

# LAB SESSION 8

## Analysis of Root Locus Technique

### Objective:

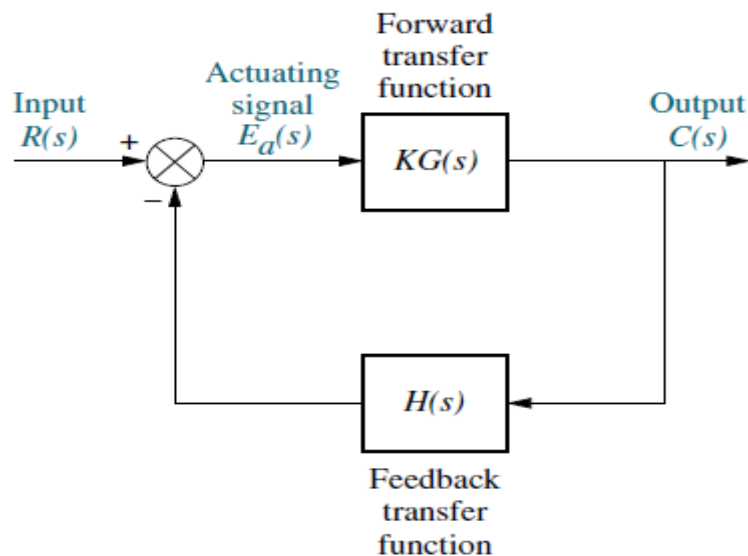
In this lab session we will explore root locus which is the graphical representation of the closed loop poles as a system parameter is varied, is a powerful method of analysis and design for stability and transient response. By using root locus technique, we will be able to achieve the qualitative description of a control system's performance. This technique shows the effect of varying gain upon percent overshoot, settling time and peak time can be vividly observed. Besides transient response using root locus technique we will learn the range of system stability, range of instability and the conditions that cause a system to break into oscillation.

### Equipment Required:

PC and MATLAB® R2017b

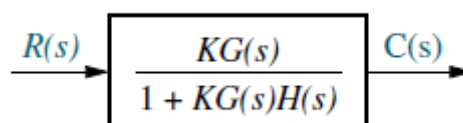
### Procedure:

By examining the properties of the root locus we will be able to rapidly sketch the root locus for higher order systems without factoring the denominator of the closed loop transfer function. Now considering a general closed loop system shown in the fig



**Fig 8.1 Closed Loop system**

The equivalent closed loop transfer function is given as



**Fig 8.2 Equivalent Transfer Function**

a pole,  $s$ , exists when the characteristic polynomial in the denominator becomes zero or

$$KG(s)H(s) = -1 = 1 \angle (2k + 1)180^\circ \quad k = 0, \pm 1, \pm 2, \pm 3 \dots \quad (1)$$

where -1 is represented in polar form as  $1 \angle (2k + 1)180^\circ$  alternatively a value of  $s$  is a closed loop pole if

$$|KG(s)H(s)| = 1 \quad (2)$$

And

$$\angle KG(s)H(s) = (2k + 1)180^\circ \quad (3)$$

equation (1) implies that a value of  $s$  is substituted into the function  $KG(s)H(s)$  a complex number of results. If the angle of the complex number is an odd multiple of  $180^\circ$  that value of  $s$  is a system pole for some value of  $K$ . the value of  $K$  is found by the given formula

$$K = \frac{1}{|H(s)||G(s)|} \quad (4)$$

Now the for finding the  $K$  we have

$$M = \frac{\prod \text{zero lengths}}{\prod \text{pole lengths}} = \frac{\prod_{i=1}^m |(s+z_i)|}{\prod_{j=1}^n |(s+p_j)|} \quad (5)$$

Now

$$K = \frac{1}{M}$$

The angle is given by

$$\theta = \sum \text{zero angles} - \sum \text{pole angles} \quad (6)$$

$$= \sum_{i=1}^m \angle(s + z_i) - \sum_{j=1}^n \angle(s + p_j) \quad (7)$$

### ***Rules for sketching the Root Locus***

#### **Number of branches:**

The number of branches of the root locus equals the number of closed loop poles.

#### **Symmetry:**

The root locus is symmetrical about real axis

#### **Real axis segment:**

On the real axis, for  $K > 0$ , the root locus exists to the left of an odd number of real axis, finite open loop poles and/or finite open loop zeros.

#### **Starting and Ending Points:**

The root locus begins at the finite and infinite poles of  $G(s)H(s)$  and ends at the finite and infinite zeros of  $G(s)H(s)$

### Behavior at Infinity:

The root locus approaches straight lines as asymptotes as the locus approaches infinity. Further the equations of the asymptotes are given by the real axis intercept and angle in radians as follows

$$\sigma_a = \frac{\sum \text{finite poles} - \sum \text{finite zeros}}{\# \text{finite poles} - \# \text{finite zeros}}$$

$$\theta_a = \frac{(2k + 1)\pi}{\# \text{finite poles} - \# \text{finite zeros}}$$

Where  $k = 0, \pm 1, \pm 2, \pm 3 \dots \dots$

### Additional Rules for Refining the sketch

#### Real-axis breakaway and break-in points:

The root locus breaks away from the real axis at a point where the gain is maximum and breaks into the real axis at a point where the gain is minimum.

#### Calculation of $j\omega$ – axis crossing:

The root locus crosses the  $j\omega$  axis at the point where  $\angle G(s)H(s) = (2k + 1)180^\circ$ . Routh Hurwitz or a search of the  $j\omega$ -axis for  $(2k + 1)180^\circ$  can be used to find the  $j\omega$ -axis crossing.

#### Angles of departure and arrival:

The root locus departs from complex, open-loop poles and arrives at complex, open loop zeros at angles that can be calculated as follows. Assume a point  $\epsilon$  close to the complex pole or zero. Add all angles drawn from all open-loop poles and zeros to this point. The sum equals  $(2k + 1)180^\circ$ . The only unknown angle is that drawn from the  $\epsilon$  close pole or zero, since the vectors drawn from all other poles and zeros can be considered drawn to the complex pole or zero that is  $\epsilon$  close to the point. Solving for the unknown angle yields the angle of departure or arrival.

#### Plotting and calibrating the root locus:

All points on the root locus satisfy the relationship  $\angle G(s)H(s) = (2k + 1)180^\circ$  the gain, K, at any point on the root locus is given by

$$K = \frac{1}{|G(s)H(s)|} = \frac{1}{M} - \frac{\prod \text{zero lengths}}{\prod \text{pole lengths}}$$

### EXERCISE 1:

Explore the command of “pzmap” in MATLAB and write about its function.

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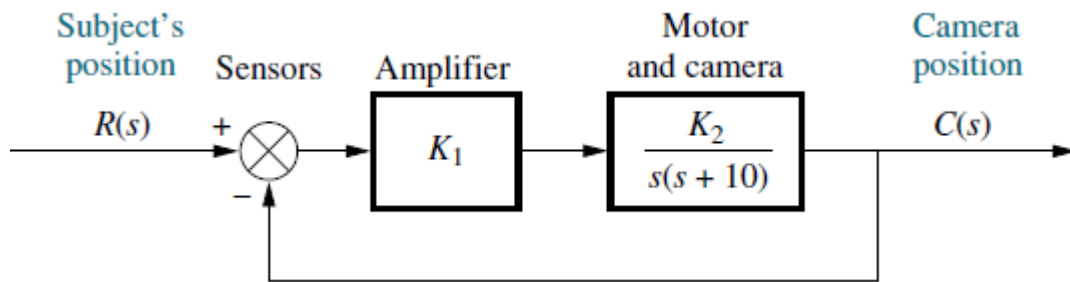
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**EXERCISE 2:**

Find the closed loop transfer function of the following system.



**Fig 8.1 Block diagram of a security camera to follow moving objects automatically**

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**EXERCISE 3:**

Write a MATLAB program for the above security camera system in which the gain of the system is varied from K=0 to K=50 with a step size of 5 and plot the pole zero diagram of the system.

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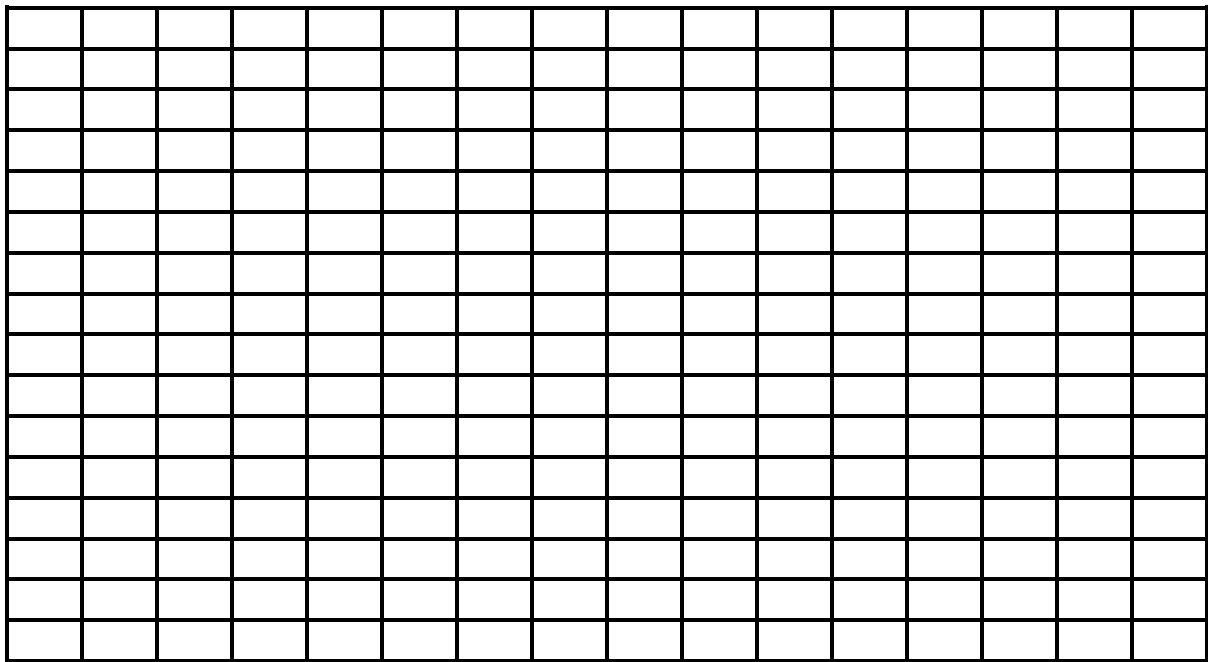
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**Fig 8.2 Pole Zero mapping of the security camera with the effect of gain**

**EXERCISE 4:**

Given a block diagram of a unity feedback system. Find the that the points,  $-2 + j3$ ,  $-2 + j(\sqrt{2}/2)$  is on its root locus or not. Also write a MATLAB code.









**EXERCISE 7:**

Apply the command you have learned in exercise 6, on the unity feedback system given in exercise 5 with the help of a MATLAB program and plot the output.

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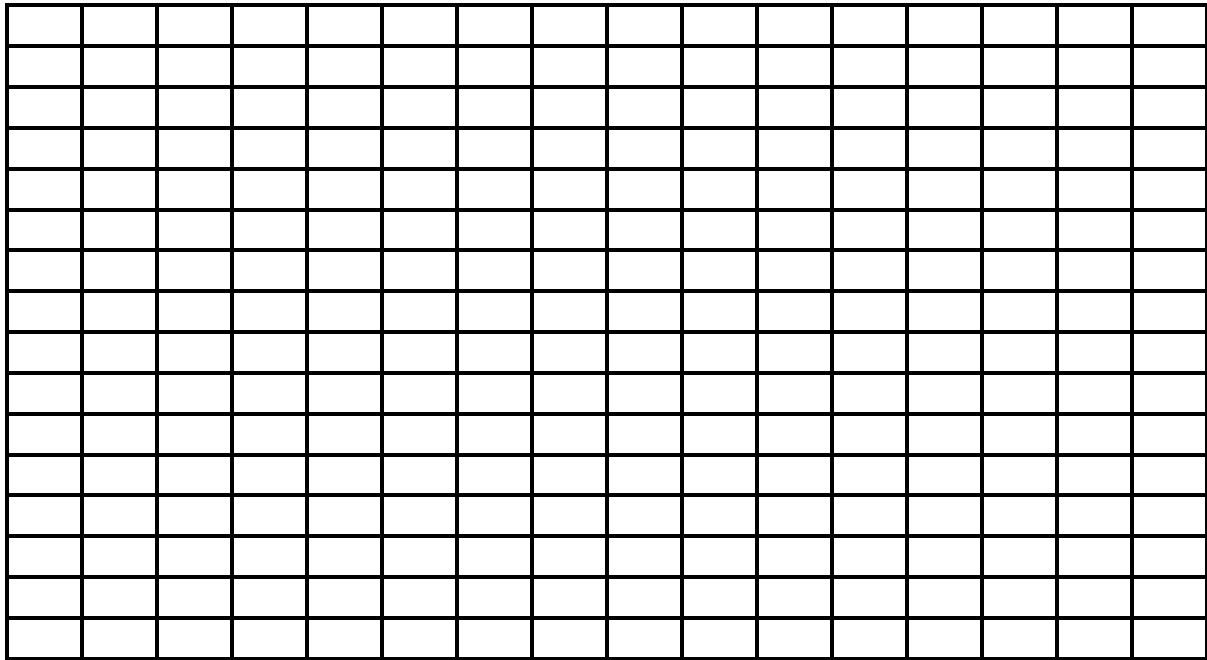
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**Fig 8.5 Root locus plot of the unity feedback system**

**EXERCISE 8:**

Given a unity feedback system that has the forward transfer function

$$G(s) = \frac{K(s + 2)}{(s^2 - 4s + 13)}$$





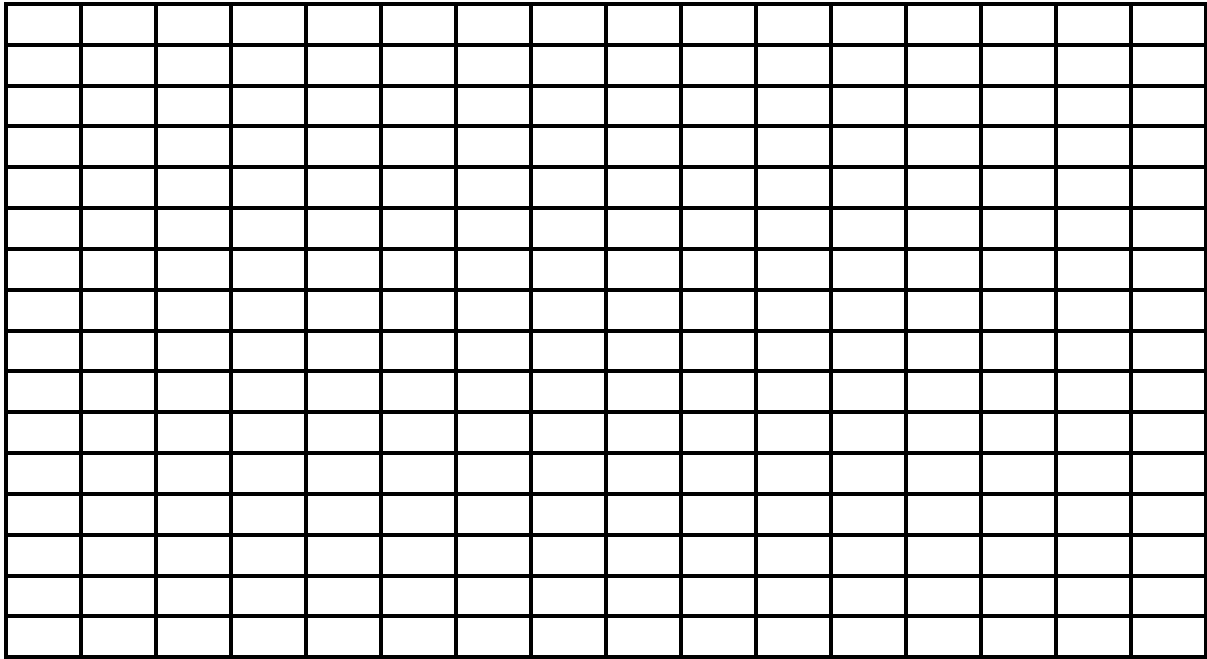


Fig 8.6 Root locus plot of the unity feedback system with refine sketching

**EXERCISE 9:**

Explore the command of “**sgrid**” in MATLAB and write about its function.

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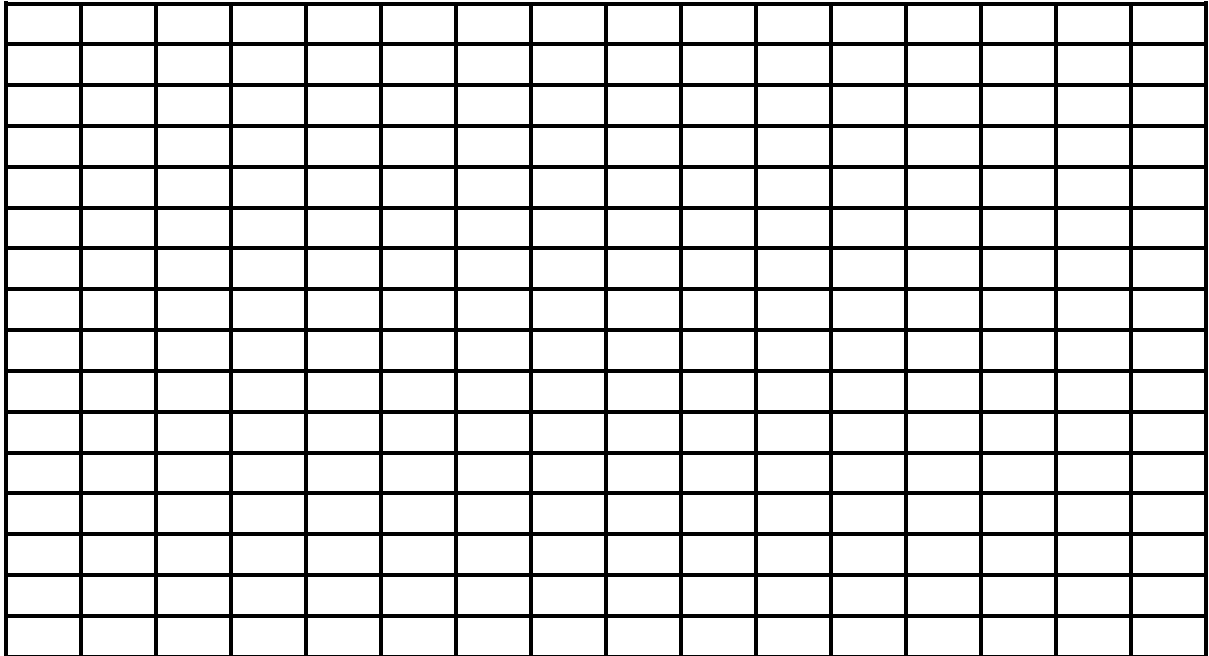
**EXERCISE 9:**

Explore the command of “**rlocfind**” in MATLAB and write about its function.

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**Fig 8.6** Root locus plot of the unity feedback system with damping ratio