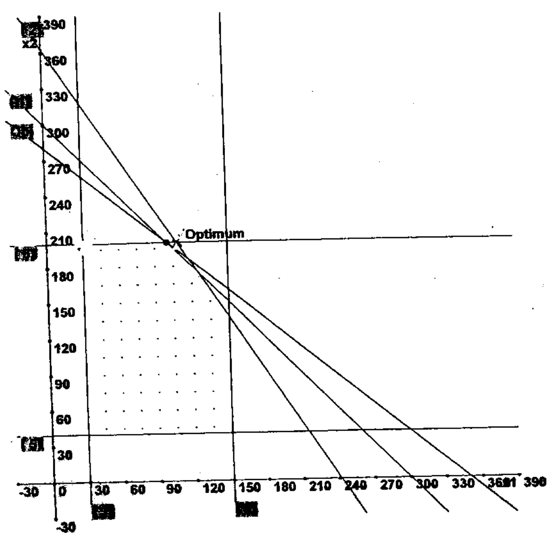
9

x_1 = units of solution A
x_2 = units of solution B

Maximize $z = 8x_1 + 10x_2$
Subject to

$$\begin{aligned}
.5x_1 + .5x_2 &\le 150\\
.6x_1 + .4x_2 &\le 145\\
x_1 &\ge 30\\
x_1 &\le 150\\
x_2 &\ge 40\\
x_2 &\le 200\\
x_1, x_2 &\ge 0
\end{aligned}$$

Summary of Optimal Solution:
Objective Value = 2800.00
x1 = 100.00
x2 = 200.00



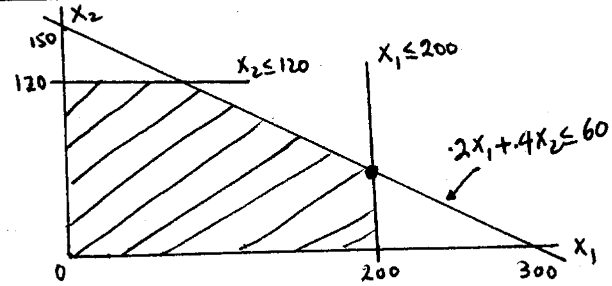
10

x_1 = nbr. of grano boxes
x_2 = nbr. of wheatie boxes

Maximize $Z = x_1 + 1.35x_2$
s.t.

$$\begin{aligned}
.2x_1 + .4x_2 &\le 60\\
x_1 &\le 200\\
x_2 &\le 120\\
x_1, x_2 &\ge 0
\end{aligned}$$

continued...



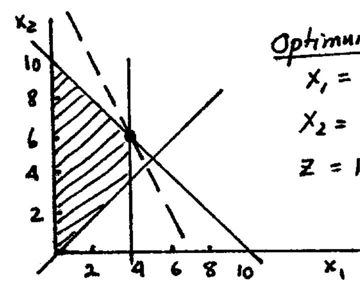
Optimum: $x_1 = 200$, $x_2 = 50$, $Z = \$267.50$
Area allocation: 67% grano, 33% wheatie

11

x_1 = play hours per day
x_2 = work hours per day

Maximize $Z = 2x_1 + x_2$
s.t.

$$\begin{aligned}
x_1 + x_2 &\le 10\\
x_1 - x_2 &\le 0\\
x_1 &\le 4\\
x_1, x_2 &\ge 0
\end{aligned}$$



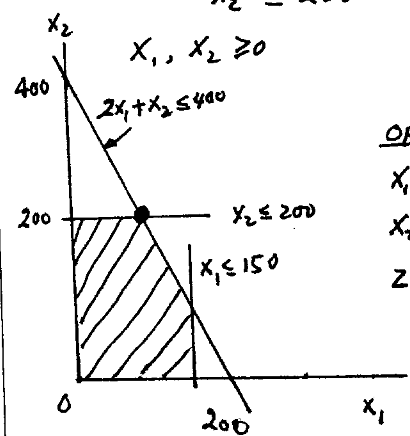
Optimum solution:
$x_1 = 4$ hours
$x_2 = 6$ hours
$z = 14$ "pleasarits"

12

x_1 = Daily nbr. of type 1 hat
x_2 = Daily nbr. of type 2 hat

Maximize $Z = 8x_1 + 5x_2$
s.t.

$$\begin{aligned}
2x_1 + x_2 &\le 400\\
x_1 &\le 150\\
x_2 &\le 200\\
x_1, x_2 &\ge 0
\end{aligned}$$



Optimum:
$x_1 = 100$ type 1
$x_2 = 200$ type 2
$Z = \$1800$

continued...

Set 2.2a

13

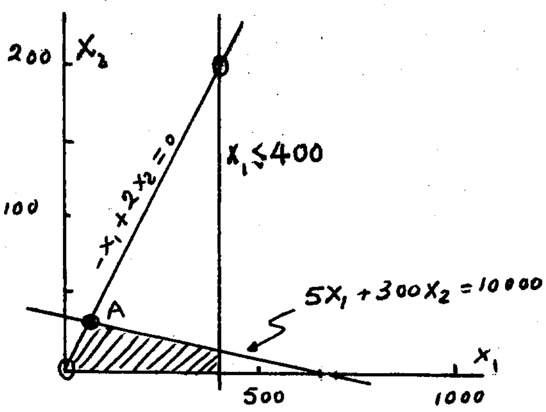
x_1 = radio minutes
x_2 = TV minutes

Maximize $z = x_1 + 25x_2$

s.t. $15x_1 + 300x_2 \le 10{,}000$

$\frac{x_1}{x_2} \ge 2$ or $-x_1 + 2x_2 \le 0$

$x_1 \le 400$, $x_1, x_2 \ge 0$



Optimum occurs at A:

x_1 = 60.61 minutes
x_2 = 30.3 minutes
z = 818.18

14

x_1 = tons of C_1 consumed per hour
x_2 = tons of C_2 consumed per hour

Maximize $z = 12000x_1 + 9000x_2$

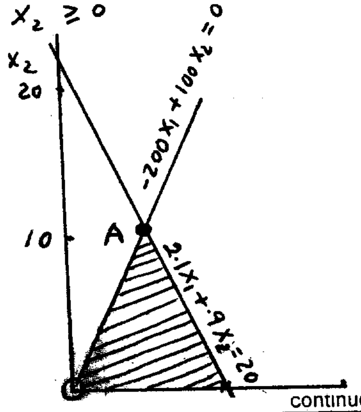
s.t.

$1800x_1 + 2100x_2 \le 2000(x_1 + x_2)$

or $-200x_1 + 100x_2 \le 0$

$2.1x_1 + .9x_2 \le 20$

$x_1, x_2 \ge 0$



continued...

(a) Optimum occurs at A:

x_1 = 5.128 tons per hour
x_2 = 10.256 tons per hour
z = 153,846 lb of steam

Optimal ratio $= \frac{5.128}{10.256} = .5$

(b) $2.1x_1 + .9x_2 \le (20+1) = 21$

Optimum z = 161538 lb of steam

$\Delta z = 161538 - 153846 = 7692$ lb

15

x_1 = Nbr. of radio commercials beyond the first
x_2 = Nbr. of TV ads beyond the first

Maximize $z = 2000x_1 + 3000x_2 + 5000 + 2000$

s.t. $300(x_1+1) + 2000(x_2+1) \le 20{,}000$

$300(x_1+1) \le .8 \times 20{,}000$

$2000(x_2+1) \le .8 \times 20{,}000$

$x_1, x_2 \ge 0$

or

Maximize $z = 2000x_1 + 3000x_2 + 7000$

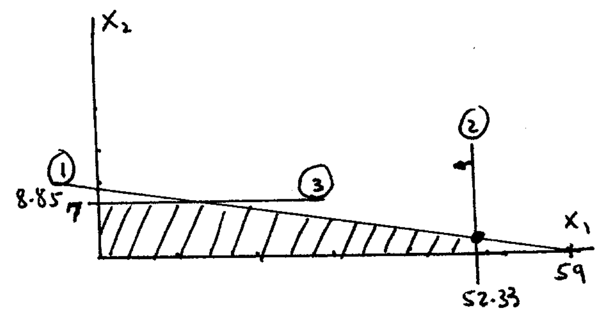
s.t.

$300x_1 + 2000x_2 \le 17700$ (1)

$300x_1 \le 15700$ (2)

$2000x_2 \le 14000$ (3)

$x_1, x_2 \ge 0$



Optimum solution:

Radio commercials = 52.33+1 = 53.33

TV ads = 1+1 = 2

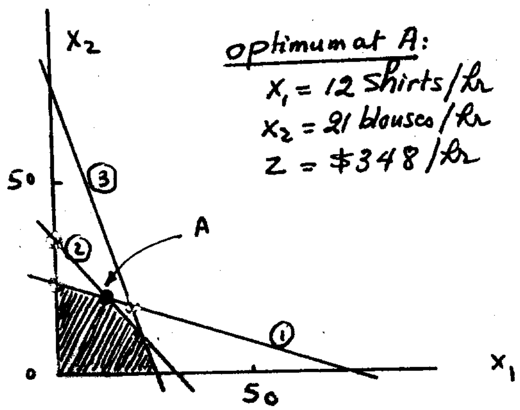
z = 107666.67+7000 = 114666.67

16

x_1 = number of shirts per hour
x_2 = number of blouses per hour

Maximize $z = 8x_1 + 12x_2$
s.t.

$$20x_1 + 60x_2 \le 25 \times 60 = 1500 \quad (1)$$
$$70x_1 + 60x_2 \le 35 \times 60 = 2100 \quad (2)$$
$$12x_1 + 4x_2 \le 5 \times 60 = 300 \quad (3)$$
$$x_1, x_2 \ge 0$$



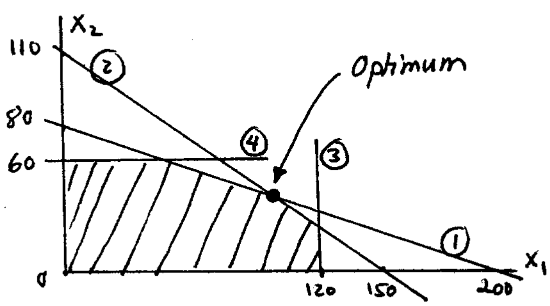
optimum at A:
$x_1 = 12$ shirts/hr
$x_2 = 21$ blouses/hr
$z = \$348$/hr

17

x_1 = Nbr. of desks per day
x_2 = Nbr. of chairs per day

Maximize $z = 50x_1 + 100x_2$

$$\frac{x_1}{200} + \frac{x_2}{80} \le 1 \quad (1)$$
$$\frac{x_1}{150} + \frac{x_2}{110} \le 1 \quad (2)$$
$$x_1 \le 120, \; x_2 \le 60 \quad (3,4)$$



Optimum:
$x_1 = 90$ desks
$x_2 = 44$ chairs
$z = \$8900$

18

x_1 = number of HiFi1 units
x_2 = number of HiFi2 units

Constraints:

$$6x_1 + 4x_2 \le 480 \times .9 = 432$$
$$5x_1 + 5x_2 \le 480 \times .86 = 412.8$$
$$4x_1 + 6x_2 \le 480 \times .88 = 422.4$$

or

$$6x_1 + 4x_2 + S_1 = 432$$
$$5x_1 + 5x_2 + S_2 = 412.8$$
$$4x_1 + 6x_2 + S_3 = 422.4$$

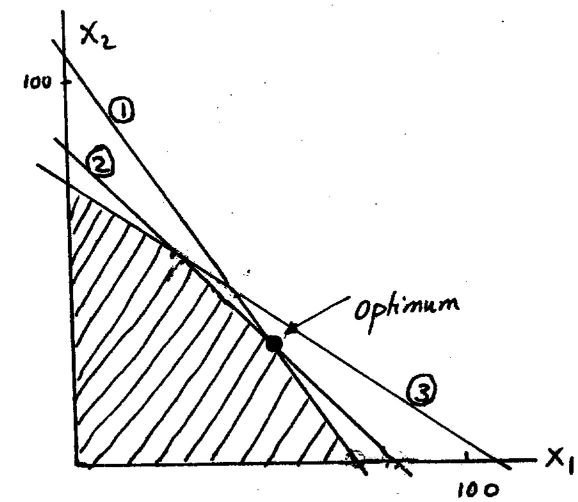
Objective function:

Minimize $S_1 + S_2 + S_3 = 1267.2 - 15x_1 - 15x_2$

Thus, $\min S_1 + S_2 + S_3 \equiv \max 15x_1 + 15x_2$

Maximize $z = 15x_1 + 15x_2$
s.t.

$$6x_1 + 4x_2 \le 432 \quad (1)$$
$$5x_1 + 5x_2 \le 412.8 \quad (2)$$
$$4x_1 + 6x_2 \le 422.4 \quad (3)$$
$$x_1, x_2 \ge 0$$

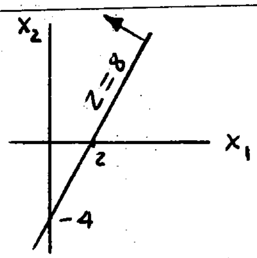


Optimum: (Problem has alternative optima)
$x_1 = 50.88$ units
$x_2 = 31.68$ units
$z = 1238.4$ minutes

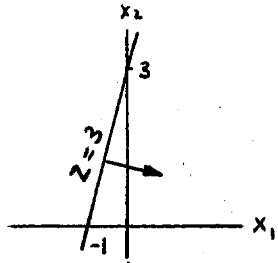
Set 2.2b

1

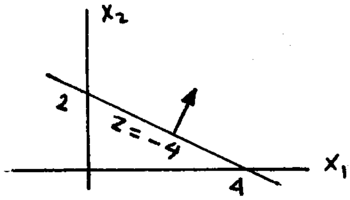
(a)



(b)

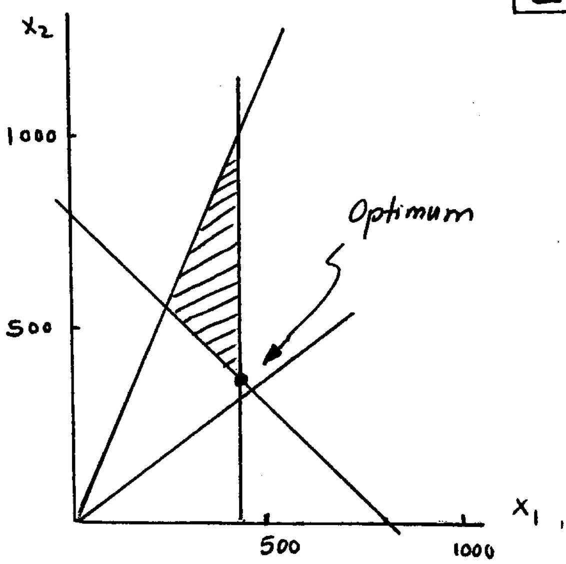


(c)



2

Additional constraint: $x_1 \leq 450$



Optimum Solution:

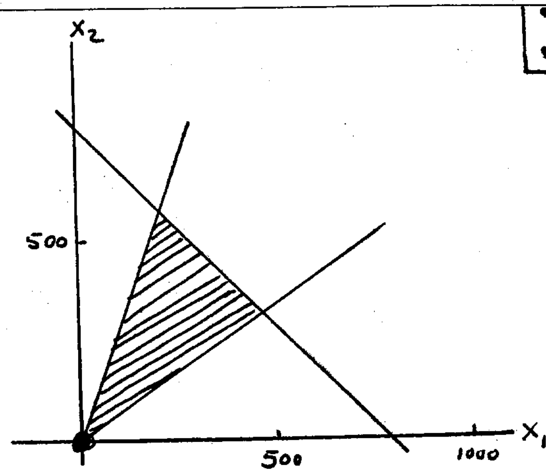
$x_1 = 450$ lb

$x_2 = 350$ lb

$z = \$450$

continued...

3



Optimum: $x_1 = 0$, $x_2 = 0$, $z = 0$, which is nonsensical

4

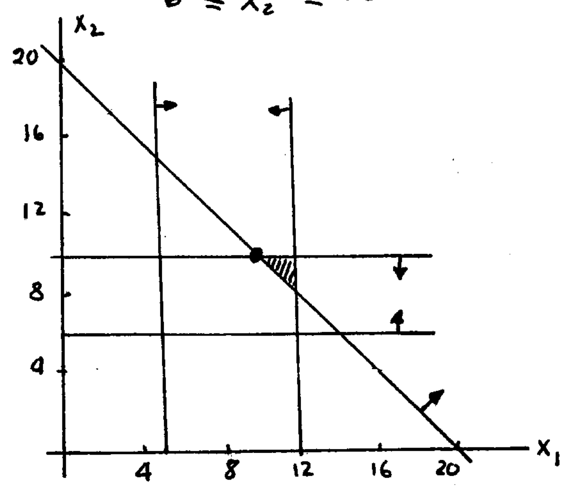
x_1 = number of hours/week in store 1

x_2 = number of hours/week in store 2

Minimize $z = 8x_1 + 6x_2$

s.t.

$$x_1 + x_2 \geq 20$$
$$5 \leq x_1 \leq 12$$
$$6 \leq x_2 \leq 10$$



Optimum:

$x_1 = 10$ hours

$x_2 = 10$ hours

$z = 140$ stress index

continued...

5

Let

$x_1 = 10^3$ bbl/day from Iran

$x_2 = 10^3$ bbl/day from Dubai

Refinery capacity $= x_1 + x_2 \ 10^3$ bbl/day

Minimize $z = x_1 + x_2$

subject to

$$x_1 \geq .4(x_1 + x_2)$$

or

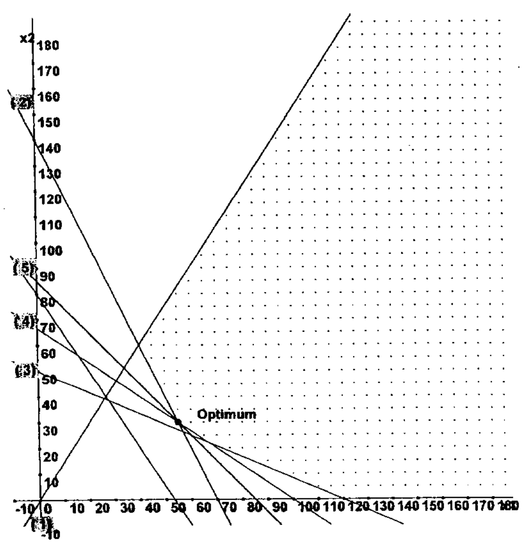
$$\begin{aligned} -.6x_1 + .4x_2 &\leq 0 \\ .2x_1 + .1x_2 &\geq 14 \\ .25x_1 + .6x_2 &\geq 30 \\ .1x_1 + .15x_2 &\geq 10 \\ .15x_1 + .1x_2 &\geq 8 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Optimum solution from TORA:

LINEAR PROGRAMMING – GRAPHICAL SOLUTION

Title: diet problem

Summary of Optimal Solution:
Objective Value = 85.00
x1 = 55.00
x2 = 30.00



6

Let

$x_1 = 10^3\$$ invested in blue chip stock

$x_2 = 10^3\$$ invested in high-tech stocks

Minimize $z = x_1 + x_2$

Subject to

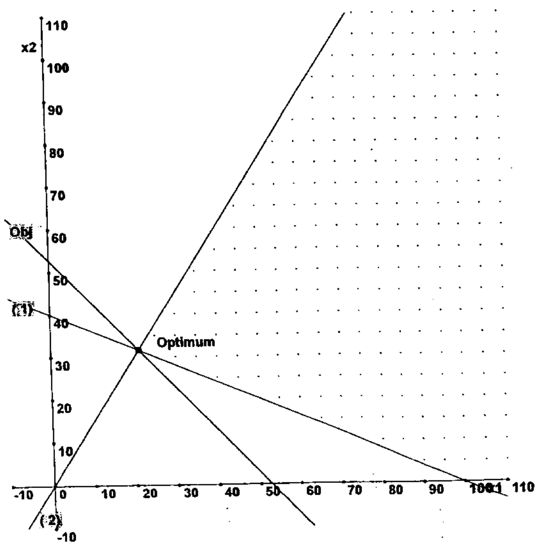
$$\begin{aligned} .1x_1 + .25x_2 &\geq 10 \\ .6x_1 - .4x_2 &\geq 0 \\ x_1, x_2 &\geq 0 \end{aligned}$$

TORA optimum solution:

LINEAR PROGRAMMING – GRAPHICAL SOLUTION

Title: diet problem

Summary of Optimal Solution:
Objective Value = 52.63
x1 = 21.05
x2 = 31.58



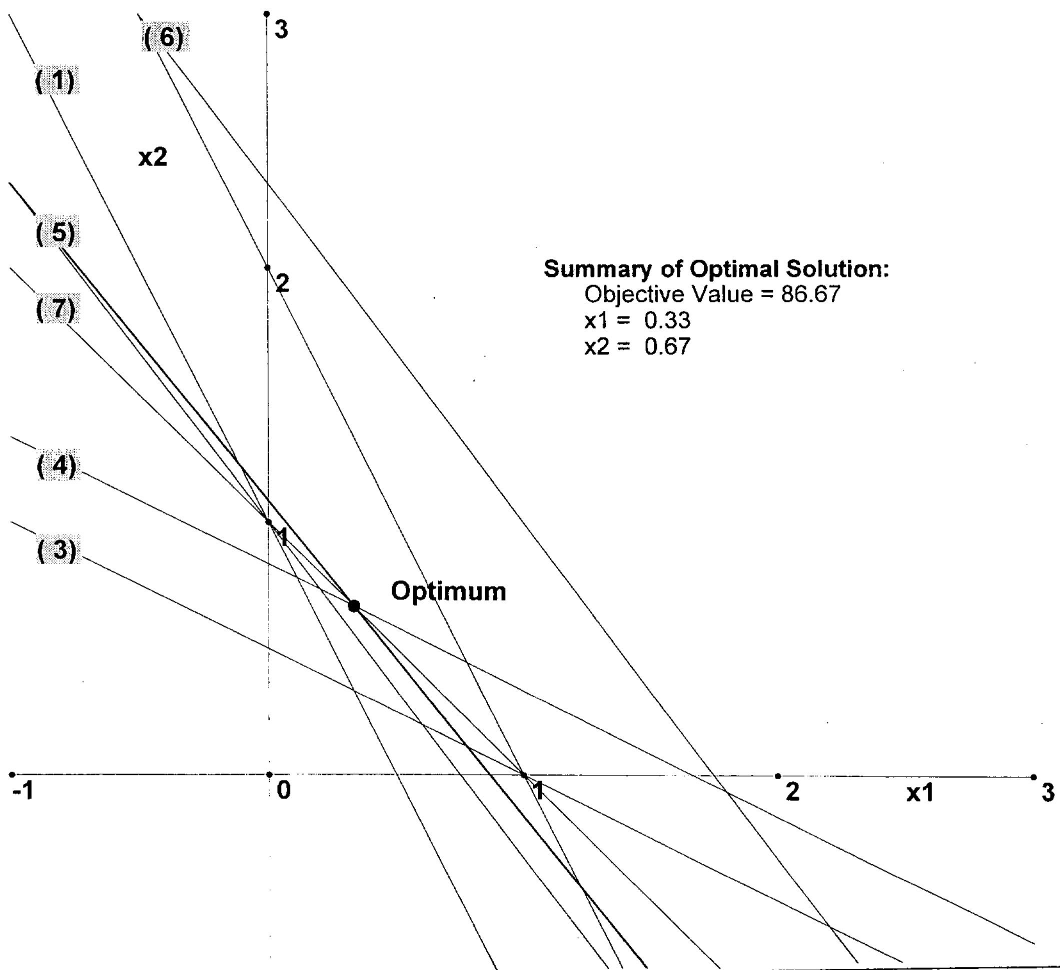
Set 2.2b

7

x_1 = Ratio of scrap A in alloy

x_2 = Ratio of scrap B in alloy

	x1	x2		
Minimize	100.00	80.00		
Subject to				
(1)	0.06	0.03	>=	0.03
(2)	0.06	0.03	<=	0.06
(3)	0.03	0.06	>=	0.03
(4)	0.03	0.06	<=	0.05
(5)	0.04	0.03	>=	0.03
(6)	0.04	0.03	<=	0.07
(7)	1.00	1.00	=	1.00



Summary of Optimal Solution:
Objective Value = 86.67
x1 = 0.33
x2 = 0.67

1

x_e = Nbr. of efficiency apartments
x_d = Nbr. of duplexes
x_s = Nbr. of single-family homes
x_r = Retail space in ft^2

Maximize $Z = 600x_e + 750x_d + 1200x_s + 100x_r$

s.t. $x_e \le 500,\ x_d \le 300,\ x_s \le 250$

$x_r \ge 10x_e + 15x_d + 18x_s$

$x_r \le 10000$

$x_d \ge \dfrac{x_e + x_s}{2}$

$x_e, x_d, x_s, x_r \ge 0$

Optimal solution:

$Z = 1{,}595{,}714.29$

$x_e = 207.14,\ x_d = 228.57$

$x_s = 250,\ x_r = 10{,}000$

LP does not guarantee integer solution. Use rounded solution or apply integer LP algorithm (Chapter 9).

2

x_i = Acquired portion of property i

Each site is represented by a separate LP. The site that yields the smaller objective value is selected.

Site 1 LP:

Minimize $Z = 25 + x_1 + 2.1x_2 + 2.35x_3 + 1.85x_4 + 2.95x_5$

s.t. $x_4 \ge .75$, all $x_i \ge 0,\ i = 1, 2, \dots, 5$

$20x_1 + 50x_2 + 50x_3 + 30x_4 + 60x_5 \ge 200$

Optimum: $Z = 34.6625$ million \$

$x_1 = .875, x_2 = x_3 = 1, x_4 = .75, x_5 = 1$

Site 2 LP:

Minimize $Z = 27 + 2.8x_1 + 1.9x_2 + 2.8x_3 + 2.5x_4$

s.t. $x_3 \ge .5,\ x_1, x_2, x_3, x_4 \ge 0$

$80x_1 + 60x_2 + 50x_3 + 70x_4 \ge 200$

Optimum: $Z = 34.35$ million \$

$x_1 = x_2 = 1,\ x_3 = x_4 = .5$

Select site 2.

3

x_{ij} = portion of project i completed in year j

Maximize $Z = .05(4x_{11} + 3x_{12} + 2x_{13}) + .07(3x_{22} + 2x_{23} + x_{24}) + .15(4x_{31} + 3x_{32} + 2x_{33} + x_{34}) + .02(2x_{43} + x_{44})$

s.t.

$\sum_{j=1}^{3} x_{1j} = 1,\ \sum_{j=3}^{4} x_{4j} = 1$

$.25 \le \sum_{j=2}^{5} x_{2j} \le 1,\ .25 \le \sum_{j=1}^{5} x_{3j} \le 1$

$5x_{11} + 15x_{31} \le 3$

$5x_{12} + 8x_{22} + 15x_{32} \le 6$

$5x_{13} + 8x_{23} + 15x_{33} + 1.2x_{43} \le 7$

$8x_{24} + 15x_{34} + 1.2x_{44} \le 7$

$8x_{25} + 15x_{35} \le 7$

Optimum:

$Z = \$523{,}750$

$x_{11} = .6,\ x_{12} = .4$

$x_{24} = .225,\ x_{25} = .025$

$x_{32} = .267,\ x_{33} = .387,\ x_{34} = .346$

$x_{43} = 1$

4

x_l = Nbr. of low income units
x_m = Nbr. of middle income units
x_u = Nbr. of upper income units
x_p = Nbr. of public housing units
x_s = Nbr. of school rooms
x_r = Nbr. of retail units
x_c = Nbr. of condemned homes

Maximize $Z = 7x_l + 12x_m + 20x_u + 5x_p + 15x_r - 10x_s - 7x_c$

s.t. $100 \le x_l \le 200,\ 125 \le x_m \le 190$

$75 \le x_u \le 260,\ 300 \le x_p \le 600$

$0 \le x_s \le 2/.045$

$.05x_l + .07x_m + .03x_u + .025x_p + .045x_s + .1x_r \le .85(50 + .25x_c)$

$x_r \ge .023x_l + .034x_m + .046x_u + .023x_p + .034x_s$

continued...

Set 2.3a

$$25x_s \ge 1.3x_l + 1.2x_m + .5x_u + 1.4x_p$$

Optimum: $z = 8290.30$ thousand \$

$x_l = 100$, $x_m = 125$, $x_u = 227.04$

$x_p = 300$, $x_s = 32.54$, $x_r = 25$

$x_c = 0$

5

x_1 = Nbr. of single-family homes

x_2 = Nbr. of double-family homes

x_3 = Nbr. of triple-family homes

x_4 = Nbr. of recreation areas

Maximize $z = 10{,}000x_1 + 12000x_2 + 15000x_3$

s.t.

$$2x_1 + 3x_2 + 4x_3 + x_4 \le .85 \times 800$$

$$\frac{x_1}{x_1+x_2+x_3} \ge .5 \text{ or } .5x_1 - .5x_2 - .5x_3 \ge 0$$

$$x_4 \ge \frac{x_1 + 2x_3 + 3x_3}{200} \text{ or } 200x_4 - x_1 - 2x_2 - 3x_3 \ge 0$$

$$1000x_1 + 1200x_2 + 1400x_3 + 800x_4 \ge 100{,}000$$

$$400x_1 + 600x_2 + 840x_3 + 450x_4 \le 200{,}000$$

$$x_1, x_2, x_3, x_4 \ge 0$$

Optimum solution:

$x_1 = 339.15$ homes

$x_2 = 0$

$x_3 = 0$

$x_4 = 1.69$ areas

$z = \$3{,}391{,}521.20$

6

New land use constraint:

$$2x_1 + 3x_2 + 4x_3 + x_4 \le .85(800 + 100)$$

New Optimum solution:

$z = \$3{,}815{,}461.35$

$x_1 = 381.54$ homes

$x_2 = x_3 = 0$

$x_4 = 1.91$ areas

$\Delta z = \$3{,}815{,}461.35 - 3{,}391{,}521.20$

$= \$423{,}940.35$

$\Delta z < \$450{,}000$, the purchasing cost of 100 acres. Hence, the purchase of the new acreage is not recommended.

1

The constraints remain unchanged, but the objective function is changed to

Maximize $z = y -$ commission

where

$$\text{commission} = .001(\text{all transactions in \$})$$

$$= .001\Big[(x_{12}+x_{13}+x_{14}+x_{15}) + \frac{1}{.769}(x_{21}+x_{23}+x_{24}+x_{25}) + \frac{1}{.625}(x_{31}+x_{32}+x_{34}+x_{35}) + \frac{1}{105}(x_{41}+x_{42}+x_{43}+x_{45}) + \frac{1}{.342}(x_{51}+x_{52}+x_{53}+x_{54})\Big]$$

Optimum solution:

	Without	With
z	5.09032	5.06211
y	5.09032	5.08986
Return	1.8064%	1.2421%

Commission $= 5.08986 - 5.06211$
$= \$27{,}750$

or, .555% of the original investment of \$5 million

2

Input I in fund type p:

$$\sum_{\substack{j=1\\ j\neq p}}^{n} r_{jp}x_{jp} \longrightarrow (p) \longrightarrow \sum_{\substack{j=1\\ j\neq p}}^{n} x_{pj}, \quad I \downarrow (p)$$

Output y in fund type q:

$$\sum_{\substack{j=1\\ j\neq q}}^{n} r_{jq}x_{jq} \longrightarrow (q) \longrightarrow \sum_{\substack{j=1\\ j\neq q}}^{n} x_{qj}, \quad (q) \downarrow y$$

For specific p and q, the model below can be used to transform any fund to any other fund. In

continued…

the present problem, $p = 1$ (\$) and $q = 2$ (€), 3 (£), 4 (¥), and 5 (KD).

General node i:

$$\sum_{\substack{j=1\\ j\neq i}}^{n} r_{ji}x_{ji} \longrightarrow (i) \longrightarrow \sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}$$

Maximize $z = y$

s.t.

$$I + \sum_{\substack{j=1\\ j\neq p}}^{n} r_{jp}x_{jp} = \sum_{\substack{j=1\\ j\neq p}}^{n} x_{pj}$$

$$\sum_{\substack{j=1\\ j\neq q}}^{n} r_{jq}x_{jq} = y + \sum_{\substack{j=1\\ j\neq q}}^{n} x_{qj}$$

$$\sum_{\substack{j=1\\ j\neq i}}^{n} r_{ji}x_{ji} = \sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}, \quad i \neq p \text{ or } q$$

$$0 \le x_{ij} \le cap_i, \text{ all } i \text{ and } j$$

Note: Solver or AMPL is ideal for solving this problem interactively. See files solver2.3b-2.xls and ampl2.3b-2.txt.

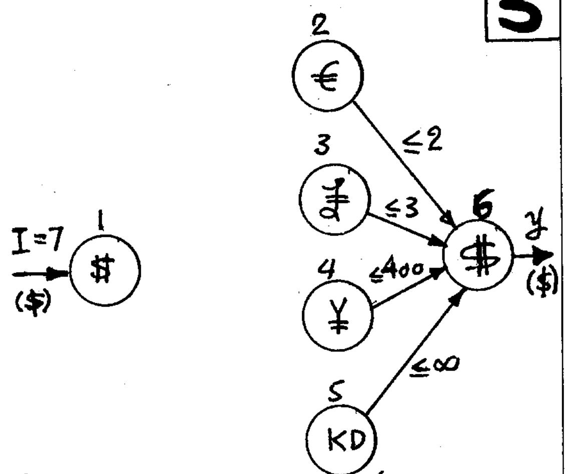
Results: (No commission)

p	q	Rate of return
\$	\$	1.8064%
\$	€	1.7966%
\$	£	1.8287%
\$	¥	2.8515%
\$	KD	1.0471%

Wide discrepancy in ¥ and KD currencies may be attributed to the fact that their exchange rates may not be consistent with the remaining rates. Nevertheless, the problem shows that there may be advantages in targeting accumulation in different currencies.

Set 2.3b

3



To formulate the objective function correctly, all output currencies are converted to a single currency (arbitrarily chosen to be \$). Thus

$$y = r_{21}x_{26} + r_{31}x_{36} + r_{41}x_{46} + r_{51}x_{56}$$

Maximize $z = y$

s.t. $x_{26} \le 2,\ x_{36} \le 3,\ x_{46} \le 400;\ x_{3j} \le 3.5$

$x_{1j} \le 5,\ x_{2j} \le 3,\ x_{4j} \le 100,\ x_{5j} \le 2.8$, all j

$$I + \sum_{i=2}^{5} x_{i1} r_{i1} = \sum_{j=2}^{5} x_{1j}$$

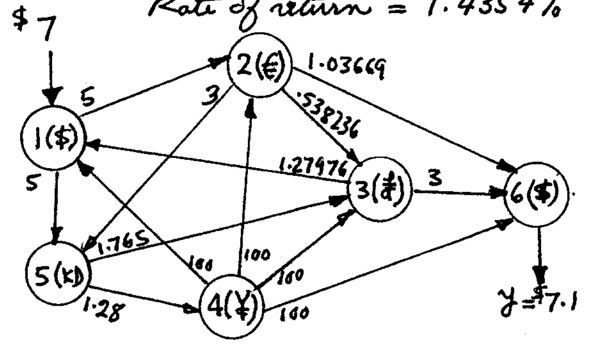
$$y = r_{21}x_{26} + r_{31}x_{36} + r_{41}x_{46} + r_{51}x_{56}$$

$$x_{i6} + \sum_{\substack{j=1\\ j\ne i}}^{5} x_{ij} = \sum_{\substack{j=1\\ j\ne i}}^{5} r_{ji}x_{ji}\ ,\quad i = 2,3,4,5$$

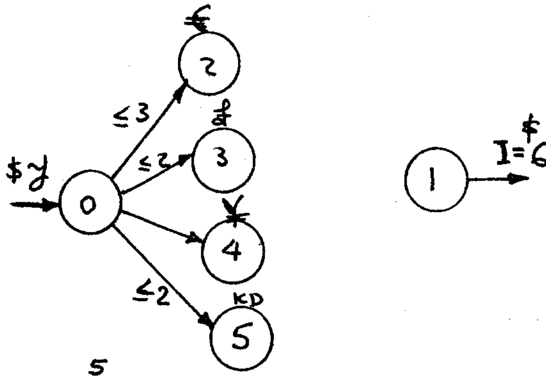
all $x_{ij} \ge 0$, $i \ne j$

Solution: Total accumulation $y = \$7.1$ million

Rate of return $= 1.4354\%$



4



$$y = \sum_{k=2}^{5} r_{k1}x_{0k} = r_{21}x_{02} + r_{31}x_{03} + r_{41}x_{04} + r_{51}x_{05}$$

Minimize $z = y$

s.t. $y = r_{21}x_{02} + r_{31}x_{03} + r_{41}x_{04} + r_{51}x_{05}$

$x_{02} \le 3,\ x_{03} \le 2,\ x_{05} \le 2$

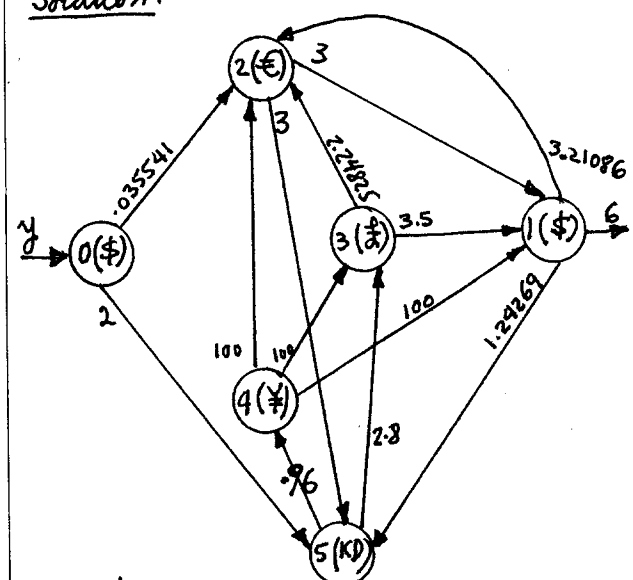
all j: $x_{1j} \le 5,\ x_{2j} \le 3,\ x_{4j} \le 3.5,\ x_{4j} \le 100,\ x_{5j} \le 2.8$

$$x_{0j} + \sum_{\substack{i=1\\ i\ne j}}^{5} r_{ij}x_{ij} = \sum_{\substack{k=1\\ k\ne j}}^{5} x_{jk}\ ,\quad j = 2,3,4,5$$

$$\sum_{i=2}^{5} r_{i1}x_{i1} = \sum_{j=2}^{5} x_{1j} + I$$

all $x_{ij} \ge 0$

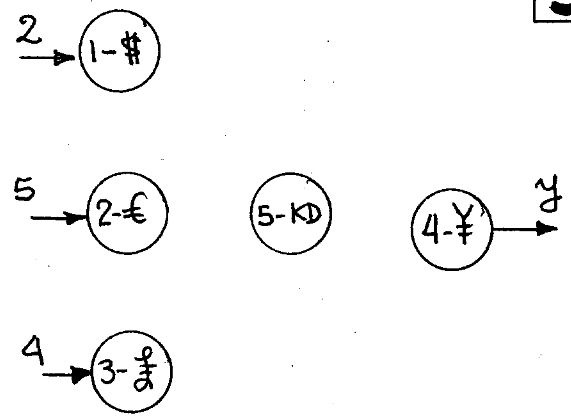
Solution:



$y = \$5.894170322$

Rate of return $= 1.7638\%$

5



Maximize $z = y$

s.t.

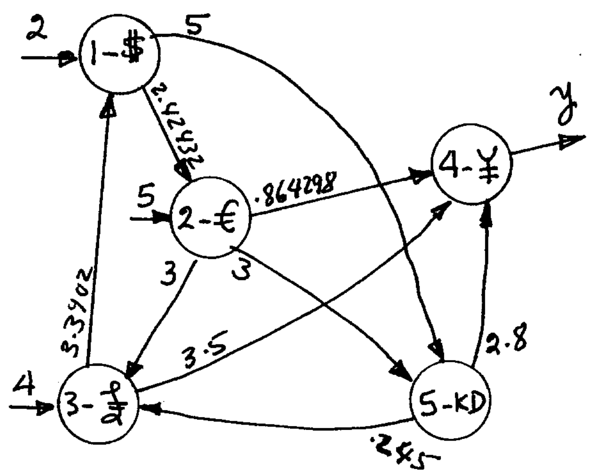
$$y = r_{14}x_{14} + r_{24}x_{24} + r_{34}x_{34} + r_{54}x_{54}$$

$$\sum_{\substack{i=1\\ i\neq j}}^{5} r_{ij}x_{ij} = \sum_{\substack{k=1\\ k\neq j}}^{5} x_{jk} - \begin{cases} 2, & j=1 \\ 5, & j=2 \\ 4, & j=3 \\ -y, & j=4 \\ 0, & j=5 \end{cases}$$

$$\left.\begin{array}{l} x_{ij} \le C_i \text{, for all } i \text{ and } j \\ x_{ij} \ge 0\text{, for all } i \text{ and } j \end{array}\right\} i \neq j$$

<u>Solution</u>: $y = 1584.91$ million¥

Rate of return $= .8853\%$



Set 2.3c

1

(a) x_i = Undertaken portion of project i

Maximize

$z = 32.4x_1 + 35.8x_2 + 17.75x_3 + 14.8x_4 + 18.2x_5 + 12.35x_6$

Subject to

$10.5x_1 + 8.3x_2 + 10.2x_3 + 7.2x_4 + 12.3x_5 + 9.2x_6 \le 60$

$14.4x_1 + 12.6x_2 + 14.2x_3 + 10.5x_4 + 10.1x_5 + 7.8x_6 \le 70$

$2.2x_1 + 9.5x_2 + 5.6x_3 + 7.5x_4 + 8.3x_5 + 6.9x_6 \le 35$

$2.4x_1 + 3.1x_2 + 4.2x_3 + 5.0x_4 + 6.3x_5 + 5.1x_6 \le 20$

$0 \le x_j \le 1, \quad j = 1, 2, \dots, 6$

TORA optimum Solution:

$x_1 = x_2 = x_3 = x_4 = 1, x_5 = .84, x_6 = 0, z = 116.06$

(b) Add the constraint $x_2 \le x_6$

TORA optimum Solution:

$x_1 = x_2 = x_3 = x_4 = x_6 = 1, x_5 = .03, z = 113.68$

(c) Let S_i be the unused funds at the end of year i and change the right-hand sides of constraints 2, 3, and 4 to $70 + S_1$, $35 + S_2$, and $20 + S_3$, respectively.

TORA optimum solution:

$x_1 = x_2 = x_3 = x_4 = x_5 = 1, \quad x_6 = .71$

$z = \$127.72$ (thousand)

The solution is interpreted as follows:

i	S_i	$S_i - S_{i-1}$	Decision
1	4.96	—	—
2	7.62	+2.66	Don't borrow from yr 1
3	4.62	−3.00	Borrow \$3 from year 2
4	0	−4.62	Borrow \$4.62 from yr 2

The effect of availing excess money for use in later years is that the first five projected are completed and 71% of project 6 is undertaken.

The total revenue increases from \$116,060 to 127,720.

(d) The slack S_i in period i is treated as an <u>unrestricted</u> variable.

TORA optimum solution: $z = \$131.30$

$S_1 = 2.3, \; S_2 = .4, \; S_3 = -5, \; S_4 = -6.1$

This means that additional funds are needed in years 3 and 4.

Increase in return $= 131.30 - 116.06 = \$15.24$

Ignoring the time value of money, the amount borrowed $5 + 6.1 - (2.3 + .4) = \$8.4$. Thus,

rate of return $= \dfrac{15.24 - 8.4}{8.4} \cong 81\%$

continued...

2

x_i = dollar investment in project i, $i = 1, 2, 3, 4$

y_j = dollar investment in bank in year j, $j = 1, 2, 3, 4, 5$

Maximize $z = y_5$

Subject to

$x_1 + x_2 + x_4 + y_1 \le 10{,}000$

$.5x_1 + .6x_2 - x_3 + .4x_4 + 1.065y_1 - y_2 = 0$

$.3x_1 + .2x_2 + .8x_3 + .6x_4 + 1.065y_2 - y_3 = 0$

$1.8x_1 + 1.5x_2 + 1.9x_3 + 1.8x_4 + 1.065y_3 - y_4 = 0$

$1.2x_1 + 1.3x_2 + .8x_3 + .95x_4 + 1.065y_4 - y_5 = 0$

All variables ≥ 0

TORA optimal solution:

$x_1 = 0, \; x_2 = \$10{,}000, \; x_3 = \$6000, \; x_4 = 0$

$y_1 = 0, \; y_2 = 0, \; y_3 = \$6800, \; y_4 = \$33{,}642$

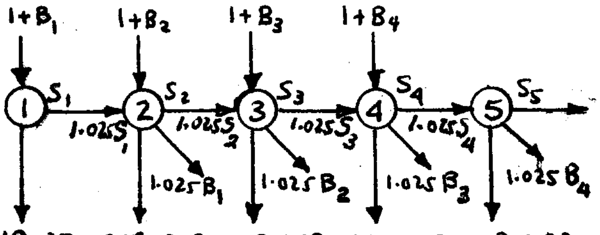
$z = \$53{,}628.73$ at the start of year 5

3

P_i = fraction undertaken of project i, $i = 1, 2$

B_j = million dollars borrowed in quarter j, $j = 1, 2, 3, 4$

S_j = surplus million dollars at the start of quarter j, $j = 1, 2, 3, 4, 5$



(a) Maximize $z = S_5$

subject to

$$P_1 + 3P_2 + S_1 - B_1 = 1$$
$$3.1P_1 + 2.5P_2 - 1.025S_1 + S_2 + 1.025B_1 - B_2 = 1$$
$$1.5P_1 - 1.5P_2 - 1.02S_2 + S_3 + 1.025B_2 - B_3 = 1$$
$$-1.8P_1 - 1.8P_2 - 1.02S_3 + S_4 + 1.025B_3 - B_4 = 1$$
$$-5P_1 - 2.8P_2 - 1.02S_4 + S_5 + 1.025B_4 = 1$$
$$0 \le P_1 \le 1, \quad 0 \le P_2 \le 1$$
$$0 \le B_j \le 1, \quad j = 1, 2, 3, 4$$

Optimum solution:

$P_1 = .7113 \qquad P_2 = 0$

$z = 5.8366$ million dollars

$B_1 = 0$, $B_2 = .9104$ million dollars

$B_3 = 1$ million dollars, $B_4 = 0$

(b) $B_1 = 0$, $S_1 = .2887$ million \$

$B_2 = .9104$, $S_2 = 0$

$B_3 = 1$, $S_3 = 0$

$B_4 = 0$, $S_4 = 1.2553$

The solution shows that $B_i \cdot S_i = 0$, meaning that you can't borrow and also end up with surplus in any quarter. The result makes sense because the cost of borrowing (2.5%) is higher than the return on surplus funds (2%)

4

Assume that the investment program ends at the start of year 11. Thus, the 6-year bond option can be exercised in years 1, 2, 3, 4, and 5 only. Similarly, the 9-year bond can be used in years 1 and 2 only. Hence, from year 6 on, the only option available is insured savings at 7.5%.

Let

I_i = insured savings investments in year i, $i = 1, 2, \ldots, 10$

G_i = 6-year bond investment in year i, $i = 1, 2, \ldots, 5$

M_i = 9-year bond investment in year i, $i = 1, 2$

The objective is to maximize total accumulation at the end of year 10; that is,

maximize $z = 1.075I_{10} + 1.079G_5 + 1.085M_2$

The constraints represent the balance equation for each year's cash flow.

$$I_1 + .98G_1 + 1.02M_1 = 2$$
$$I_2 + .98G_2 + 1.02M_2 = 2 + 1.075I_1 + .079G_1 + .085M_1$$
$$I_3 + .98G_3 = 2.5 + 1.075I_2 + .079(G_1 + G_2) + .085(M_1 + M_2)$$
$$I_4 + .98G_4 = 2.5 + 1.075I_3 + .079(G_1 + G_2 + G_3) + .085(M_1 + M_2)$$
$$I_5 + .98G_5 = 3 + 1.075I_4 + .079(G_1 + G_2 + G_3 + G_4) + .085(M_1 + M_2)$$
$$I_6 = 3.5 + 1.075I_5 + .079(G_1 + G_2 + G_3 + G_4 + G_5) + .085(M_1 + M_2)$$

continued...

Set 2.3c

$$I_7 = 3.5 + 1.075\, I_6 + 1.079\, G_1 + .079\,(G_2 + G_3 + G_4 + G_5) + .085\,(M_1 + M_2)$$

$$I_8 = 4 + 1.075\, I_7 + 1.079\, G_2 + .079\,(G_3 + G_4 + G_5) + .085\,(M_1 + M_2)$$

$$I_9 = 4 + 1.075\, I_8 + 1.079\, G_3 + .079\,(G_4 + G_5) + .085\,(M_1 + M_2)$$

$$I_{10} = 5 + 1.075\, I_9 + 1.079\, G_4 + .079\, G_5 + 1.085\, M_1 + .085\, M_2$$

all variables ≥ 0

*** OPTIMUM SOLUTION SUMMARY ***

Title: Problem 26a-14
Final iteration No: 14
Objective value (max) = 46.8500

Variable	Value	Obj Coeff	Obj Val Contrib
x1 I1	0.0000	0.0000	0.0000
x2 I2	0.0000	0.0000	0.0000
x3 I3	0.0000	0.0000	0.0000
x4 I4	0.0000	0.0000	0.0000
x5 I5	0.0000	0.0000	0.0000
x6 I6	4.6331	0.0000	0.0000
x7 I7	9.6137	0.0000	0.0000
x8 I8	15.4678	0.0000	0.0000
x9 I9	24.6663	0.0000	0.0000
x10 I10	37.5201	1.0750	40.3341
x11 G1	0.0000	0.0000	0.0000
x12 G2	0.0000	0.0000	0.0000
x13 G3	2.9053	0.0000	0.0000
x14 G4	3.1395	0.0000	0.0000
x15 G5	3.9028	1.0790	4.2111
x16 M1	1.9608	0.0000	0.0000
x17 M2	2.1242	1.0850	2.3047

Constraint	RHS	Slack(-)/Surplus(+)
1 (=)	2.0000	0.0000
2 (=)	2.0000	0.0000
3 (=)	2.5000	0.0000
4 (=)	2.5000	0.0000
5 (=)	3.0000	0.0000
6 (=)	3.5000	0.0000
7 (=)	3.5000	0.0000
8 (=)	4.0000	0.0000
9 (=)	4.0000	0.0000
10 (=)	5.0000	0.0000

Year	Recommendation
1	Invest all in 9-yr bond
2	Invest all in 9-yr. bond
3	Invest all in 6-yr bond
4	Invest all in 6-yr bond
5	Invest all in 6-yr bond
7	Invest all in insured savings
8	Invest all in insured savings
9	Invest all in insured savings
10	Invest all in insured savings

5

x_{iA} = amount invested in year i, plan A (1000\$)

x_{iB} = amount invested in year i, plan B (1000\$)

Maximize $z = 3x_{2B} + 1.7x_{3A}$

subject to

$$x_{1A} + x_{1B} \leq 100$$

$$-1.7x_{1A} + x_{2A} + x_{2B} = 0$$

$$-3x_{1B} - 1.7x_{2A} + x_{3A} = 0$$

$$x_{iA}, x_{iB} \geq 0 \text{ for } i = 1, 2, 3$$

*** OPTIMUM SOLUTION SUMMARY ***

Title: Problem 2.6a-15
Final iteration No: 4
Objective value (max) = 510.0000
==> ALTERNATIVE solution detected at x2

Variable	Value	Obj Coeff	Obj Val Contrib
x1 x1A	100.0000	0.0000	0.0000
x2 x1B	0.0000	0.0000	0.0000
x3 x2A	0.0000	0.0000	0.0000
x4 x2B	170.0000	3.0000	510.0000
x5 x3A	0.0000	1.7000	0.0000

Constraint	RHS	Slack(-)/Surplus(+)
1 (<)	100.0000	0.0000-
2 (<)	0.0000	0.0000-
3 (<)	0.0000	0.0000-

Optimum solution: Invest \$100,000 in A in yr 1 and \$170,000 in B in yr 2.
Alternative optimum: Invest \$100,000 in B in yr 1 and \$300,000 in A in yr 3.

6

x_i = dollars allocated to choice i, $i = 1, 2, 3, 4$

y = minimum return

$$\text{Maximize } z = \min \begin{cases} -3x_1 + 4x_2 - 7x_3 + 15x_4 \\ 5x_1 - 3x_2 + 9x_3 + 4x_4 \\ 3x_1 - 9x_2 + 10x_3 - 8x_4 \end{cases}$$

subject to

$$x_1 + x_2 + x_3 + x_4 \leq 500$$

$$x_1, x_2, x_3, x_4 \geq 0$$

The problem can be converted to a linear program as

continued...

Maximize $z = y$

subject to

$$-3x_1 + 4x_2 - 7x_3 + 15x_4 \geq y$$
$$5x_1 - 3x_2 + 9x_3 + 4x_4 \geq y$$
$$3x_1 - 9x_2 + 10x_3 - 8x_4 \geq y$$
$$x_1 + x_2 + x_3 + x_4 \leq 500$$
$$x_1, x_2, x_3, x_4 \geq 0$$
y unrestricted

*** OPTIMUM SOLUTION SUMMARY ***

Title:
Final iteration No: 5
Objective value (max) = 1175.0000

Variable	Value	Obj Coeff	Obj Val Contrib
x1	0.0000	0.0000	0.0000
x2	0.0000	0.0000	0.0000
x3	287.5000	0.0000	0.0000
x4	212.5000	0.0000	0.0000
x5 y	1175.0000	1.0000	1175.0000

Constraint	RHS	Slack(-)/Surplus(+)
1 (>)	0.0000	0.0000+
2 (>)	0.0000	2262.5000+
3 (>)	0.0000	0.0000+
4 (<)	500.0000	0.0000-

Allocate \$287.50 to choice 3 and \$212.50 to choice 4. Return = \$1175.00

7

$$i = \begin{cases} 1, & \text{regular savings} \\ 2, & \text{3-month CD} \\ 3, & \text{6-month CD} \end{cases}$$

x_{it} = Deposit in plan i at start of month t

$$t = \begin{cases} 1, 2, \dots, 12 & \text{if } i = 1 \\ 1, 2, \dots, 10 & \text{if } i = 2 \\ 1, 2, \dots, 7 & \text{if } i = 3 \end{cases}$$

y_1 = initial amount on hand to insure a feasible solution

r_i = interest rate for plan $i = 1, 2, 3$

$$J_i = \begin{cases} 12, & i = 1 \\ 10, & i = 2 \\ 7, & i = 3 \end{cases}$$

continued...

$$P_i = \begin{cases} 1, & i = 1 \\ 3, & i = 2 \\ 6, & i = 3 \end{cases}$$

d_t = \$ demand for period t

$$\text{Maximize } z = \sum_{\substack{t=1 \\ t-P_i>0}}^{12} \sum_{i=1}^{3} r_i x_{i,t-P_i} - y_1$$

s.t.

$$y_1 - x_{11} - x_{21} - x_{31} \geq d_1$$
$$1000 + \sum_{\substack{i=1 \\ t-P_i>0}}^{3} (1+r_i) x_{i,t-P_i} - \sum_{\substack{i=1 \\ t \leq J_i}}^{3} x_{it} \geq d_t, \quad t = 2, \dots, 12$$
$$x_{it}, y_1 \geq 0$$

Solution: (see file ampl2.3c-7.txt)

y_1 = \$1200, $z = -1136.29$

Interest amount = 1200 − 1136.29 = \$63.71

Deposits:

t	x_{1t}	x_{2t}	x_{3t}
1	0	0	0
2	0	200	0
3	286.48	313.53	0
4	0	587.43	0
5	314.37	289.30	0
6	0	734.69	0
7	0	98.20	0
8	0	294.60	
9	0	848.16	
10	0	0	
11	0		
12	0		

Set 2.3d

1

x_{W1} = # wrenches/wk using regular time
x_{W2} = # wrenches/wk using overtime
x_{W3} = # wrenches/wk using subcontracting
x_{C1} = # chisels/wk using regular time
x_{C2} = # chisels/wk using overtime
x_{C3} = # chisels/wk using subcontracting

$$\text{Minimize } z = 2x_{W1} + 2.8x_{W2} + 3x_{W3} + 2.1x_{C1} + 3.2x_{C2} + 4.2x_{C3}$$

Subject to

$$x_{W1} \le 550,\ x_{W2} \le 250$$

$$x_{C1} \le 620,\ x_{C2} \le 280$$

$$\frac{x_{C1} + x_{C2} + x_{C3}}{x_{W1} + x_{W2} + x_{W3}} \ge 2$$

or

$$2x_{W1} + 2x_{W2} + 2x_{W3} - x_{C1} - x_{C2} - x_{C3} \le 0$$

$$x_{W1} + x_{W2} + x_{W3} \ge 1500$$

$$x_{C1} + x_{C2} + x_{C3} \ge 1200$$

all variables ≥ 0

(a) Optimum from TORA:

$$x_{W1} = 550,\ x_{W2} = 250,\ x_{W3} = 700$$

$$x_{C1} = 620,\ x_{C2} = 280,\ x_{C3} = 2100$$

$$z = \$14{,}918$$

(b) Increasing marginal cost ensures that regular time capacity is used before that of overtime, and that overtime capacity is used before that of subcontracting. If the marginal cost function is not monotonically increasing, additional constraints are needed to ensure that the capacity restriction is satisfied.

continued...

2

x_j = number of units produced of product j, $j = 1, 2, 3, 4$

<u>Profit per unit</u>:

Product 1 $= 75 - 2\times10 - 3\times5 - 7\times4 = \$12$
Product 2 $= 70 - 3\times10 - 2\times5 - 3\times4 = \$18$
Product 3 $= 55 - 4\times10 - 1\times5 - 2\times4 = \$2$
Product 4 $= 45 - 2\times10 - 2\times5 - 1\times4 = \$11$

$$\text{Maximize } z = 12x_1 + 18x_2 + 2x_3 + 11x_4$$

s.t.

$$2x_1 + 3x_2 + 4x_3 + 2x_4 \le 500$$

$$3x_1 + 2x_2 + x_3 + 2x_4 \le 380$$

$$7x_1 + 3x_2 + 2x_3 + x_4 \le 450$$

$$x_1, x_2, x_3, x_4 \ge 0$$

TORA Solution:

$$x_1 = 0,\ x_2 = 133.33,\ x_3 = 0,\ x_4 = 50$$

$$z = \$2950$$

3

x_j = number of units of model j

$$\text{Maximize } z = 30x_1 + 20x_2 + 50x_3$$

Subject to

① $2x_1 + 3x_2 + 5x_3 \le 4000$

② $4x_1 + 2x_2 + 7x_3 \le 6000$

③ $x_1 + .5x_2 + \frac{1}{3}x_3 \le 1500$

④ $\frac{x_1}{3} = \frac{x_2}{2}$, or $2x_1 - 3x_2 = 0$

⑤ $\frac{x_2}{2} = \frac{x_3}{5}$, or $5x_2 - 2x_3 = 0$

$$x_1 \ge 200,\ x_2 \ge 200,\ x_3 \ge 150$$

*** OPTIMUM SOLUTION SUMMARY ***

Title: Problem 2.6a-12
Final iteration No: 4
Objective value (max) =41081.0820

Variable	Value	Obj Coeff	Obj Val Contrib
x1	324.3243	30.0000	9729.7305
x2	216.2162	20.0000	4324.3242
x3	540.5405	50.0000	27027.0273

Constraint	RHS	Slack(-)/Surplus(+)
1 (<)	4000.0000	0.0000-
2 (<)	6000.0000	486.4865-
3 (<)	1500.0000	887.3875-
4 (=)	0.0000	0.0000
5 (=)	0.0000	0.0000
LB-x1	200.0000	124.3243+
LB-x2	200.0000	16.2162+
LB-x3	150.0000	390.5405+

4

x_{ij} = Nbr. cartons in month i from supplier j

I_i = End inventory in period i, $I_0 = 0$

c_{ij} = Price per unit of x_{ij}

h = Holding cost/unit/month

C = Supplier capacity/month

d_i = Demand for month i

$i = 1, 2, 3, \quad j = 1, 2$

$$\text{Minimize } z = \sum_{i=1}^{3}\sum_{j=1}^{2} c_{ij}\, x_{ij} + \frac{h}{2}\left(\sum_{i=1}^{3}\left(\sum_{j=1}^{2} x_{ij} + I_{i-1} + I_i\right)\right)$$

s.t. $x_{ij} \le C$, all i and j

$$\sum_{j=1}^{2} x_{ij} + I_{i-1} - I_i = d_i, \text{ all } i$$

Optimum solution:

i	x_{i1}	x_{i2}	I
1	400	100	0
2	400	400	200
3	200	0	0

Total cost = \$167,450.

6

x_{ij} = Qty of product i in month j, $i = 1, 2$, $j = 1, 2, 3$

I_{ij} = End inventory of product i in month j

$$\text{Minimize } z = 30(x_{11}+x_{12}+x_{13}) + 28(x_{21}+x_{22}+x_{23}) + .9(I_{11}+I_{12}+I_{13}) + .75(I_{21}+I_{22}+I_{23})$$

s.t.

$$(x_{1j}/1.25) + x_{2j} \le \begin{cases} 3000, & j=1 \\ 3500, & j=2 \\ 3000, & j=3 \end{cases}$$

$$I_{1,j-1} + x_{1j} - I_{1j} = \begin{cases} 500, & j=1 \\ 5000, & j=2 \\ 750, & j=3 \end{cases}$$

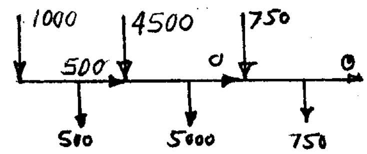
$$I_{2,j-1} + x_{2j} - I_{2j} = \begin{cases} 1000, & j=1 \\ 1200, & j=2 \\ 1200, & j=3 \end{cases}$$

$I_{i0} = 0, \; i = 1, 2$

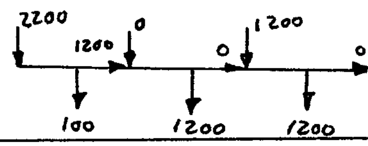
$x_{ij}, \; I_{ij} \ge 0$

Optimum solution: Cost = \$284,050

Product 1:



Product 2:



5

x_i = Production amount in quarter i

I_i = End inventory for quarter i

$$\text{Minimize } z = 20x_1 + 22x_2 + 24x_3 + 26x_4 + 3.5(I_1 + I_2 + I_3)$$

s.t.

$x_1 = 300 + I_1$

$I_1 + x_2 = 400 + I_2$

$I_2 + x_3 = 450 + I_3$

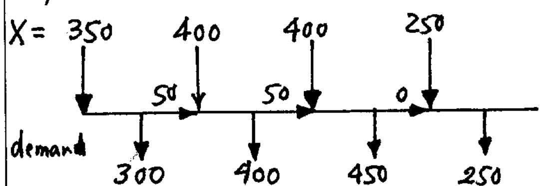
$I_3 + x_4 = 250$

$x_i \le 400, \; i = 1,2,3,4$

$I_i \le 100, \; i = 1,2,3$

$I_0 = I_4 = 0$

Optimum solution:



Total cost = \$32,250

7

x_{ij} = Qty by operation i in month j

$i = 1, 2, \quad j = 1, 2, 3$

$$\text{Minimize } z = .2\sum_{j=1}^{3} I_{1j} + .4\sum_{j=1}^{3} I_{2j} + 10x_{11} + 12x_{12} + 11x_{13} + 15x_{21} + 18x_{22} + 16x_{23}$$

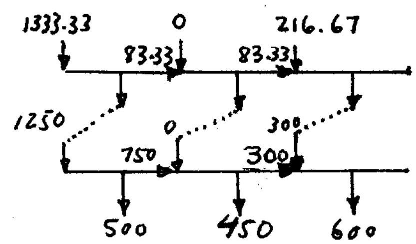
$.6x_{11} \le 800$, $.6x_{12} \le 700$, $.6x_{13} \le 550$

$.8x_{21} \le 1000$, $.8x_{22} \le 850$, $.8x_{23} \le 700$

$$\left.\begin{aligned} x_{1j} + I_{1,j-1} &= x_{2j} + I_{1j} \\ x_{2j} + I_{2,j-1} &= I_{2j} + d_j \end{aligned}\right\} j = 1, 2, 3$$

$I_{i0} = 0, \; i = 1, 2$

Solution: Cost = \$39,720



I_{ij} = Entering inv. of op. i in month j

8

x_j = Units of product j, $j = 1, 2$

y_i^- = Unused hours of machine i $\Big\}$ $i = 1, 2$

y_i^+ = Overtime hours of machine i

Maximize $z = 110x_1 + 118x_2 - 100(y_1^+ + y_2^+)$

s.t.

$$\frac{x_1}{5} + \frac{x_2}{5} + y_1^- - y_1^+ = 8$$

$$\frac{x_1}{8} + \frac{x_2}{4} + y_2^- - y_2^+ = 8$$

$$y_1^+ \le 4, \quad y_2^+ \le 4$$

$$x_1, x_2, y_1^-, y_1^+, y_2^-, y_2^+ \ge 0$$

<u>Solution</u>:

Revenue = \$6,232

$x_1 = 56$, $y_1^+ = 4$ hrs

$x_2 = 4$, $y_2^+ = 0$

$y_1^-, y_2^- = 0$

1

x_s = tons of strawberry / day
x_g = tons of grapes / day
x_a = tons of apples / day

x_A = cans of drink A / day
x_B = cans of drink B / day
x_C = cans of drink C / day
(Each can holds one lb)

x_{sA} = lb of strawberry used in drink A / day
x_{sB} = lb of strawberry used in drink B / day
x_{gA} = lb of grapes used in drink A / day
x_{gB} = lb of grapes used in drink B / day
x_{gC} = lb of grapes used in drink C / day
x_{aB} = lb of apples used in drink B / day
x_{aC} = lb of apples used in drink C / day

Maximize $z = 1.15x_A + 1.25x_B + 1.2x_C - 200x_s - 100x_g - 90x_a$

s.t.

$x_s \le 200,\ x_g \le 100,\ x_a \le 150$
$x_{sA} + x_{sB} = 1500x_s$
$x_{gA} + x_{gB} + x_{gC} = 1200x_g$
$x_{aB} + x_{aC} = 1000x_a$
$x_A = x_{sA} + x_{gA}$
$x_B = x_{sB} + x_{gB} + x_{aB}$
$x_C = x_{gC} + x_{aC}$
$x_{sA} = x_{gA}$,
$x_{sB} = x_{gB}$, $x_{gB} = .5x_{aB}$
$3x_{gC} = 2x_{aC}$
all variables ≥ 0

Optimum solution:

$x_A = 90,000$ cans, $x_B = 300,000$ cans, $x_C = 0$

x_{ij}:

i \ j	A	B	C
s	45,000	75,000	0
g	45,000	75,000	0
a	0	150,000	0
	90,000	300,000	0

$x_s = 80$ tons, $x_g = 100$ tons, $x_a = 150$ tons
$z = \$439,000/\text{day}$

2

x_s = lb of screws per package
x_b = lb of bolts per package
x_n = lb of nuts per package
x_w = lb of washers per package

Minimize $z = 1.1x_s + 1.5x_b + \frac{70}{80}x_n + \frac{20}{30}x_w$

s.t. $Y = x_s + x_b + x_n + x_w$
$x_s \ge .1Y$
$x_b \ge .25Y,\ \frac{x_b}{50} \le x_w,\ \frac{x_b}{10} \le x_n$
$x_n \le .15Y$
$x_w \le .1Y$
$Y \ge 1$

All variables are nonnegative

Optimum solution:

$Y = 1,\ x_s = .5,\ x_b = .25,\ x_n = .15,\ x_w = .1$
Cost = \$1.12

3

$x_{o,(A,B,C)}$ = lb of oats in cereals A, B, C
$x_{r,(A,C)}$ = lb of raisins in cereals A, C
$x_{c,(B,C)}$ = lb of coconuts in cereals B, C
$x_{a,(A,B,C)}$ = lb of almond in cereals A, B, C

$Y_o = x_{oA} + x_{oB} + x_{oC}$
$Y_r = x_{rA} + x_{rC}$
$Y_c = x_{cB} + x_{cC}$
$Y_a = x_{aA} + x_{aB} + x_{aC}$
$W_A = x_{oA} + x_{rA} + x_{aA}$
$W_B = x_{oB} + x_{cB} + x_{aB}$
$W_C = x_{oC} + x_{rC} + x_{cC} + x_{aC}$

Maximize $z = \frac{1}{5}(2W_A + 2.5W_B + 3W_C) - \frac{1}{2000}(100Y_o + 120Y_r + 110Y_c + 200Y_n)$

s.t. $W_A \le 500 \times 5 = 2500$
$W_B \le 600 \times 5 = 3000$
$W_C \le 500 \times 5 = 4000$

continued...

Set 2.3e

$Y_o \le 5 \times 2000 = 10{,}000$

$Y_r \le 2 \times 2000 = 4{,}000$

$Y_c \le 1 \times 2000 = 2{,}000$

$Y_a \le 1 \times 2000 = 2{,}000$

$X_{oA} = \frac{50}{5} X_{rA}$, $X_{oA} = \frac{50}{2} X_{aA}$

$X_{oB} = \frac{60}{2} X_{cB}$, $X_{oB} = \frac{60}{3} X_{aB}$

$X_{oC} = \frac{60}{3} X_{rC}$, $X_{oC} = \frac{60}{4} X_{cC}$, $X_{oC} = \frac{60}{2} X_{aC}$

all variables are nonnegative.

Optimum solution: $Z = \$5384.84/\text{day}$

$W_A = 2500$ lb or 500 boxes/day

$W_B = 3000$ lb or 600 boxes

$W_C = 5793.45$ lb or $\simeq 1158$ boxes

$X_o = 10{,}000$ lb or 5 tons/day

$X_r = 471.19$ lb or .236 ton

$X_c = 428.16$ lb or .214 ton

$X_a = 394.11$ lb or .197 ton

4

X_{Ai} = bbl of gasoline A in fuel i

X_{Bi} = bbl of gasoline B in fuel i

X_{Ci} = bbl of gasoline C in fuel i

X_{Di} = bbl of gasoline D in fuel i

$i = 1, 2$

$Y_A = X_{A1} + X_{A2}$

$Y_B = X_{B1} + X_{B2}$

$Y_C = X_{C1} + X_{C2}$

$Y_D = X_{D1} + X_{D2}$

$F_1 = X_{A1} + X_{B1} + X_{C1} + X_{D1}$

$F_2 = X_{A2} + X_{B2} + X_{C2} + X_{D2}$

Maximize $Z = 200F1 + 250F2 - (120Y_A + 90Y_B + 100Y_C + 150Y_D)$

s.t.

$X_{A1} = X_{B1}$, $X_{A1} = .5X_{C1}$, $X_{A1} = .25X_{D1}$

$X_{A2} = X_{B2}$, $X_{A2} = 2X_{C2}$, $X_{A2} = \frac{2}{3}X_{D2}$

$Y_A \le 1000$, $Y_B \le 1200$, $Y_C \le 900$, $Y_D \le 1500$

$F_1 \ge 200$, $F_2 \ge 400$

Optimum solution: $Z = \$495{,}416.67$

$Y_A = 958.33$ bbl/day

$Y_B = 958.33$ bbl/day

$Y_C = 516.67$ bbl/day

$Y_D = 1500$ bbl/day

$F1 = 200$ bbl/day

$F_2 = 3733.33$ bbl/day

5

A = bbl of crude A/day

B = bbl of crude B/day

R = bbl of regular gasoline/day

P = bbl of premium gasoline/day

J = bbl of jet gasoline/day

Maximize $Z = 50(R - R^+) + 70(P - P^+) + 120(J - J^+) - (10R^- + 15P^- + 20J^-) - (2R^+ + 3P^+ + 4J^+) - (30A + 40B)$

s.t.

$A \le 2500$, $B \le 3000$

$R = .2A + .25B$, $R + R^- - R^+ = 500$

$P = .1A + .3B$, $P + P^- - P^+ = 700$

$J = .25A + .1B$, $J + J^- - J^+ = 400$

All variables ≥ 0

Optimum solution:

$Z = \$21{,}852.94$

$A = 1176.47$ bbl/day

$B = 1058.82$ bbl/day

$R = 500$ bbl/day

$P = 435.29$ bbl/day

$J = 400$ bbl/day

continued...

6

NR = bbl/day of naphta used in regular
NP = bbl/day of naphta used in premium
NJ = bbl/day of naphta used in jet
LR = bbl/day of light used in regular
LP = bbl/day of light used in premium
LJ = bbl/day of light used in jet

Using the other notation in Problem 5,

$$\text{Maximize } z = 50(R - R^+) + 70(P - P^+) + 120(J - J^+) - (10R^- + 15P^- + 20J^-) - (2R^+ + 3P^+ + 4J^+) - (30A + 40B)$$

s.t.

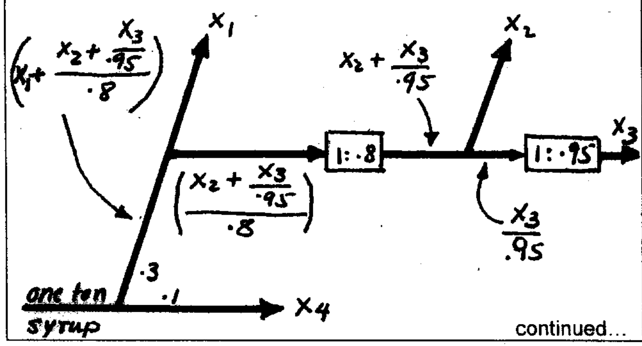
$$A \le 2500, \quad B \le 3000$$
$$R + R^- - R^+ = 500$$
$$P + P^- - P^+ = 700$$
$$J + J^- - J^+ = 400$$
$$.35A + .45B = NR + NP + NJ$$
$$.6A + .5B = LR + LP + LJ$$
$$R = NR + LR$$
$$P = NP + LP$$
$$J = NJ + LJ$$

all variables are nonnegative

Optimum solution: $z = \$71,473.68$
$A = 1684.21$, $B = 0$
$R = 500$, $P = 700$, $J = 400$

7

x_1 = tons of brown sugar per week
x_2 = tons of white sugar per week
x_3 = tons of powdered sugar per week
x_4 = tons of molasses per week



continued...

$$\text{Maximize } Z = 150x_1 + 200x_2 + 230x_3 + 35x_4$$

s.t.

$$x_4 \le 4000 \times .1$$
or
$$x_4 \le 400$$
$$x_1 + \left(\frac{x_2 + \frac{x_3}{.95}}{.8}\right) \le .3 \times 4000$$
or
$$.76x_1 + .95x_2 + x_3 \le 912$$
$$x_1 \ge 25, \ x_2 \ge 25$$
$$x_3 \ge 25, \ x_4 \ge 0$$

Optimum solution from TORA:
$x_1 = 25$ tons per week
$x_2 = 25$ tons per week
$x_3 = 869.25$ tons per week
$x_4 = 400$ tons per week
$Z = \$222,677.50$

8

A = bbl/hr of stock A
B = bbl/hr of stock B
Y_{Ai} = bbl/hr of A used in gasoline i
Y_{Bi} = bbl/hr of B used in gasoline i } $i = 1, 2$

$$\text{Maximize } z = 7(Y_{A1} + Y_{B1}) + 10(Y_{A2} + Y_{B2})$$

s.t.

$$A = Y_{A1} + Y_{A2}, \quad A \le 450$$
$$B = Y_{B1} + Y_{B2}, \quad B \le 700$$
$$98Y_{A1} + 89Y_{B1} \ge 91(Y_{A1} + Y_{B1})$$
$$98Y_{A2} + 89Y_{B2} \ge 93(Y_{A2} + Y_{B2})$$
$$10Y_{A1} + 8Y_{B1} \le 12(Y_{A1} + Y_{B1})$$
$$10Y_{A2} + 8Y_{B2} \le 12(Y_{A2} + Y_{B2})$$

all variables are nonnegative

Optimum solution:
$Z = \$10,675$
$A = 450$ bbl/hr
$B = 700$ bbl/hr
Gasoline 1 production $= Y_{A1} + Y_{B1} = 61.11 + 213.89 = 275$ bbl/hr
Gasoline 2 production $= Y_{A2} + Y_{B2} = 388.89 + 486.11 = 875$ bbl/hr

Set 2.3e

9

S = tons of steel scrap / day
A = tons of alum. scrap / day
C = tons of cast iron scrap / day
Ab = tons of alum. briquettes / day
Sb = tons silicon briquettes / day
a = tons of alum. / day
g = tons of graphite / day
s = tons of silicon / day
aI = tons of alum. in ingot I / day
aII = tons of alum. in ingot II / day
gI = tons of graphite in ingot I / day
gII = tons of graphite in ingot II / day
sI = tons of silicon in ingot I / day
sII = tons of silicon in ingot II / day
I_1 = tons of ingot I / day
I_2 = tons of ingot II / day.

Minimize $Z = 100S + 150A + 75C + 900Ab + 380Sb$

s.t. $S \le 1000$, $A \le 500$, $C \le 2500$

$$a = .1S + .95A + Ab$$
$$g = .05S + .01A + .15C$$
$$s = .04S + .02A + .08C + S_b$$
$$I_1 = aI + gI + sI$$
$$I_2 = aII + gII + sII$$
$$a_I + a_{II} \le a,\ sI + sII \le s,\ gI + gII \le g$$
$$.081 I_1 \le aI \le .108 I_1$$
$$.015 I_1 \le gI \le .03 I_1$$
$$.025 I_1 \le sI < \infty$$
$$.062 I_2 \le aII \le .089 I_2$$
$$.041 I_2 \le gII \le \infty$$
$$.028 I_2 \le sII \le .041 I_2$$
$$I_1 \ge 130,\quad I_2 \ge 250$$

Optimum solution:

$$Z = \$117{,}435.65$$
$$S = 0,\ A = 38.2,\ C = 1489.41$$
$$Ab = Sb = 0$$
$$I_1 = 130,\quad I_2 = 250$$
$$a = 36.29,\ g = 223.79,\ s = 119.92$$

10

x_{ij} = tons of ore i allocated to alloy k
w_k = tons of alloy k produced

Maximize $Z = 200\, w_A + 300\, w_B$
$- 30(x_{1A} + x_{1B})$
$- 40(x_{2A} + x_{2B})$
$- 50(x_{3A} + x_{3B})$

Subject to

Specification constraints:

$$.2 x_{1A} + .1 x_{2A} + .05 x_{3A} \le .8 w_A \quad (1)$$
$$.1 x_{1A} + .2 x_{2A} + .05 x_{3A} \le .3 w_A \quad (2)$$
$$.3 x_{1A} + .3 x_{2A} + .2 x_{3A} \ge .5 w_A \quad (3)$$
$$.1 x_{1B} + .2 x_{2B} + .05 x_{3B} \ge .4 w_B \quad (4)$$
$$.1 x_{1B} + .2 x_{2B} + .05 x_{3B} \le .6 w_B \quad (5)$$
$$.3 x_{1B} + .3 x_{2B} + .7 x_{3B} \ge .3 w_B \quad (6)$$
$$.3 x_{1B} + .3 x_{2B} + .2 x_{3B} \le .7 w_B \quad (7)$$

Ore constraints:

$$x_{1A} + x_{1B} \le 1000$$
$$x_{2A} + x_{2B} \le 2000$$
$$x_{3A} + x_{3B} \le 3000$$

```
*** OPTIMUM SOLUTION SUMMARY ***
Title: Problem 26a-17
Final iteration No: 12
Objective value (max) =400000.0000
```

Variable	Value	Obj Coeff	Obj Val Contrib
x1 wA	1799.9999	200.0000	359999.9688
x2 wB	1000.0001	300.0000	300000.0312
x3 x1A	1000.0000	-30.0000	-30000.0000
x4 x1B	0.0000	-30.0000	-0.0000
x5 x2A	0.0000	-40.0000	-0.0000
x6 x2B	2000.0001	-40.0000	-80000.0078
x7 x3A	3000.0000	-50.0000	-150000.0000
x8 x3B	0.0000	-50.0000	-0.0000

Constraint	RHS	Slack(-)/Surplus(+)
1 (<)	0.0000	1090.0000-
2 (<)	0.0000	290.0000-
3 (>)	0.0000	0.0000+
4 (>)	0.0000	0.0000+
5 (<)	0.0000	200.0000-
6 (>)	0.0000	300.0002+
7 (<)	0.0000	100.0000-
8 (<)	1000.0000	0.0000-
9 (<)	2000.0000	0.0000-
10 (<)	3000.0000	0.0000-

Solution:

Produce 1800 tons of alloy A and 1000 tons of alloy B.

1

h = Regular pay hour

8-hr pay = $8h$

12-hr pay = $12h + \frac{4h}{2} = 14h$

x_i = Nbr 8-hr buses starting in period i

y_i = Nbr. of 12-hr buses starting in period i

Minimize $z = h\left(8\sum_{i=1}^{6} x_i + 14\sum_{i=1}^{6} y_i\right)$

s.t.

x_1	x_2	x_3	x_4	x_5	x_6	y_1	y_2	y_3	y_4	y_5	y_6	
1					1	1				1	1	≥ 4
1	1					1	1				1	≥ 8
	1	1				1	1	1				≥ 10
		1	1				1	1	1			≥ 7
			1	1				1	1	1		≥ 12
				1	1				1	1	1	≥ 4

Solution: $z = 196h$

$x_1 = 4,\ x_2 = 4,\ x_4 = 2,\ x_5 = 4,\ x_3 = x_6 = 0$

$y_3 = 6,\ y_1 = y_2 = y_4 = y_5 = y_6 = 0$

For 8-hr only buses, solution is

$z = 208h$

$x_1 = x_2 = 4,\ x_3 = 6,\ x_4 = 1,\ x_5 = 11,\ x_6 = 0$

(8-hr + 12-hr) buses is cheaper.

2

x_i = Nbr. of volunteers starting in hour i

Minimize $z = \sum_{i=1}^{14} x_i$

s.t.

(8:00) $x_1 \geq 4$

(9:00) $x_1 + x_2 \geq 4$

(10:00) $x_1 + x_2 + x_3 \geq 6$

(11:00) $x_2 + x_3 + x_4 \geq 6$

(12:00) $x_3 + x_4 + x_5 \geq 8$

(1:00) $x_4 + x_5 + x_6 \geq 8$

(2:00) $x_5 + x_6 + x_7 \geq 6$

(3:00) $x_6 + x_7 + x_8 \geq 6$

(4:00) $x_7 + x_8 + x_9 \geq 4$

(5:00) $x_8 + x_9 + x_{10} \geq 4$

(6:00) $x_9 + x_{10} + x_{11} \geq 6$

(7:00) $x_{10} + x_{11} + x_{12} \geq 6$

(8:00) $x_{11} + x_{12} + x_{13} \geq 8$

(9:00) $x_{12} + x_{13} \geq 8$

All $x_j \geq 0$

continued...

Solution: z = 32 volunteers

$x_1 = 4,\ x_3 = 2,\ x_4 = 6,\ x_6 = 2,\ x_7 = 4,\ x_{10} = 6,\ x_{12} = 8$

all other $x_i = 0$

3

Same formulation as in Problem 2 with the added constraints $x_5 = 0,\ x_{11} = 0$

Optimum solution remains the same

4

x_i = Nbr. of casuals starting on day i

($i = 1$: Monday, $i = 7$: Sunday)

Minimize $z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$

s.t.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
M	1			1	1	1	1	≥ 20
T	1	1			1	1	1	≥ 14
W	1	1	1			1	1	≥ 10
Th	1	1	1	1			1	≥ 15
F	1	1	1	1	1			≥ 18
Sat		1	1	1	1	1		≥ 10
Sun			1	1	1	1	1	≥ 12

Solution: z = 20 workers

$x_1 = 8,\ x_4 = 6,\ x_5 = 4,\ x_6 = 1,\ x_7 = 1$

5

x_i = Nbr. Students starting at hour i

$i = 1$ (8:01), $i = 9$ (4:01), $x_5 = 0$

Minimize $z = x_1 + x_2 + x_3 + x_4 + x_6 + x_7 + x_8 + x_9$

s.t.

	x_1	x_2	x_3	x_4	x_6	x_7	x_8	x_9	
8:01	1								≥ 2
9:01	1	1							≥ 2
10:01	1	1	1						≥ 3
11:01		1	1	1					≥ 4
12:01			1	1					≥ 4
1:01				1	1				≥ 3
2:01					1	1			≥ 3
3:01					1	1	1		≥ 3
4:01						1	1	1	≥ 3

Solution: z = 9 students

$x_1 = 2,\ x_3 = 1,\ x_4 = 3,\ x_7 = 3$

Set 2.3f

6

Let x_i = Nbr. starting on day i and lasting for 7 days

y_{ij}= Nbr. starting shift on day i and *starting* their 2 days off on day j, $i \neq j$

Thus, of the x_1 workers who start on Monday, y_{12} will take T and W off, y_{13} will take W and Th off, and so on, as the following table shows.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
1	**start on Mon**	y_{12}	$y_{12}+y_{13}$	$y_{13}+y_{14}$	$y_{14}+y_{15}$	$y_{15}+y_{16}$	y_{16}
2	y27	**Tue**	y23	y23+y24	y24+y25	y25+y26	y26+y27
3	y31+y37	y31	**Wed**	y34	y34+y35	y35+y36	y36+y37
4	y41+y47	y41+y42	y42	**Th**	y45	y45+y46	y46+y47
5	y51+y57	y51+y52	y52+y53	y53	**Fri**	y56	y56+y57
6	y61+y67	y61+y62	y62+y63	y63+y64	y64	**Sat**	y67
7	y71	y71+y72	y72+y73	y73+y74	y74+y75	y75	**Su**

Minimize $z = x_1+x_2+x_3+ x_4+x_5+x_6+x_7$

Each employee has 2 days off: x_i = sum{j in 1..7,j≠i}y_{ij}

Mon (1) constraint: s -(y27 + y31+ y37 + y41+y47+ y51+ y57 + y61+y67 + y71)>= 12

Tue (2) constraint: s -(y12 + y31 + y41+ y42 + y51+y52 + y61+y62 + y71+y72>= 18

Wed (3) constraint: s -(y12+y13 + y23 + y42 + y52+y53 + y62+y63 + y72+y73>= 20

Th (4) constraint: s -(y13+y14 + y23 +y24+ y24 + y53 + y63+y64 + y73+y74>= 28

Fri (5) constraint: s -(y14+y15 + y24 +y25+ y34 +y35 + y45+ y64 + y74+y75>= 32

Sat(6) constraint: s -(y15+y16+ y25 +y26+ y35 + y36 + y45+y46+ y56+y75>= 40

Sun(7) constraint: s -(y16+ y26 +y27+ y36 + y37 +y46+y47+ y56+y57+y67)>= 40

continued

Solution: 42 employees

Starting		Nbr off						
On	Nbr	M	Tu	Wed	Th	Fri	Sat	Sun
M	16		16	16				
Tu	8				8	8		
Wed	8	8	8					
Th	0							
Fri	6			6	6			
Sat	2	2						2
Sun	2					2	2	
Nbr off		10	24	22	14	10	2	2
Nbr at work		32	18	20	28	32	40	40
Surplus above minimum		20	0	0	0	0	0	0

Set 2.3g

1

	Setting 1	Setting 3	Number produced	Surplus rolls
5'	0	2	200	50
7'	1	0	200	0
9'	1	1	300	0
Loss/ foot	4	1		
No. rolls	200	100		

Trim loss area =

$L(200\times 4 + 100\times 1 + 50\times 5) = 1150L\ ft^2$

(b) 15' standard roll:

	Setting 1	2	3	4
5'	3	1	1	0
7'	0	1	0	2
9'	0	0	1	0
trim loss per ft	0	3	1	1

(c) $x_1 + x_2 + 2x_5 \geq 120$

New solution calls for decreasing the number of standard 20'-rolls by 30

(d) $x_1 + x_3 + 2x_6 \geq 240$

New solution calls for increasing the number of standard 20'-rolls by 50

2

x_i = Space(in^2) allocated to cereal i

Maximize $z = 1.1x_1 + 1.3x_2 + 1.08x_3 + 1.25x_4 + 1.2x_5$

s.t.

$16x_1 + 24x_2 + 18x_3 + 22x_4 + 20x_5 \leq 5000$

$x_1 \leq 100,\ x_2 \leq 85,\ x_3 \leq 140,\ x_4 \leq 80,\ x_5 \leq 90$

$x_i \geq 0$ for all $i = 1, 2, .., 5$

Solution:

$z = \$\ 314/\text{day}$

$x_1 = 100,\ x_3 = 140,\ x_5 = 44$

$x_2 = x_4 = 0$

3

x_i = Nbr. of ads for issue i, $i = 1, 2, 3, 4$

Minimize $z = S_1^- + S_2^- + S_3^- + S_4^-$

s.t.

$(-30,000 + 60,000 + 30,000)x_1 + S_1^- - S_1^+ = .51\times 400,000$

$(80,000 + 30,000 - 45,000)x_2 + S_2^- - S_2^+ = .51\times 400,000$

$(40,000 + 10,000)x_3 + S_3^- - S_3^+ = .51\times 400,000$

$(90,000 - 25,000)x_4 + S_4^- - S_4^+ = .51\times 400,000$

$1500(x_1 + x_2 + x_3 + x_4) \leq 100,000$

$x_1, x_2, x_3, x_4 \geq 0$

Solution:

$x_1 = 3.4,\ x_2 = 3.14,\ x_3 = 4.08,\ x_4 = 3.14$

4

x_{ij} = Units of part j produced by department i, $i = 1, 2, 3$, $j = 1, 2$

Maximize $z = \min\{x_{11} + x_{21},\ x_{12} + x_{22},\ x_{13} + x_{23}\}$

or

Maximize $z = y$

s.t.

$y \leq x_{11} + x_{21}$

$y \leq x_{12} + x_{22}$

$y \leq x_{13} + x_{23}$

$\frac{x_{11}}{8} + \frac{x_{12}}{5} + \frac{x_{13}}{10} \leq 100$

$\frac{x_{21}}{6} + \frac{x_{22}}{12} + \frac{x_{23}}{4} \leq 80$

all $x_{ij} \geq 0$

Solution:

Nbr. of assembly units $= y = 556.2 \simeq 557$

$x_{11} = 354.78,\ x_{12} = 0$

$x_{21} = 556.52,\ x_{22} = 201.74$

$x_{31} = 556.52,\ x_{32} = 0$

5

x_i = tons of coal i, $i = 1, 2, 3$

Minimize $z = 30x_1 + 35x_2 + 33x_3$

s.t.

$2500x_1 + 1500x_2 + 1600x_3 \leq 2000(x_1 + x_2 + x_3)$

$x_1 \leq 30,\ x_2 \leq 30,\ x_3 \leq 30$

$x_1 + x_2 + x_3 \geq 50$

Solution: $z = \$1361.11$

$x_1 = 22.22$ tons, $x_2 = 0$, $x_3 = 27.78$ tons.

6

t_i = Green time in secs for highway i, $i = 1, 2, 3$

Maximize $z = 3\left(\frac{500}{3600}\right)t_1 + 4\left(\frac{600}{3600}\right)t_2 + 5\left(\frac{400}{3600}\right)t_3$

s.t.

$\left(\frac{500}{3600}\right)t_1 + \left(\frac{600}{3600}\right)t_2 + \left(\frac{400}{3600}\right)t_3 \le \frac{510}{3600}\left(2.2\times 60 - 3\times 10\right)$

$t_1 + t_2 + t_3 + 3\times 10 \le 2.2\times 60$, $t_1 \ge 25, t_2 \ge 25, t_3 \ge 25$

Solution: $z = \$58.04/hr$

$t_1 = 25$, $t_2 = 43.6$, $t_3 = 33.4$ Sec

7

y_i = observation i

Define straight line as

$\hat{y}_i = a\, i + b$, a, b unrestricted

$$\text{Minimize } z = \sum_{i=1}^{10} |y_i - \hat{y}_i| = \sum_{i=1}^{10} |y_i - ai - b|$$

Let $d_i = |y_i - ai - b|$

Minimize $z = d_1 + d_2 + \cdots + d_{10}$

s.t.

$y_i - ai - b \le d_i$

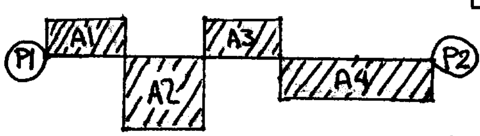
$y_i - ai - b \ge -d_i$

a, b, unrestricted

$d_i \ge 0$

Solution: $\hat{y}_i = 2.85714\, i + 6.42857$

8



$A1 = 2\times 1760\times 10\times 50 = 1760$ (thousand) Yd^3

$A2 = 3520$, $A3 = 1760$, $A4 = 3520$

Distances (center to center) in miles:

$$\begin{array}{c|cc} & A2 & A4 \\ \hline A1 & 2 & 7 \\ A3 & 2 & 3 \\ P1 & 3 & 8 \\ P2 & 7 & 2 \end{array}$$

continued…

Cost ($) per cubic yd:

$$\begin{array}{l|cc} & (5)\ A2 & (6)\ A4 \\ \hline (1)\ A1 & .2+2\times.15 = .50 & .20+7\times.15 = 1.25 \\ (2)\ A3 & .20+2\times.15 = .50 & .20+3\times.15 = .65 \\ (3)\ P1 & 1.70+3\times.15 = 2.15 & 1.70+8\times.15 = 2.90 \\ (4)\ P3 & 2.10+7\times.15 = 3.15 & 2.10+2\times.15 = 2.40 \end{array}$$

Using the code $A1 \equiv 1$, $A3 \equiv 2$, $P1 \equiv 3$, $P2 \equiv 4$, $A2 \equiv 5$, $A4 \equiv 6$, let

$x_{ij} = 10^3 Yd^3$ from source i to destination j

$i = 1, 2, 3, 4$, $j = 5, 6$

Minimize $z = 1000(.5x_{15} + 1.25x_{16} + .5x_{25} + .65x_{26} + 2.15x_{35} + 2.9x_{36} + 3.15x_{45} + 2.4x_{46})$

s.t.

$x_{15} + x_{16} \le 1760$ $\quad x_{35} + x_{36} \le 20{,}000$

$x_{25} + x_{26} \le 1760$ $\quad x_{45} + x_{46} \le 15{,}000$

$x_{15} + x_{25} + x_{35} + x_{45} \ge 3520$

$x_{16} + x_{26} + x_{36} + x_{46} \ge 3520$

Solution:

$A1 \to A2$: $x_{15} = 1760$ (1000 Cu Yd)

$A1 \to A4$: $x_{16} = 0$

$A3 \to A2$: $x_{25} = 0$

$A3 \to A4$: $x_{26} = 1760$

$P1 \to A2$: $x_{35} = 1760$

$P1 \to A4$: $x_{36} = 0$

$P2 \to A2$: $x_{45} = 0$

$P2 \to A4$: $x_{46} = 1760$

Cost $= \$10{,}032{,}000$

9

x_{ij} = Blue regulars on front i in defense line j, $i=$

y_{ij} = Blue reserves on front i in defense line j.

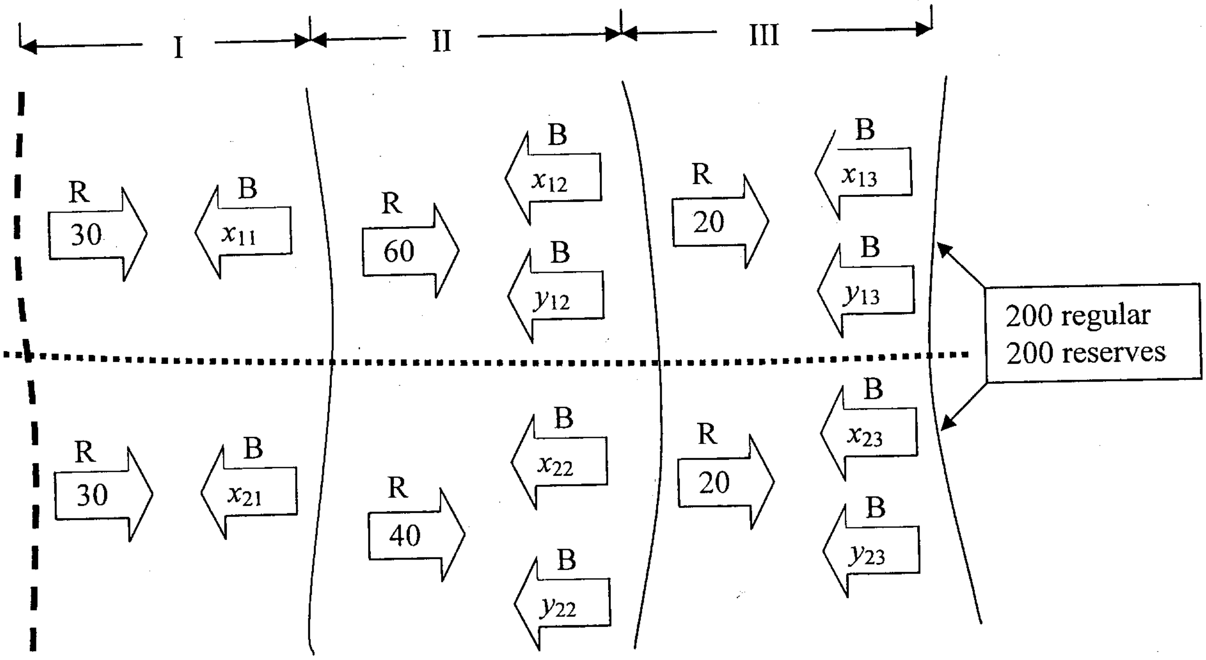
t_{ij} = Delay days on front i in defense line j.

Maximize $z = \min\{t_{11} + t_{12} + t_{13},\ t_{21} + t_{22} + t_{23}\}$

or

continued…

Set 2.3g

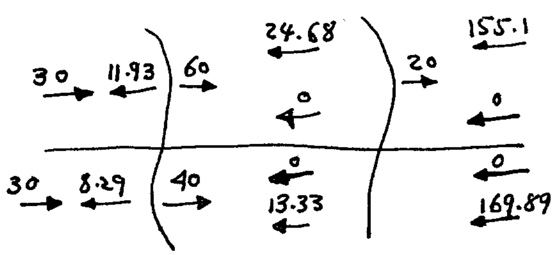


Maximize $z = T$

s.t.

$$T \le t_{11} + t_{12} + t_{13}$$
$$T \le t_{21} + t_{22} + t_{23}$$
$$x_{11} + x_{12} + x_{13} + x_{21} + x_{22} + x_{23} \le 200$$
$$y_{12} + y_{13} + y_{22} + y_{23} \le 200$$
$$t_{11} = .5 + 8.8\,\frac{x_{11}}{30}$$
$$t_{12} = .75 + 7.9\,\frac{x_{12} + y_{12}}{60}$$
$$t_{13} = .55 + 10.2\,\frac{x_{13} + y_{13}}{20}$$
$$t_{21} = 1.1 + 10.5\,\frac{x_{21}}{30}$$
$$t_{22} = 1.3 + 8.1\,\frac{x_{22} + y_{22}}{40}$$
$$t_{23} = 1.5 + 9.2\,\frac{x_{23} + y_{23}}{20}$$

Solution: Battle duration = 87.65 days



10

x_i = Efficiency of plant i

Minimize $z = .2(500)x_1 + .25(3000)x_2 + .15(6000)x_3 + .18(1000)x_4$

s.t.

$$500(1-x_1) \le .00085 \times 215{,}000$$
$$.94(500)(1-x_1) + 3000(1-x_2) \le .0009 \times 220{,}000$$
$$.94^2(500)(1-x_1) + .94(3000)(1-x_2) + 6000(1-x_3) \le .0008 \times 200{,}000$$
$$.94^3(500)(1-x_1) + .94^2(300)(1-x_2) + .94(6000)(1-x_3) + 1000(1-x_4) \le .0008 \times 210{,}000$$
$$0 \le x_1 \le .99$$
$$0 \le x_2 \le .99$$
$$0 \le x_3 \le .99$$
$$0 \le x_4 \le .99$$

Solution

Cost per hour = \$1891.41

Plant 1 efficiency = .99

Plant 2 efficiency = .9661

Plant 3 efficiency = .99

Plant 4 efficiency = .9824

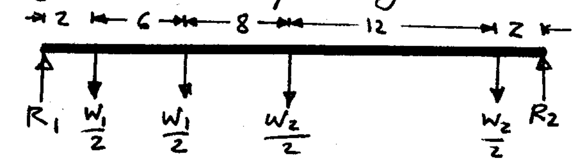
continued...

11

W_i = Capacity of yoke i (Kips)

R_1 = Reaction in Kips at left end

R_2 = Reaction in Kips at right end



Maximize $Z = W_1 + W_2$

s.t.

$$R_1 + R_2 = W_1 + W_2$$

$$2\left(\frac{W_1}{2}\right) + 8\left(\frac{W_1}{2}\right) + 16\left(\frac{W_2}{2}\right) + 28\left(\frac{W_2}{2}\right) = 30 R_2$$

$$R_1 \le 25, \quad R_2 \le 25$$

$$\frac{W_1}{2} \le 20, \quad \frac{W_2}{2} \le 20$$

Solution:

$W_1 = 20.59$ Kips

$W_2 = 29.41$ Kips

12

X_{ij} = Nbr. of aircraft of type i allocated to route j ($i = 1,2,3,4$, $j = 1,2,3,4$)

S_j = Nbr. of passengers not served on route j, $j = 1,2,3,4$

$$\begin{aligned} \text{Minimize } Z = & 1000(3x_{11}) + 1100(2x_{12}) \\ & + 1200(2x_{13}) + 1500(x_{14}) \\ & + 800(4x_{21}) + 900(3x_{22}) \\ & + 1000(3x_{23}) + 1000(2x_{24}) \\ & + 600(5x_{31}) + 800(5x_{32}) \\ & + 800(4x_{33}) + 900(2x_{34}) \\ & + 40S_1 + 50S_2 + 45S_3 + 70S_4 \end{aligned}$$

subject to

$$\sum_{j=1}^{4} x_{1j} \le 5, \quad \sum_{j=1}^{4} x_{2j} \le 8, \quad \sum_{j=1}^{4} x_{3j} \le 10$$

$$50(3x_{11}) + 30(4x_{21}) + 20(5x_{31}) + S_1 = 1000$$

$$50(2x_{12}) + 30(3x_{22}) + 20(5x_{32}) + S_2 = 2000$$

$$50(2x_{13}) + 30(3x_{23}) + 20(4x_{33}) + S_3 = 900$$

$$50(x_{14}) + 30(2x_{24}) + 20(2x_{34}) + S_4 = 1200$$

All x_{ij} and $S_j \ge 0$

continued...

*** OPTIMUM SOLUTION SUMMARY ***

Title: Problem 26a-16
Final iteration No: 16
Objective value (min) =221900.0000
==> ALTERNATIVE solution detected at x13

Variable	Value	Obj Coeff	Obj Val Contrib
x1 x11	5.0000	3000.0000	14999.9990
x2 x12	0.0000	2200.0000	0.0000
x3 x13	0.0000	2400.0000	0.0000
x4 x14	0.0000	1500.0000	0.0000
x5 x21	0.0000	3200.0000	0.0000
x6 x22	0.0000	2700.0000	0.0000
x7 x23	0.0000	3000.0000	0.0000
x8 x24	8.0000	2000.0000	15999.9990
x9 x31	2.5000	3000.0000	7500.0015
x10 x32	7.5000	4000.0000	29999.9980
x11 x33	0.0000	3200.0000	0.0000
x12 x34	0.0000	1800.0000	0.0000
x13 s1	0.0000	40.0000	0.0000
x14 s2	1250.0000	50.0000	62500.0000
x15 s3	899.9998	45.0000	40499.9922
x16 s4	720.0001	70.0000	50400.0078

Constraint	RHS	Slack(-)/Surplus(+)
1 (<)	5.0000	0.0000-
2 (<)	8.0000	0.0000-
3 (<)	10.0000	0.0000-
4 (=)	1000.0000	0.0000
5 (=)	2000.0000	0.0000
6 (=)	900.0000	0.0000
7 (=)	1200.0000	0.0000

Solution:

Aircraft Type	Route	Nbr. aircraft
1	1	5
2	4	8
3	1	2.5
3	2	7.5

Fractional solution must be rounded.

Cost = \$221,900