LAB SESSION 7

Analysis of System Type and Steady State Errors

Objective:

In this lab session we will explore the steady state errors which is the difference between the input and the output for a prescribed test input as limit $t \to \infty$. Test inputs used for steady state error analysis are step, ramp and parabola. In which step input represents the constant position, ramp inputs represent the constant-velocity inputs and parabolas represents constant acceleration inputs to position control systems to determine the steady-state error performance.

Equipment Required:

PC and MATLAB® R2017b

Procedure:

The static errors can be calculated from the system's closed loop transfer function,T_s, or the open loop transfer function,$G(s)$, for a unity fed back system. The static error for a closed loop system shown in the fig can be calculated from the eq (1).

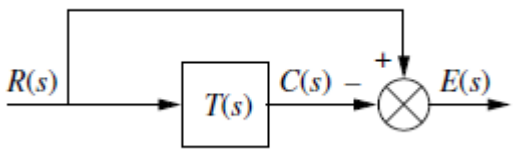

Fig 7.1 General representation

Fig 7.2 A representation for unity feedback systems

$$E(s) = R(s)[1 - T(s)] \tag{1}$$

As we can calculate the error for any instant of time from the equation (1) but most of the time we are interested in the final value of error, $e(\infty)$, which calculate by the given equation.

$$e(\infty) = \lim_{s \to 0} sR(s)[1 - T(s)] \tag{2}$$

Furthermore, we can find the static error from the system open loop transfer function by using the following equation.

$$E[s] = \frac{R(s)}{1+G(s)} \tag{3}$$

And the final value of error is given by

$$e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)} \tag{4}$$

Every system is tested for its statics error for the three basic signals which are step, ramp and parabola. To quantify the errors in terms of steady state error performance we used the definitions of static error constants. The definitions of static error constant are

For a step input,$u(t)$,

$$e(\infty) = e_{step}(\infty) = \frac{1}{1+\lim_{s\to 0} G(s)} \tag{5}$$

For a ramp input,$tu(t)$,

$$e(\infty) = e_{ramp}(\infty) = \frac{1}{\lim_{s\to 0} sG(s)} \tag{6}$$

For a parabolic input,$\frac{1}{2}t^2u(t)$

$$e(\infty) = e_{parabola}(\infty) = \frac{1}{\lim_{s\to 0} s^2G(s)} \tag{7}$$

The three terms in the denominator from the eq(5)-eq (7) are taken to the limit determine the steady state error. These limits are called static error constants and their individual names are

Positions constant

$$K_p = \lim_{s\to 0} G(s) \tag{8}$$

Velocity constant

$$K_v = \lim_{s\to 0} sG(s) \tag{9}$$

Acceleration constant

$$K_a = \lim_{s\to 0} s^2G(s) \tag{10}$$

These quantities are dependent on the $G(s)$, can assume the values of zero, finite constants or infinity. Since the statics error constant appears in the denominator of the steady state error eq (5) through eq (7), the value of the steady error decrease as the static error constant increases.

Th values of the static error constants depend upon the form of $G(s)$, especially the number of pure integrators in the forward path. Since the steady state errors are dependent upon the number of integrators, we give a name to this system attribute, given a system in the fig

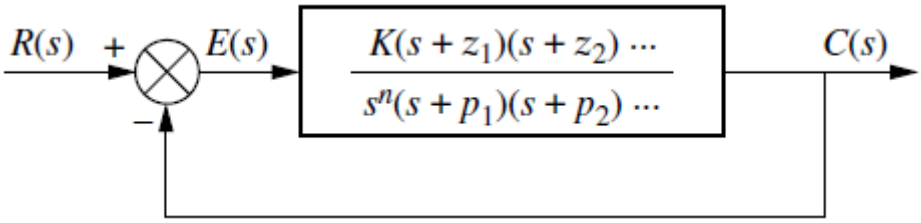

Fig 7.3 Feedback control system for defining the system type

We define the system type to be the value of n in the denominator or equivalently, the number of pure integrators in the forward path. Therefore, a system with n=0 is a type zero system, n=1 is a type one and n=2 is a type 2 system. The complete static errors are depicted in the following table against the system type.

		Type 0		Type 1		Type 2	
Input	**Steady-state error formula**	**Static error constant**	**Error**	**Static error constant**	**Error**	**Static error constant**	**Error**
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p = \text{Constant}$	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v = \text{Constant}$	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a = \text{Constant}$	$\frac{1}{K_a}$

EXERCISE 1:

Find the steady errors for the inputs of $5\,u(t), 5tu(t), 5t^2u(t)$ to the system shown in the fig. the function u(t) is the unit step.

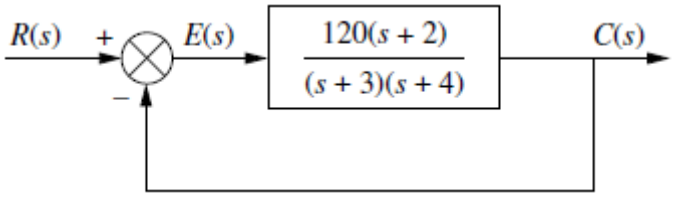

Fig 7.4 Feed back control system

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EXERCISE 2:

Draw the following closed loop system in the Simulink and find the steady state errors and plot them for the different inputs of $5\,u(t), 5tu(t), 5t^2u(t)$, compare it with exercise1 and comment on it.

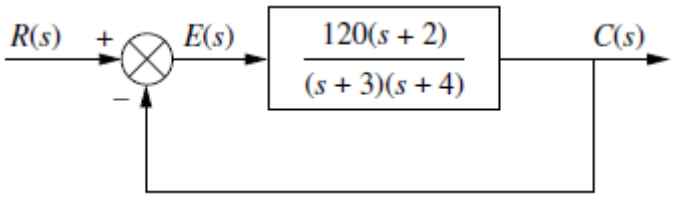

Fig 7.4 Feedback control system

Fig 7.5 Simulink diagram of the closed loop system

Fig 7.6 steady state error for *5u(t)*

Fig 7.7 steady state error for *5tu(t)*

Fig 7.8 steady state error for $5t^2u(t)$

EXERCISE 3:

Find the steady errors for the inputs of $5\,u(t), 5tu(t), 5t^2u(t)$ to the system shown in the fig. the function u(t) is the unit step.

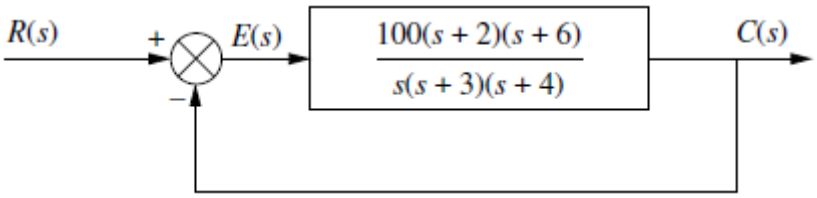

Fig 7.9 Feedback control system

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EXERCISE 4:

Draw the following closed loop system in the Simulink and find the steady state errors and plot them for the different inputs of $5\,u(t), 5tu(t), 5t^2u(t)$, compare it with exercise 2 and comment on it.

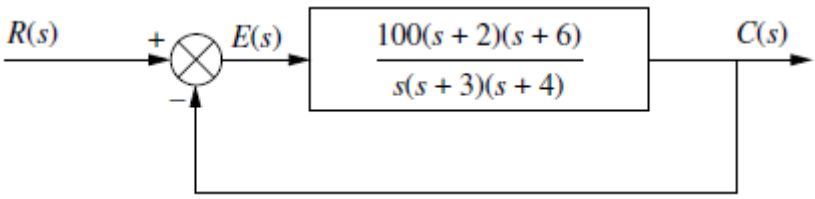

Fig 7.10 Feedback control system

Fig 7.11 Simulink diagram of the closed loop system

Fig 7.12 steady state error for *5u(t)*

Fig 7.13 steady state error for *5tu(t)*

Fig 7.14 steady state error for $5t^2u(t)$

EXERCISE 5:

Write a MATLAB program for the given system to find the system stability, evaluate the static error constants and steady-state errors

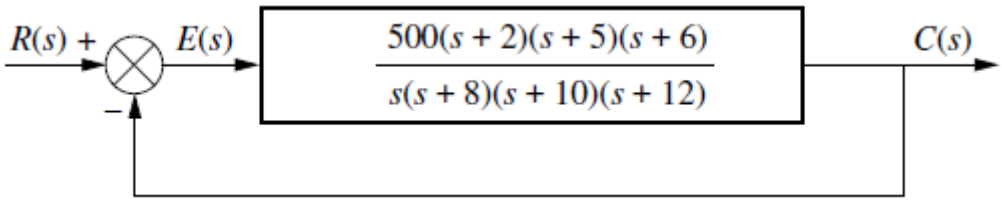

Fig 7.15 Feedback control system

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EXERCISE 6:

For the given control system in the fig write a MATLAB program to find the system type, value of K so that there is 10% error in steady state and plot its output.

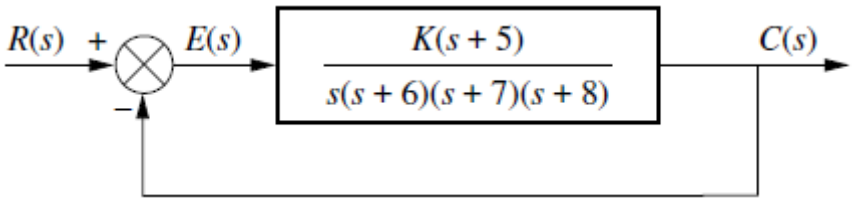

Fig 7.16 Feedback control system

Fig 7.17 Output of the Feedback system