Review 2 Essentials of fluid dynamics

CONTENTS

LEARNING AIMS

After reading this Review, you should be familiar with the basic equations and terminology of fluid mechanics that are used in other chapters of this book, and with the physical principles that lie behind those equations.

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§R2.1 INTRODUCTION

Transferring energy to and from a moving fluid is the basis of hydro, wind, wave and some solar power systems, and of meteorology. We review here the fluid dynamics we use in our analysis of these applications. Readers seeking further explanation should refer to the references listed in the Bibliography.

We start with the basic laws of the *conservation of mass, energy and momentum*. The term *fluid* includes both liquids and gases, which, unlike solids, do not remain in equilibrium when subjected to shearing forces. The hydrodynamic distinction between liquids and gases is that gases are more easily compressed, whereas liquids have volumes varying only slightly with temperature and pressure. However, for air, flowing at speeds <100 m/s and not subject to large variations in pressure or temperature, density change is negligible. Therefore, for wind power (and where applicable in other renewable energy systems) moving air is treated as an *incompressible* fluid, i.e. as though it is a liquid. Thisconsiderably simplifies the analysis without introducing significant error.

Many important fluid flows are *steady*, in the sense that the particular flow *pattern* at a location does not vary with time. (Of course the fluid itself is moving!) The flow itself may be represented by *streamlines*, parallel with the instantaneous velocity vectors at each point, which can represent either *laminar or turbulent flow* (§R2.5). However, even in turbulence, the streamlines remain within well-defined (though imaginary) *stream tubes*.

§R2.2 CONSERVATION OF ENERGY: BERNOULLI'S EQUATION

Consider steady, incompressible flow. At first, we assume that no work is done by the moving fluid (e.g. on a turbine).

(a) No heat input

Fig. R2.1 shows a section of a stream tube between heights z_1 and z_2. Assume no energy exchange of heat or work across the stream tubes, as is often the case. The tube is narrow in comparison with other dimensions, so z is considered constant over each cross-section of the tube. A fluid mass $m = \rho A_1 u_1 \Delta t$ enters the control volume at 1, and an equal mass $m = \rho A_2 u_2 \Delta t$ leaves at 2 (where ρ is the density of the fluid, treated as constant). So:

change in potential energy + change in pressure forces
= change in kinetic energy + friction

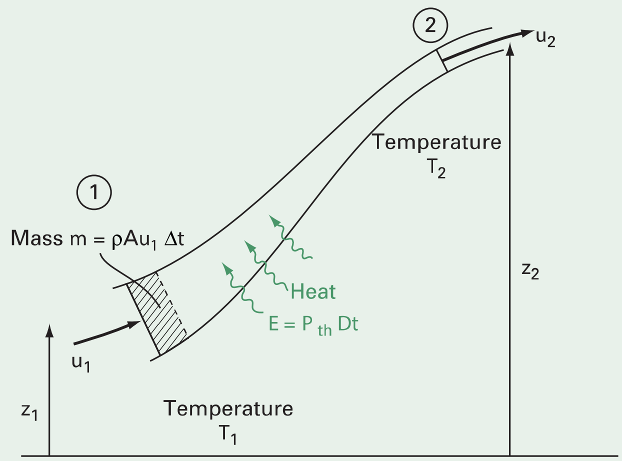

Fig. R2.1
Illustrating conservation of energy in fluid flow: a stream tube rises from height z_1 to z_2. In some cases, thermal power P_{th} may be added to the flow as heat.

$$mg(z_1 - z_2) + [(p_1A_1)(u_1\Delta t) - (p_2A_2)(u_2\Delta t)] = \tfrac{1}{2}m(u_2^2 - u_1^2) + E_f \qquad \text{(R2.1)}$$

where (i) pressure force p_1A_1 acts through a distance $u_1\Delta t$, and similarly for p_2A_2, and (ii) E_f is the heat generated internally by friction.

Neglecting fluid friction E_f and rearranging terms, yields:

$$(p_1/\rho) + gz_1 + \tfrac{1}{2}u_1^2 = (p_2/\rho) + gz_2 + \tfrac{1}{2}u_2^2 \qquad \text{(R2.2)}$$

or, equivalently,

$$\frac{p}{\rho g} + z + \frac{u^2}{2g} = \text{constant along a streamline, with no loss of energy.} \qquad \text{(R2.3)}$$

Either of these forms of the equation is called *Bernoulli's equation.*

The sum of the terms on the left of (R 2.3) as dimensions of length and is called the total *head* of fluid (H), with particular relevance for hydropower.

Note that R (2.2) and R (2.3) apply to fluids treated as ideal, i.e. with zero viscosity, zero compressibility and zero thermal conductivity and with no internal heat sources. These approximations work well for almost all the calculations in this book about wind and hydro turbines. (The assumption of zero viscosity, or equivalently zero internal friction, is usually valid except very near to solid surfaces: see §R2.4.) The energy equation may be modified to include non-ideal characteristics as for combustion engines and other thermal devices (e.g. high-temperature-concentrating solar collectors (see Bibliography)).

(b) With heat input

In solar heating systems and heat exchangers, heat $E = P_{th}\Delta t$ is added as an energy input in Fig. R2.1. The mass m coming into the control volume at temperature T_1 may be considered to have heat content mcT_1 (where c is the specific heat capacity of the fluid), and that going out has heat content mcT_2. This gives an equation corresponding to (R2.2), namely:

$$(p_1/\rho)+gz_1+\tfrac{1}{2}u_1^2+cT_1+(P_{th}/\rho Q)=(p_2/\rho)+gz_2+\tfrac{1}{2}u_2^2+cT_2 \qquad \text{(R2.4)}$$

where the volume flow rate

$$Q = Au \qquad \text{(R2.5)}$$

In most heating systems, thermal contributions dominate the energy balance, with the fluid movements insignificant. So, for practical purposes, (R2.4) reduces to:

$$P_{th} = \rho c Q(T_2 - T_1) \qquad \text{(R2.6)}$$

§R2.3 CONSERVATION OF MOMENTUM

Newton's second law of motion may be generalized from particles to fluids: 'At any instant in steady flow, the resultant force acting on a moving fluid within a fixed volume equals the net outflow of momentum from that volume.' This is known as the *momentum theorem.* Newton's third law (action and reaction) may be applied to fluids in a similar manner.

For example, consider fluid passing across a turbine in a pipe. In Fig. R2.2, fluid flowing at speed u_1 into the left of the control surface carries momentum ρu_1 *per unit volume* in the direction of flow, and exits at right at speed u_2. The momentum theorem tells us that the rate of change of momentum equals the force, F, on the fluid and the reaction, $-F$, is the force exerted on the turbine and pipe by the fluid. So:

$$F = \rho\,(A_2\,u_2{}^2 - A_1\,u_1{}^2) = \dot{m}u_2 - \dot{m}u_1 \qquad \text{(R2.7)}$$

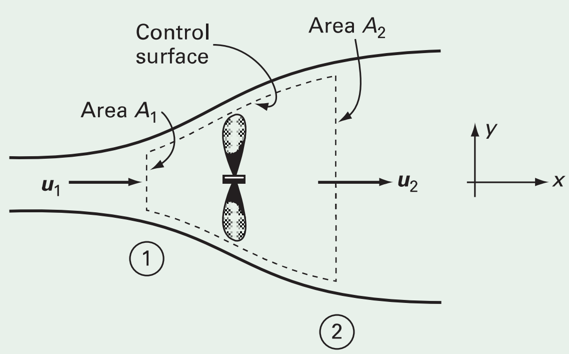

Fig. R2.2

A turbine in a pipe. The dotted line shows the control surface over which the momentum theorem is applied.

where $\dot{m} = |\rho A_1 u_1| = |\rho A_2 u_2|$ is the mass flow (always taken as positive) and the signs in (R2.7) indicate directions, which are obvious in this case. In more complex cases, such as inside a turbine, the momentum and forces must be treated as vectors (i.e. direction matters!).

§R2.4 VISCOSITY

Consider two parallel plates, with fluid between them and the top plate moving at a velocity u_1 relative to the bottom one (Fig. R2.3). The axes have x in the direction of motion, and y across the gap between the plates. It is found experimentally that *fluid does not slip at a solid surface*, i.e. the fluid immediately adjacent to each plate has the same speed and direction of movement as the plate.

At microscopic scale, the random motion of molecules in the fluid transfers larger momentum (acquired from the top plate) downward and smaller momentum (acquired from the bottom plate) upward. This *diffusion of momentum* limits the velocity gradient that the fluid can sustain, producing an internal friction opposing the horizontal slip in the flow. It is found that the shear stress (i.e. the force per unit area, in the direction indicated in Fig. R2.3) is

$$\tau = \mu(\partial u / \partial y) \tag{R2.8}$$

where μ is the *dynamic viscosity* (unit N s m^{-2}). This viscosity is independent of τ and $\partial u/\partial y$, and depends only on the composition and temperature of the fluid.

A closely related fluid parameter is *kinematic viscosity*:

$$\nu = \mu / \rho \tag{R2.9}$$

In incompressible fluids, the flow pattern often depends more directly on ν than on μ. By combining (R2.8) and (R2.9), we find that the units of kinetic viscosity ν are:

$$\frac{(\text{kg m s}^{-2})\text{m}^{-2}}{\text{kg m}^{-3}}\frac{\text{m}}{\text{m s}^{-1}} = \text{m}^2\,\text{s}^{-1}$$

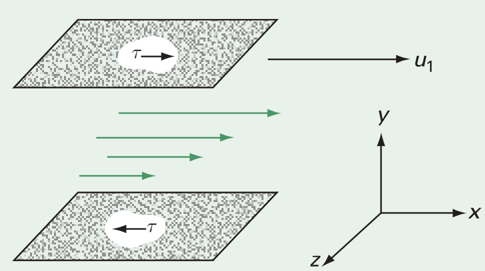

Fig. R2.3
Flow between two parallel plates.

Thus ν has the character of a diffusivity; i.e. changes in momentum diffuse a distance x in time ~ x^2/ν (compare thermal diffusivity κ defined in §R3.3). Typical values of ν are given in Appendix B, Tables B.1 and B.2.

§R2.5 TURBULENCE

Turbulent flow occurs because most fluid motion is unstable. Suppose fluid is initially flowing through a pipe in an orderly, stable manner, as in the path lines shown in Fig. R2.4(a). We consider a small moving volume of the fluid, which we refer to here as a 'blob' or 'packet'. Something will disturb the motion (e.g. an oscillation or a knock on the pipe), causing small forces to act on the blobs. If these are moving rapidly enough, fluid friction will not be sufficient to keep them in their original paths, thus causing instability in the flow. The disturbed elements then disturb other nearby blobs of fluid from their original paths, and soon the entire flow is in the semi-chaotic state called *turbulence*, illustrated in Fig. R2.4(b). Water flowing from a tap or smoke rising from a taper often shows this change from smooth (laminar) flow to turbulence. Wind in the open environment is always turbulent and only becomes laminar as it meets the leading edge of aerodynamic blades or wings.

The non-dimensional *Reynolds number*

$$\mathscr{R} = uX/\nu \tag{R2.10}$$

is key to determining whether a flow is laminar or turbulent; it represents the ratio of fluid momentum (arising from 'inertia forces') to viscous friction. Here u is the mean speed of the flow, X is a nominated *characteristic length* of the system (for pipes, their diameter), and ν is the kinematic viscosity of the fluid. Only flows with relatively small values of $\mathscr{R}$ will be laminar; most practical flows are turbulent with larger values of $\mathscr{R}$. For instance, in pipes, flow is likely to be turbulent if $\mathscr{R} >$ ~2300.

In turbulent flow, the effect of the sideways motions of the fluid is to transport fluid of low speed from near a solid surface (e.g. the wall of a pipe)

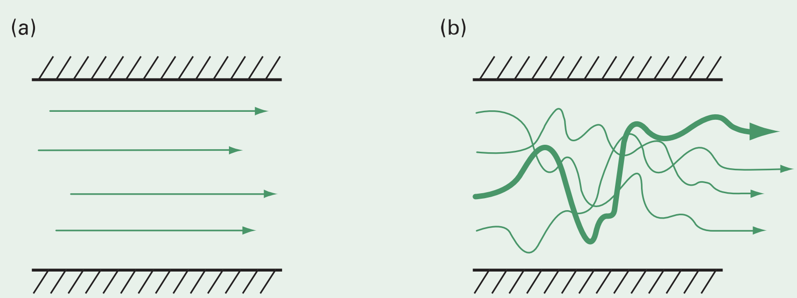

Fig. R2.4
Path lines of flow in a pipe:
a laminar,
b turbulent.

towards the main part of the flowing fluid and fluid of high speed in the opposite direction. The momentum so transferred by blobs of fluid is greater than that transferred by molecular motion because a blob of fluid may move a long way (e.g. half-way across a pipe) in a single jump. This transfer of momentum from fluid to a static solid surface creates a significant friction force opposing the motion of the fluid. Thus, the presence of turbulence in pipes *increases* friction as compared with laminar flow.

If the walls of the pipe are hotter than the incoming fluid, these rapid inward and outward motions transfer heat rapidly to the bulk of the fluid. An element of cold fluid can jump from the center of the pipe, pick up heat by conduction from the hot wall, and then carry it much more rapidly back into the center of the pipe than could molecular conduction. Thus *turbulence* likewise increases heat transfer (see §R3.4). Criteria for laminar or turbulent flow in heat transfer are discussed in Review 3.

§R2.6 FRICTION IN PIPE FLOW

Due to friction, otherwise useful energy and pressure are said to be 'lost' or 'dissipated' when a fluid flows through pipes; for instance, in the pipework leading to a hydroelectric turbine. Let Δp be the pressure overcoming friction as fluid moves at average speed u, through the pipe of length L and diameter D. Observation indicates that Δp increases as L increases and D decreases. Bernoulli's equation shows that the quantity $\frac{1}{2}\rho u^2$ has the same dimensions as p, i.e. kg/(ms^2). All this can be expressed in the equation:

$$\Delta p = 2f(L/D)(\rho u^2) \qquad \text{(R2.11)}$$

Here f is a dimensionless *pipe friction factor* that changes value with experimental conditions. (*Caution*: (1) In some other books $f' = 4f$ is called the friction factor, and an equivalent equation is used instead of (R2.11); in this book we use only f. (2) Neither of the 'friction factors' f or f' is related to the 'friction coefficient' describing the friction between two *solid* surfaces.) As with many non-dimensional factors in engineering, the magnitude of f characterizes the physical conditions independently of the scale, depending only on the *pattern* of flow, i.e. the *shape* of the streamlines.

The friction factor f is the *proportion* of the kinetic energy $\frac{1}{2}\rho u^2$ entering unit area of the pipe that has to be applied as external work (Δp) to overcome frictional forces. It depends on (a) the dimensionless Reynolds number $\mathscr{R}$ of (R2.10) and (b) the ratio of the height, ξ, of the surface irregularities (roughness) to the diameter of the pipe, D. Fig. R2.5 plots a series of curves of friction factor versus Reynolds number, with one curve for each roughness ratio ξ/D.

Provided that the appropriate value of ξ is used, these curves give a reasonable estimate of pipe friction. Typical values of ξ are given in Table R2.1, but it should be realized that the roughness of a pipe tends to increase with age and, very noticeably, with accretion of sediments and encrustations. This applies in many circumstances, including heating systems in factories and buildings, and arteries in the human body. Note the exceptional smoothness of clean plastic materials and coatings (e.g. on wind turbine blades).

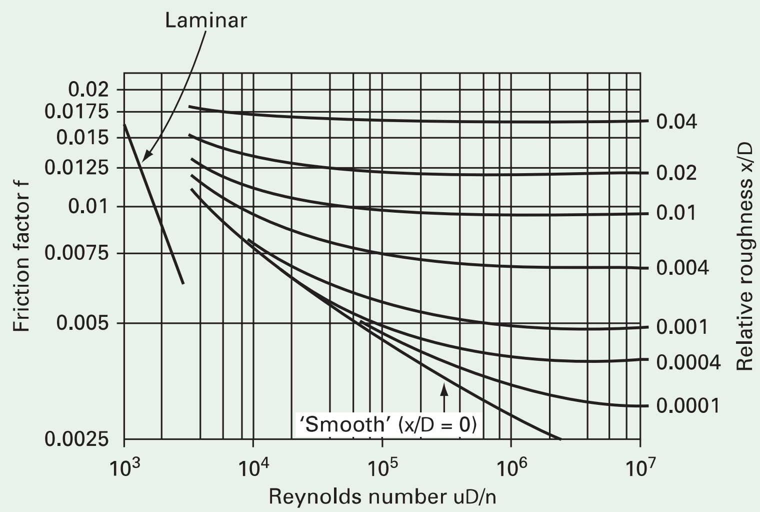

Fig. R2.5
Chart of friction factor *f* for pipe flow (see (R2.11)).

Table R2.1 Approximate pipe roughness ξ

Material	ξ*/mm*
Glass, PVC and most other plastics	0.0015
Cast iron	0.25
New steel	0.1
Smoothed concrete	0.4

WORKED EXAMPLE R2.1

What is the head loss due to friction when water flows through a concrete pipe of length 200 m and diameter 0.30 m at a volume flow rate of 0.10 m^3/s?

Solution

The mean water speed is:

$$u = Q / A = \frac{0.1\,m^3s^{-1}}{\pi(0.15\,m)^2} = 1.4\,ms^{-1}$$

From (R2.10), the Reynolds number

$$\mathcal{R} = \frac{uD}{\nu} = \frac{(1.4\ \mathrm{ms^{-1}})(0.3\ \mathrm{m})}{1.0\times10^{-6}\ \mathrm{m^2s^{-1}}} = 0.4\times10^6$$

where the value of ν is taken from Appendix B, Table B.2. Since $\mathcal{R} >> 2000$, flow is turbulent.
For concrete (from Table R2.1), $\xi = 0.4$ mm. Thus the ratio

$$\xi / D = \frac{0.4\ \mathrm{mm}}{300\ \mathrm{mm}} = 0.0013$$

For these values of $\mathcal{R}$ and ξ / D, Fig. R2.5 gives

$f = 0.0050$,

Expressing (R2.11) in terms of the head loss due to friction,

$$H_f = \Delta p / \rho g = 2fLu^2 / Dg \quad \text{(R2.12)}$$

Hence:

$$H_f = \frac{(2)(5.0\times10^{-3})(200\ \mathrm{m})(1.4\ \mathrm{ms^{-1}})^2}{(0.3\ \mathrm{m})(9.8\ \mathrm{ms^{-2}})}$$
$$= 1.3\ \mathrm{m}$$

Fig. R2.5 shows only one curve for $R < 2000$ indicating the flow is laminar; the 'pattern' of the moving water is independent of the pipe internal surface in this range of Reynolds number. In laminar flow it is possible to calculate the pressure drop Δp explicitly from (R2.8), and hence it may be shown that the friction factor is:

$$f = 16\nu / (uD) \quad \text{(laminar)} \quad \text{(R2.13)}$$

§R2.7 LIFT AND DRAG FORCES

Lift and drag forces apply to any solid object immersed in a fluid flow (e.g. wings on an aircraft or blades on a wind turbine rotor).

In Fig. R2.6(a) a solid object is immersed in a fluid flowing from left to right (relative to the object). However, due to intricacies of the flow pattern passing the object, the resulting force on the object is unlikely to be parallel to the upstream flow. If the total (vector) force exerted on the body is $\boldsymbol{F}$, the *drag* force F_D is the component of that force in the direction of the upstream flow and the *lift* force F_L is the component normal to the flow. It is the lift force that twists and turns the object.

An important special case is an *airfoil*. This is a smooth structure of width (chord) much less than its length (span), and thickness much less

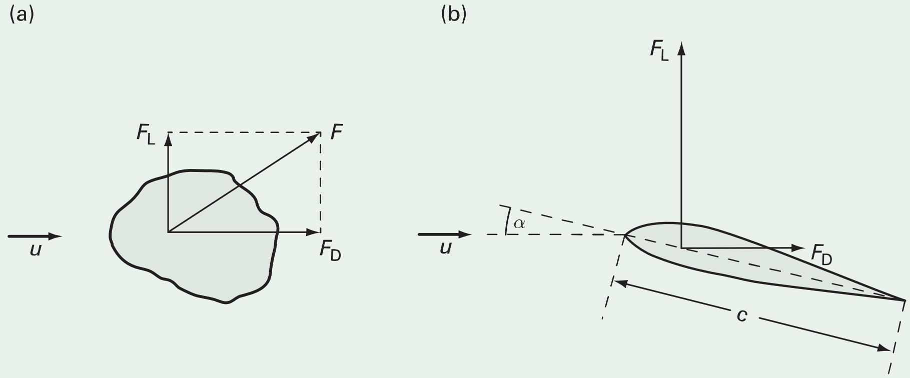

Fig. R2.6

Sketches to illustrate forces on an object immersed in a fluid flow.

a Any object: lift force F_D (parallel to stream velocity u), lift force F_L (normal to F_D), total (vector) force F.

b Special case of an airfoil (e.g. wind turbine blade) at angle of attack α.

than its chord, having a relatively sharp trailing edge and more curved on the top than on the bottom. Examples are an aircraft wing or wind turbine blade (Fig. R2.6(b)). The airfoil shape and the smooth surfaces encourage laminar air flow such that with the airfoil set at a small angle to the inflow of air, the lift force is much larger than the drag force. In the operating range of Reynolds number (typically $>10^5$), the flow around the airfoil is close to ideal (i.e. zero viscosity, zero compressibility) except in a thin 'boundary layer' close to the surface. This greatly simplifies modeling of lift and drag, as described in textbooks on aerodynamics (see Bibliography).

With aircraft, the lift force overcomes gravitational forces and the airplane does not drop. To understand the action of wind turbine blades, the lift and drag forces have to be resolved in and out of the plane of rotation; doing so shows that the net result is a force turning the blade across the upstream wind direction; see §8.6.1 for a fuller discussion.

Lift and drag of an airfoil are characterized by two non-dimensional parameters:

the *lift coefficient* $C_L = F'_L / (\frac{1}{2}\rho u^2 c)$ R2.14)

and

the *drag coefficient* $C_D = F'_D / (\frac{1}{2}\rho u^2 c)$ (R2.15)

where F'_L and F'_D are respectively the lift and drag forces per unit length of span, and c is the length of the chord line (see Fig. R2.6).

Both C_L and C_D are functions of the Reynolds number $\mathscr{R}$, and of the *angle of attack* α, which is the angle between the incident air flow and the chord line between the leading and trailing edges. (In the airplane context, α is often called the 'angle of incidence'.) Fig. R2.7 shows a typical variation of C_L and C_D with α for a particular aerofoil in its working range of $\mathscr{R}$. For small values of α, C_L is directly proportional to α; note the changing ratio between lift and drag forces in the top two diagrams of

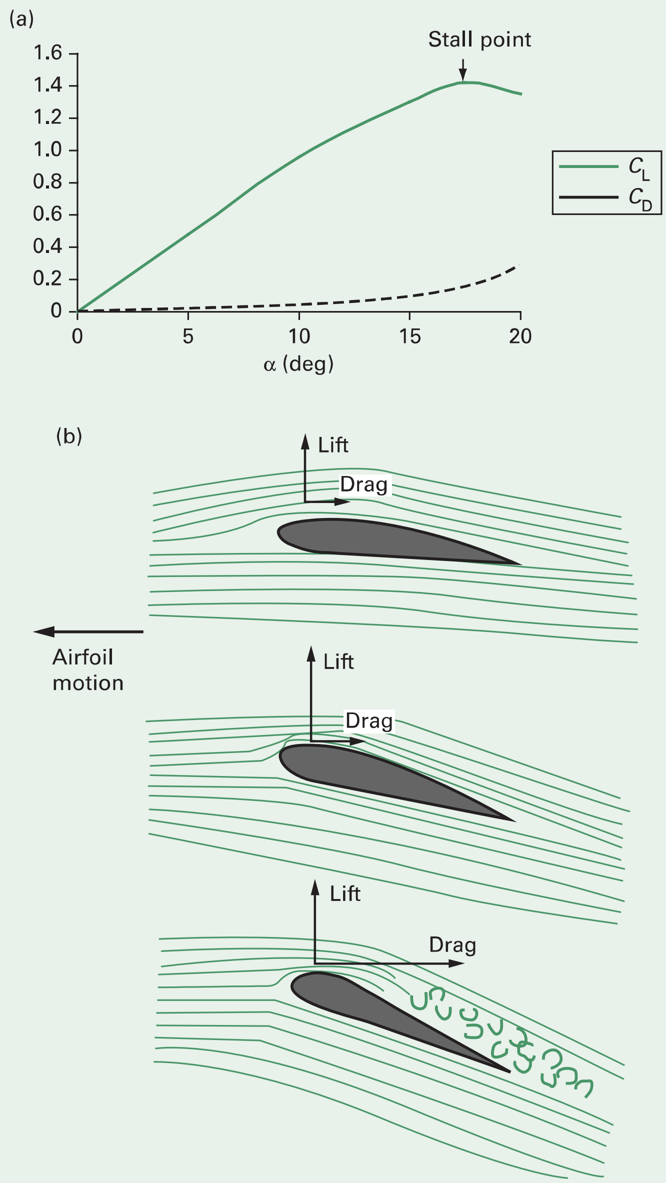

Fig. R2.7

Variation of lift and drag coefficients with angle of attack α for a typical aerofoil in its working range:

a graph of C_L and C_D against angle of attack α. If α ~5°, conditions are far from stall and of acceptable drag.

b streamlines of flow.

Fig. R2.7(b). For some value of α between 10° and 20°, the lift decreases, and the aerofoil becomes stalled, with the flow separating from the top surface and the drag increasing substantially, as in the bottom diagram of Fig. R2.7(b).

QUICK QUESTIONS

Note: Answers to these questions are in the text of the relevant section of this chapter, or may be readily inferred from it.

1. Why can air be treated as incompressible in most renewable energy applications?
2. Write down Bernouilli's equation. Does it relate primarily to the speed, the momentum, or the energy of a fluid?
3. Distinguish dynamic viscosity from kinenatic viscosity.
4. Define Reynolds number. Why is it so important in calculations of fluid flow?
5. What is the difference between turbulent flow and laminar flow?
6. Why is pipe friction greater in than in laminar flow?
7. Define lift and drag.
8. Why are aircraft wings usually thin compared to their length or width?
9. Compare the density of air and water, and discuss the effect on turbine design.
10. What is the speed of a moving fluid at a smooth boundary surface?

BIBLIOGRAPHY

The following selection from the many books and websites on fluid mechanics may prove useful. There are many other good books besides those listed. For work on turbo-machinery, books written for engineers are usually more useful than those written for mathematicians, who too often ignore friction and forces. Since the basics of fluid dynamics have not changed, old textbooks may still be useful, especially if they use SI units (which many older books do not). However, modern engineering practice makes much use of computer software packages, with the danger of misuse if basic principles are not understood by the user.

Books

Batchelor, G.K. (1967) *An Introduction to Fluid Dynamics*, Cambridge University Press, Cambridge. Classic text, reissued unchanged in 2000. A most precise statement of the foundations (see especially ch. 3), with many examples. Repays careful reading, but perhaps unsuitable for beginners.

Çengel, Y.A. and Cimbala, J. (2009, 2nd edn) *Fluid Mechanics: Fundamentals and applications*, McGraw-Hill, New York. Clear and detailed explanations with emphasis on physical principles. Very student-friendly with exemplary accompanying learning aids.

Francis, J.R. (1974, 4th edn) *A Textbook of Fluid Mechanics*, Edward Arnold, London. Clear writing makes this easy reading for beginners. More engineering detail than Kay and Nedderman.