## LAB SESSION 9

# Analysis of Integral Compensation (PI) and Lag Compensated Systems

## **Objective:**

In this lab session we will learn how to introduce the flexibility in our design for the desired transient response while maintaining the required percent overshoot, settling time. Analysis of the compensation technique in which we will use the lag compensator to improve the system's response.

## **Equipment Required:**

PC and MATLAB® R2017b

## **Pre-Lab Requisites:**

- Hands on experience on Simulink and MATLAB Programming
- Students must have knowledge of basic configurations of systems and their MATLAB commands

## **Procedure:**

There are two configurations of compensation are used, one is cascade compensation and feedback compensation as shown in the fig

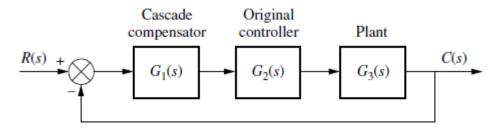


Fig 9.1 Cascaded compensation configuration

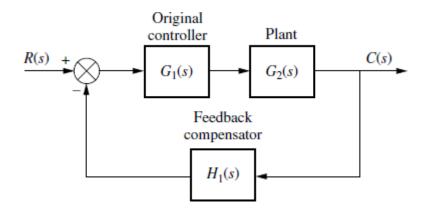
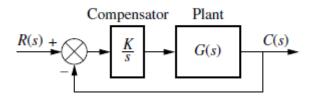


Fig 9.2 Feedback Compensation Configuration

In this lab session we will deal with the cascaded compensation in which further we will deal, firstly, with the ideal integral compensation, which uses a pure integration to place an open loop, forward path pole at the origin, thus increasing the system type and reducing the error to zero. Systems that feed the error forward to the plant are called **proportional control** systems. Systems feed the integral of the error to the plant are called **integral control** systems. Finally, the system feed derivative of the error to the plant are called **derivative control** system.

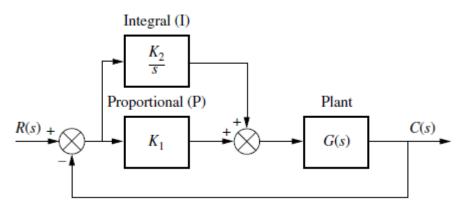
Firstly, in this experiment proportional plus integral (PI) controller is examined, as we feed the error (proportional) plus the integral of the error to the plant it will change the type of the system and introducing a zero near to the pole will maintain the system previous root locus. Thus, we have improved the steady-state error without appreciably affecting the transient response.

The schematic diagram of an integral control is shown below.



**Fig 9.1 Integral Control** 

The schematic diagram of a Proportional-Integral (PI) control is shown below.



**Fig 9.2 Proportional Plus Integral Control** 

Secondly, we examine the lag compensator in which we don't have an active integrator and it comprises of passive components. In this technique the pole and zero are moved to the left, close to the origin. You can guess that this pole placement, although does not increase the system type, does yield an improvement in the static error constant over an uncompensated system.

The lag compensation system's schematic diagram is shown in the following figure.

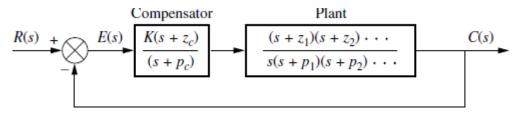
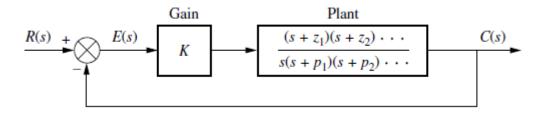


Fig 9.3 Lag Compensated Type 1 System

Consider the following uncompensated system.



#### Fig 9.4 Uncompensated System

The static error constant,  $K_{\nu o}$  for the uncompensated system is given as follows

$$K_{\nu o} = \frac{K z_1 z_2 \dots}{p_1 p_2 \dots} \tag{1}$$

Now with the lag compensation the new static error  $K_{vn}$  is given as follows

$$K_{\nu o} = \frac{(K z_1 z_2 \dots)(z_c)}{(p_1 p_2 \dots)(p_c)}$$
(2)

Now by substituting eq (1) in eq (2) we have

$$K_{vn} = K_{vo} \frac{Z_c}{p_c} > K_{vo} \tag{3}$$

Equation 3 states that the improvement in the compensated system's  $K_v$  over the uncompensated system's  $K_v$  is equal to the ratio of the magnitude of the compensator zero to the compensator pole.

In conclusion the ideal compensator drives the steady state error to zero, a lag compensator with a pole that is not at origin will improve the static error constantly by a factor equal to  $\frac{z_c}{p_c}$ .

#### EXERCISE 1:

Plot the root locus and step response using MATLAB of the following uncompensated system.

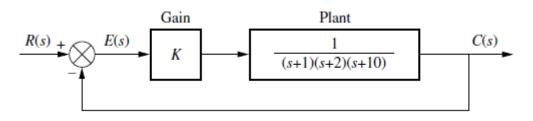


Fig 9.5 Closed loop system before compensation


Fig 9.6 Root locus for uncompensated closed loop system

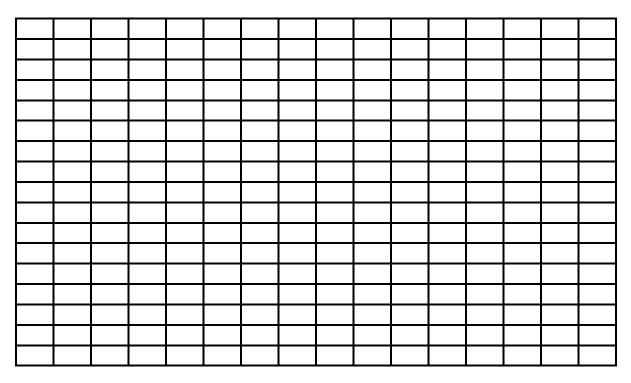


Fig 9.7 Step response for uncompensated closed loop system

## EXERCISE 2:

Draw the root locus of the following system and step response for the following system by introducing an integrator in series with the system transfer function.

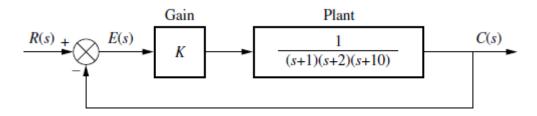


Fig 9.8 Closed loop system with integrator

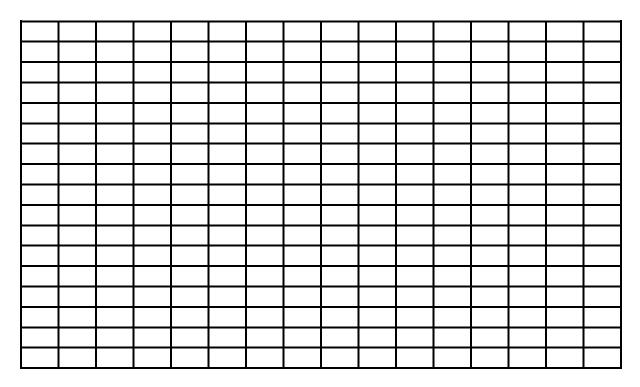


Fig 9.9 Root locus for the closed loop system with integrator

Fig 9.10 Step Response for the closed loop system with integrator

## EXERCISE 3:

Plot the Root Locus and the step response for the following compensated system.

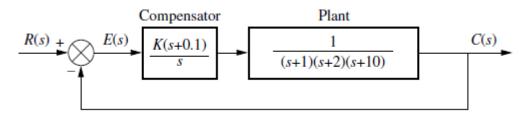


Fig 9.11 Closed Loop system after ideal integral compensation

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Fig 9.12 Root locus for the ideal integral compensation

Fig 9.13 Step Response for the ideal integral compensation

## **EXERCISE 4:**

Plot the step response on the for both uncompensated and compensated system and comment on the out put response of the system.


Fig 9.14 Step Response of both compensated and uncompensated systems

## **EXERCISE 5:**

Design a lag compensator for the following system to improve the steady state error by a factor of 10 if the system is operating with a damping ratio of 0.174.

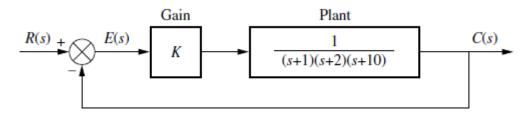
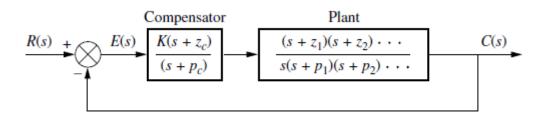


Fig 9.15 Closed loop system for lag compensator design



#### Fig 9.16 General Diagram of lag compensator

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Fig 9.17 Root Locus for the Lag compensated system

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## Fig 9.18 Step Response for the Lag compensated and uncompensated system

Parameter	Uncompensated	Lag compensated
Plant and Compensator	$\frac{K}{(s+1)(s+2)(s+10)}$	$\frac{K(s+0.111)}{(s+1)(s+2)(s+10)(s+0.01)}$
К		
K <sub>p</sub>		
$e(\infty)$		
Dominant Second Order poles		
Third Pole		
Fourth Pole		
Zero		