LAB SESSION 4

Analysis of the Transient and Steady State Responses for the First and Second Order Systems

Objective:

In this experiment we analyzed the systems for their transient and steady state responses to see if these characteristics yields the desired behavior. After discussion the valuable analysis and design tool, pole zeros, we will analyze our systems to find the step response for the first and the second order systems. Furthermore, we analyze the effects of the additional poles and zeros on the system.

Equipment Required:

PC and MATLAB® R2017b

Procedure:

The poles and zeros analysis of a system give us a rapid and productive results, if we find the result by inspection, we sometime refer to this method as qualitative method. The poles of a system are the roots of the denominator of the transfer function which causes the transfer function to become infinite and the roots of the denominator of the transfer function that are common to roots of the numerator. The zeros of the transfer function are the values numerator which causes the transfer function to become zero and the numerator of the transfer function that are common to the roots of the denominator.

EXERCISE 1:

Explore the **"zpk"** command in the MATLAB and write down its function.

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EXERCISE2:

Find the poles and zeros of the following transfer function using the matlab command.

$$T(s) = \frac{s^2 + 3s + 2}{s^4 + 4s^3 + 6s^2 + 3s + 2}$$

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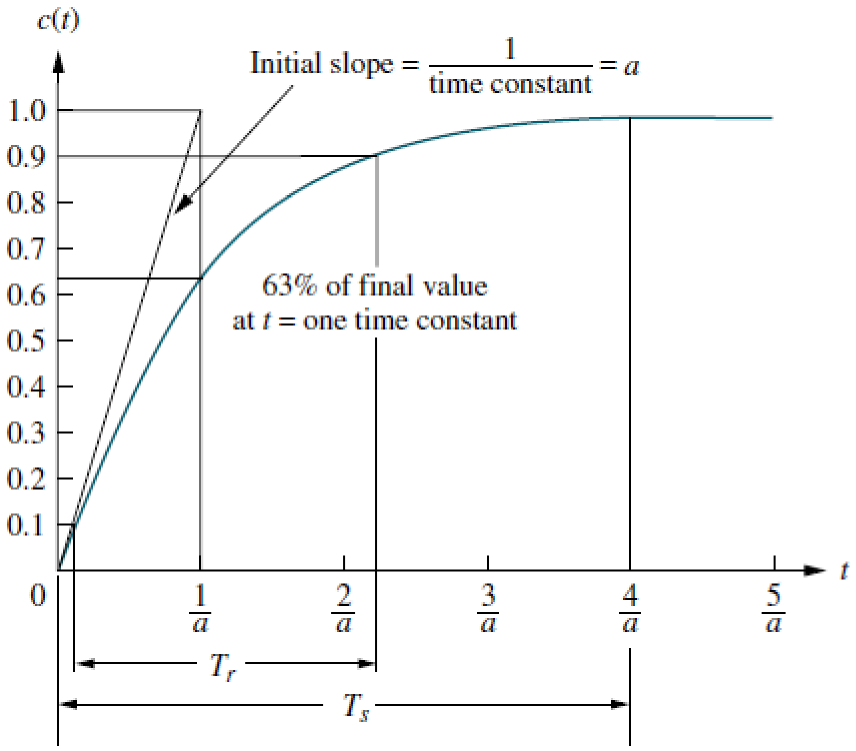
Let's assume that we have given a unit step in to a first order system the complete solution of a first order system is given by the following equation

$$c(t) = c_f(t) + c_n(t) \tag{1}$$

$$c(t) = 1 - e^{-at}$$

Were the input pole at the origin generates the forced response $c_f(t) = 1$ and te system pole at $-a$ generated the natural response $c_n(t) = -e^{-at}$. The response has been plotted in the following fig



Now we examine the significance of the parameter $\boldsymbol{a}$, the only parameter is needed to describe the transient response. Wen $t = 1/a$,

$$e^{-at}|_{t=1/a} = e^{-1} = 0.37 \quad (2)$$

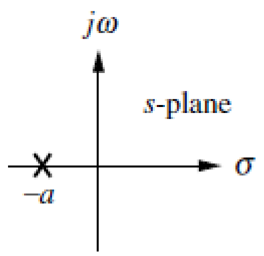
Or

$$c(t)|_{t=1/a} = 1 - e^{-at}|_{t=1/a} = 1 - 0.37 = 0.63 \quad (3)$$

Now by using the eq (1), eq(2) and eq(3) we define tree transient response performance specifications.

<u>Time constant:</u>

We call the $1/a$ te time constant of te response. The time constant can be described as the time for e^{-at} to decay 37% of its initial value. Alternatively, from eq (3) it is defined as the step response to reach 63% of its final value. The reciprocal of the time constant as the units (1/seconds), or frequency. Thus, we call the parameter a the exponential frequency. Since the derivative of $e^{-at}$is $-a$ wen $t = 0$,a is te initial rate of cane of the exponential at $t = 0$. Thus, the time constant can be considered a transient response specification for a first order system, since it is related to the speed at which the system responds to a step input. The time constant can be evaluated from the pole plot as sown in the figure 4.4b.



since the pole of the transfer function is at $-a$, we can say te pole is located at the reciprocal of the time constant, and farter the pole from the imaginary axis, the faster the transient response.

The transient response specifications, such as rise time,T_r, and settling time,T_S is given as follows

<u>Rise Time T_r:</u>

Rise time is defined as the time for the waveform to reach from 0.1 to 0.9 of its final values. Solving the equation (1) we have

$$T_r = \frac{2.2}{a}$$

<u>Settling Time T_s:</u>

Settling time is defined as the time for the response to reach, and witin,2% of its final value. Solving the equation (1) we have

$$T_s = \frac{4}{a}$$

EXERCISE3:

Find the time constant T_c ,settling time T_s and rise time T_r for the following transfer function.

$$T(s) = \frac{50}{s + 50}$$

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EXERCISE4:

Plot the step response for the following first order system using MATLAB. From your plots, find the time constant, rise time, and settling time.

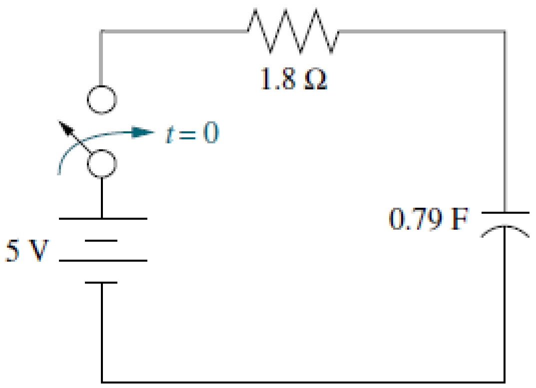

Figure 4.1 First Order Electrical System

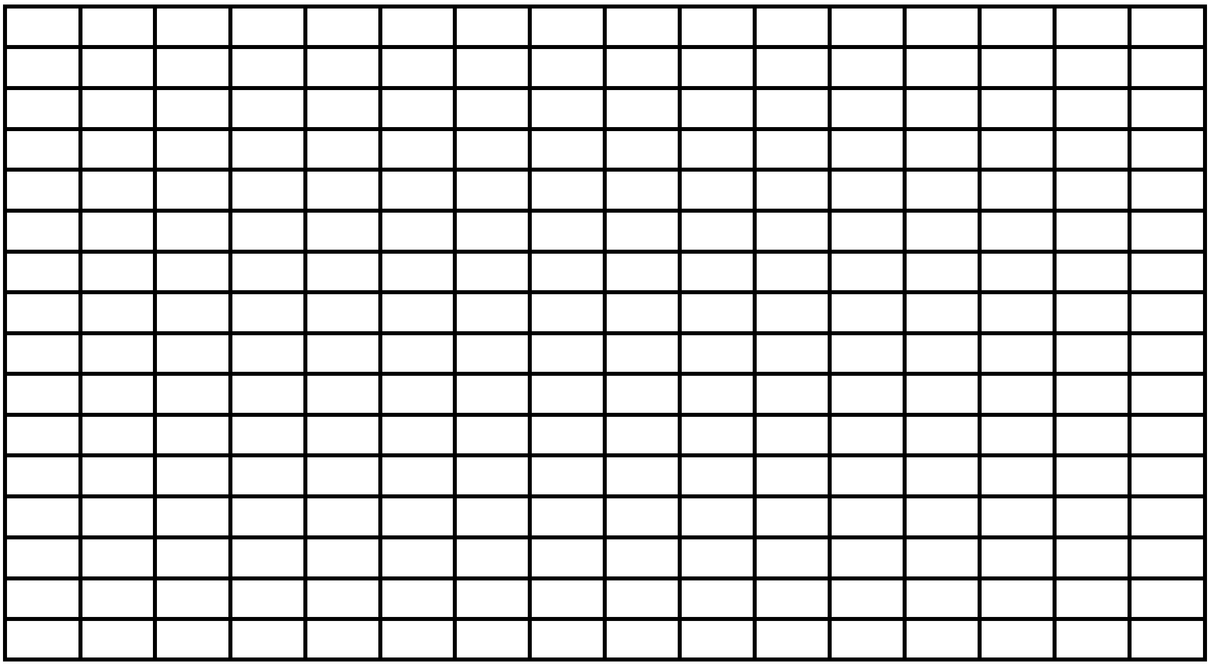

Figure 4.2 Step response of the Electrical system

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EXERCISE5:

Plot the step response for the following first order system using MATLAB. From your plots, find the time constant, rise time, and settling time. Use *M=1* and *M=2*.

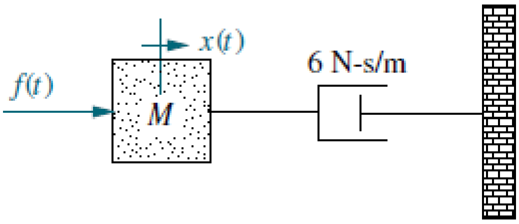


Figure 4.3 First Order Mechanical System

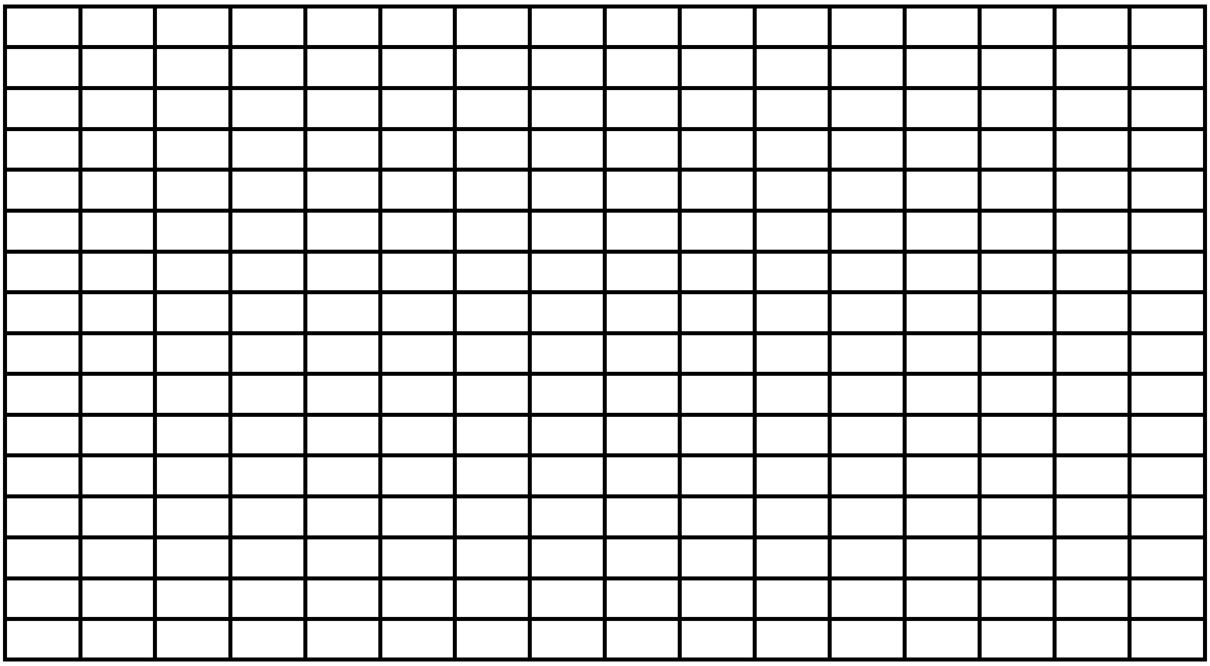


Figure 4.4 Step response of the Mechanical system

Second Order Systems:

Compared to the simplicity of a first-order system, a second-order system exhibits a wide range of responses that must be analysed and described. Whereas varying a first-order system's parameter simply changes the speed of the response, changes in the parameters of a second-order system can change the form of the response. For example, a second-order system can display characteristics much like a first-order system, or, depending on component values, display damped or pure oscillations for its transient response. A simple second order system is given in the form of transfer function as

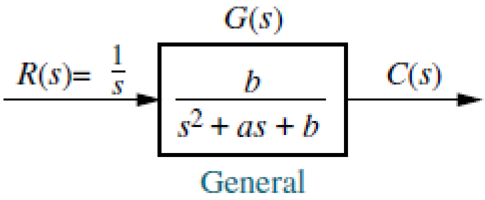


Figure 4.5 A general second order system

Now by changing the values of the $a\ and\ b$ and giving a unit step input, we get the different responses like

i. Over damped
ii. Underdamped
iii. Undamped
iv. Critically damped

Overdamped responses:

Poles: Two real at σ_1; σ_2
Natural response: Two exponentials with time constants equal to the reciprocal of the pole locations, or

$$c(t) = K_1 e^{-\sigma 1t} + K_2 e^{-\sigma 2t}$$

Underdamped responses:

Poles: Two complex at $-\sigma_d \pm j\omega_d$
Natural response: Damped sinusoid with an exponential envelope whose time constant is equal to the reciprocal of the pole's real part. The radian frequency of the sinusoid, the damped frequency of oscillation, is equal to the imaginary part of the poles, or

$$c(t) = Ae^{-\sigma_d t} \cos(\omega_d t - \varphi)$$

Undamped responses:

Poles: Two imaginary at $\pm j\omega_1$
Natural response: Undamped sinusoid with radian frequency equal to the imaginary part of the poles, or

$$c(t) = \mathrm{A}\cos(\omega_1 t - \varphi)$$

Critically damped responses:

Poles: Two real at $-\sigma_1$
Natural response: One term is an exponential whose time constant is equal to the reciprocal of the pole location. Another term is the product of time, t, and an exponential with time constant equal to the reciprocal of the pole location

$$c(t) = K_1 e^{-\sigma 1t} + K_2 e^{-\sigma 1t}$$

The combined responses are shown in the following fig

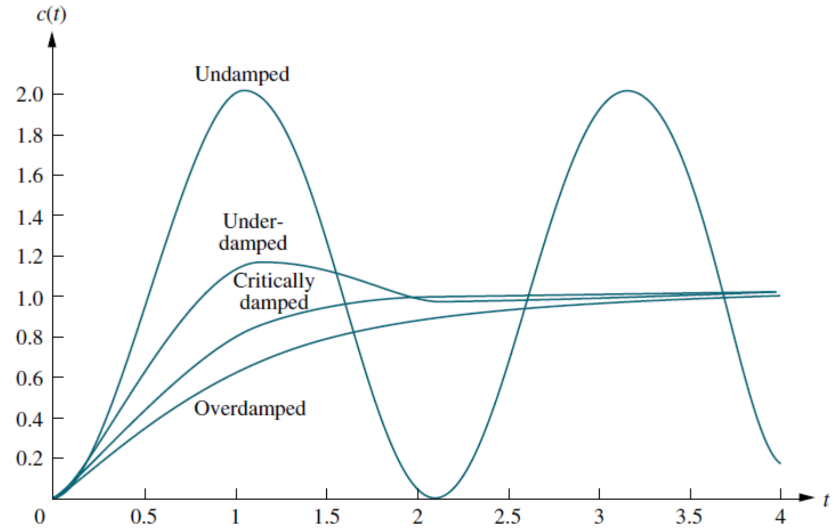
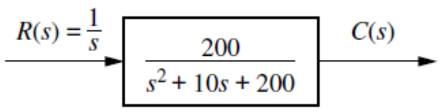


Figure 4.6 Step Responses second order system damping cases

EXERCISE6:

For the given system find the poles and plot the step response of the system and comment on the system output.



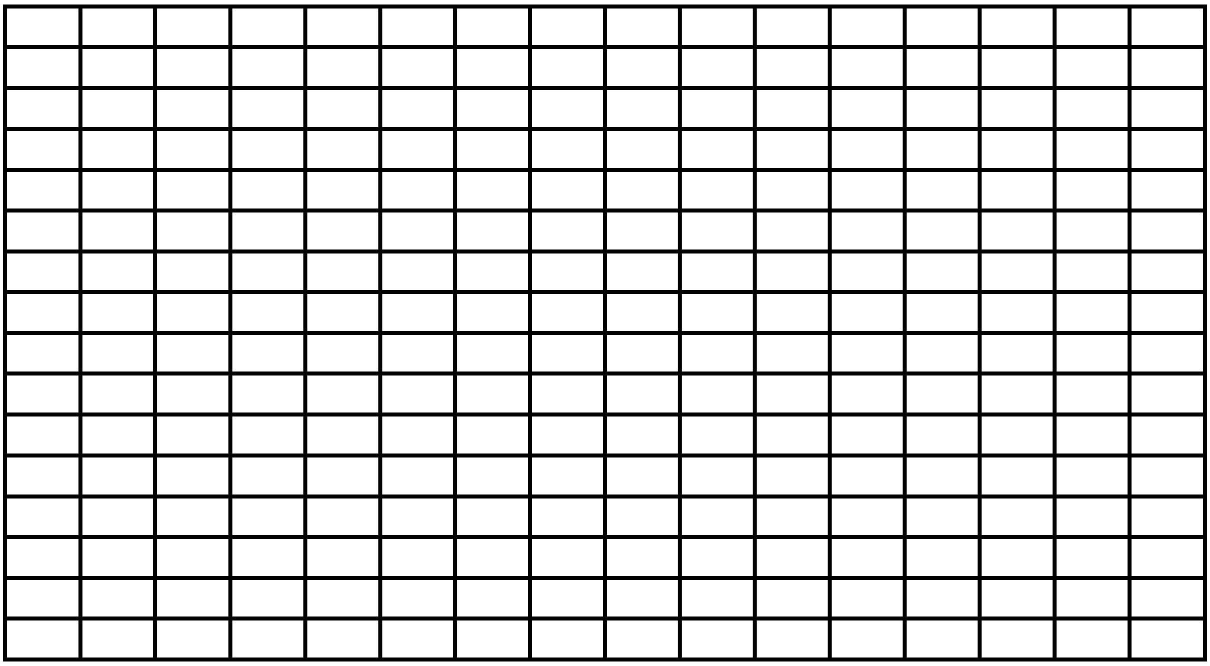


Figure 4.6 Step response of system

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Now, we define two physically meaningful specifications for second-order systems. These quantities can be used to describe the characteristics of the second-order transient response just as time constants describe the first-order system response. The two quantities are called natural frequency and damping ratio.

Natural Frequency, ω_n:

The natural frequency of a second-order system is the frequency of oscillation of the system without damping. For example, the frequency of oscillation of a series RLC circuit with the resistance shorted would be the natural frequency.

Damping Ratio, ξ:

A viable definition for this quantity is one that compares the exponential decay frequency of the envelope to the natural frequency. This ratio is constant regardless of the time scale of the response. Also, the reciprocal, which is proportional to the ratio of the natural period to the exponential time constant, remains the same regardless of the time base.
We define the damping ratio, ξ, to be

$$\xi = \frac{Exponential\ decay\ frequency}{Natural\ Frequency(rad/sec)} = \frac{1}{2\pi}\frac{Natural\ period(seconds)}{Exponential\ time\ constant}$$

Now our second order system will be

$$G(s) = \frac{{\omega_n}^2}{s^2+2\xi\omega_n+{\omega_n}^2}$$

EXERCISE7:

For the given transfer function, find ξ and ω_n

$$G(s) = \frac{36}{s^2+4.2s+36}$$

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Now that we have defined ζ and ω_n, let us relate these quantities to the pole location.

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

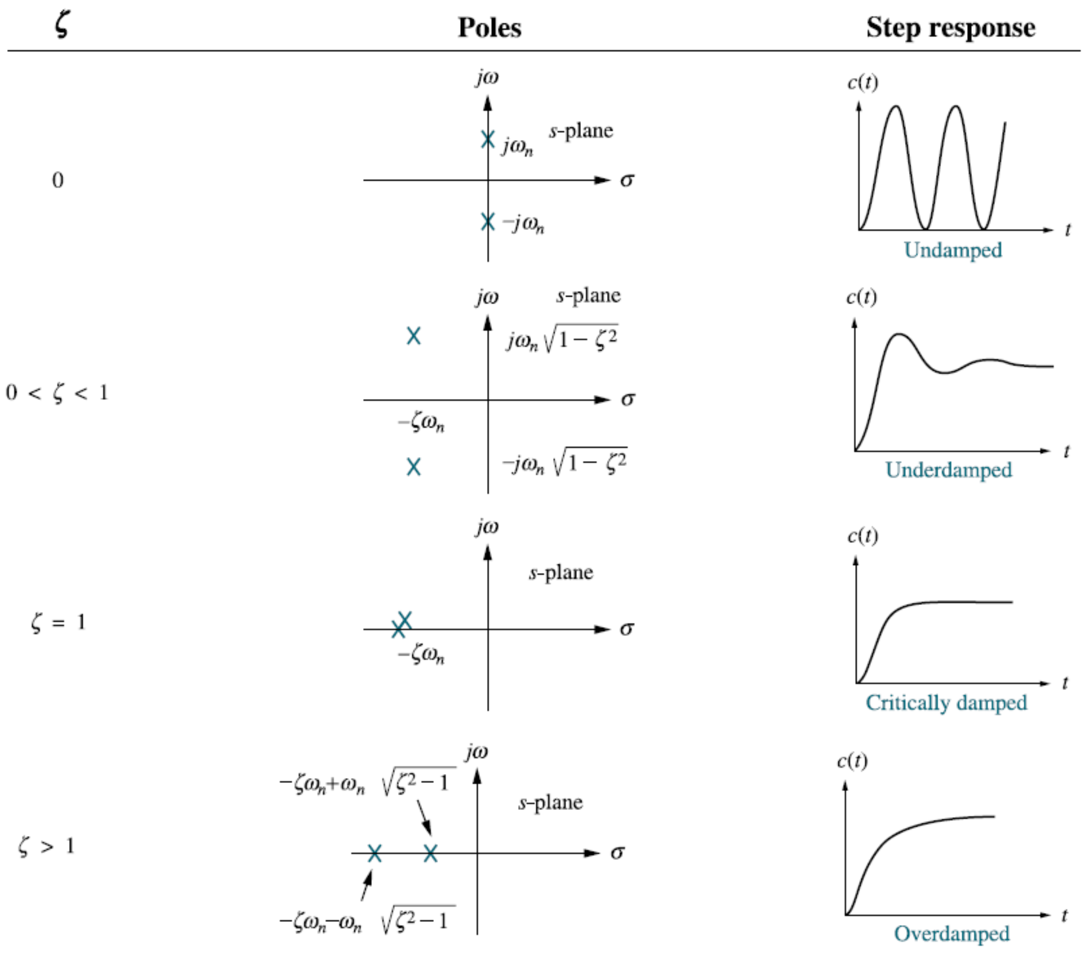

Figure 4.7 second order response as a function of damping ratio

EXERCISE8:

Find the value of the ζ in the given systems, comment on the output response and plot the output of the systems.

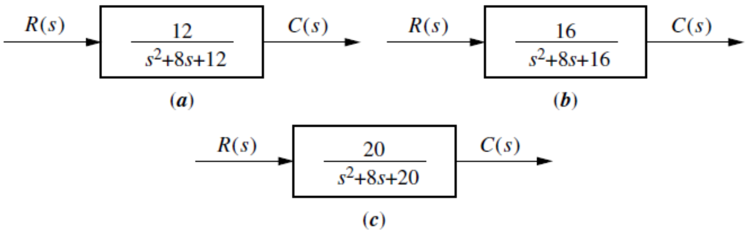

Figure 4.7 Step response of the system (a)

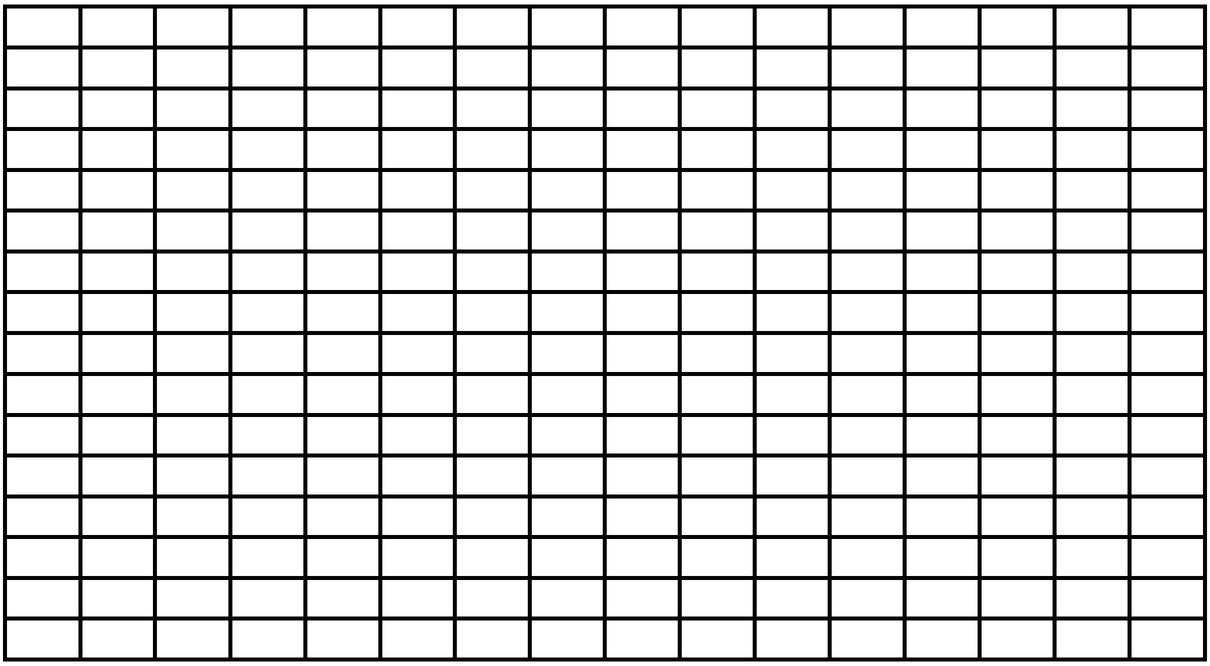

Figure 4.8 Step response of the system (b)

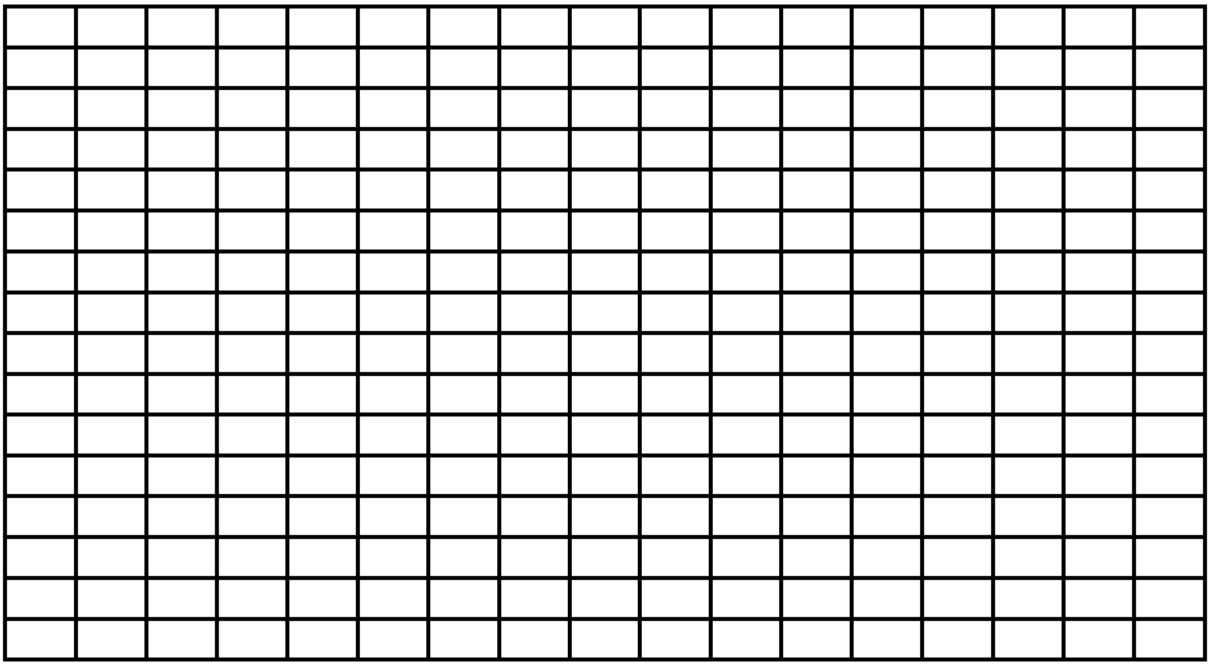

Figure 4.9 Step response of the system (c)

We have defined two parameters associated with second-order systems, ξ andω_n. Other parameters associated with the underdamped response are rise time, peak time, percent overshoot, and settling time. These specifications are defined as follows, see also Figure.

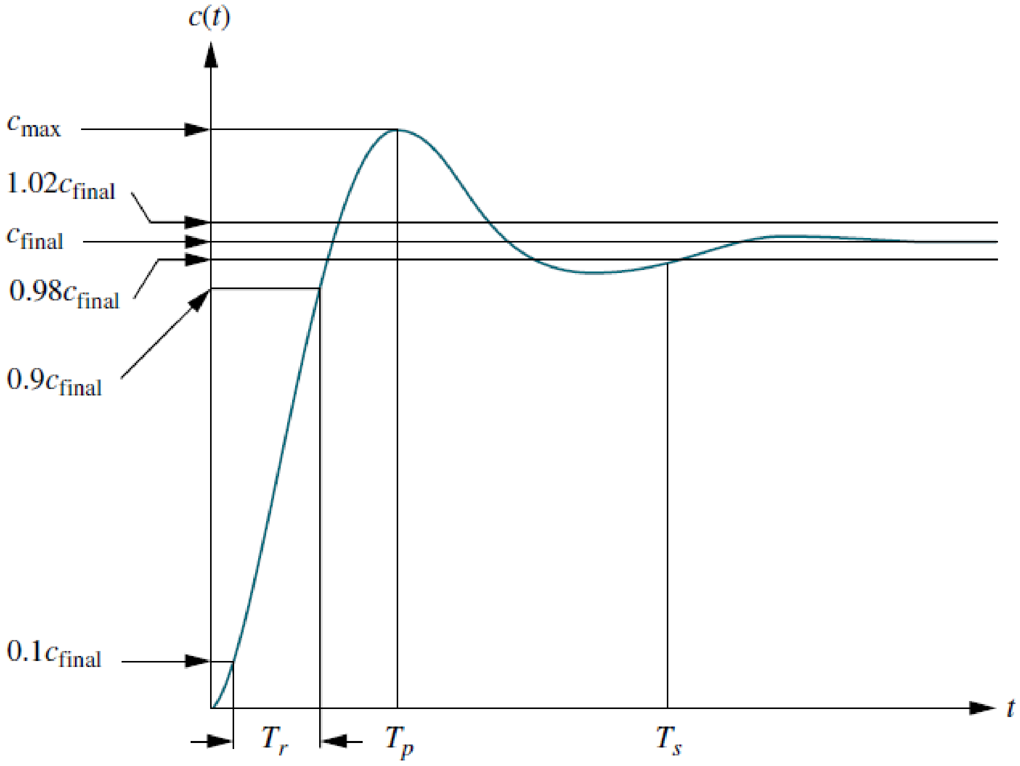

Figure 4.10 Second order underdamped response specifications

Rise time, Tr:

The time required for the waveform to go from 0.1 of the final value to 0.9 of the final value.

Peak time, TP:

The time required to reach the first, or maximum, peak. Which is find as follows

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

Percent overshoot, %OS:

The amount that the waveform overshoots the SteadyState, or final, value at the peak time, expressed as a percentage of the steady state value. Which is found by using as follows

$$\%OS = \frac{c_{max} - c_{final}}{c_{final}} \times 100$$

To find %OS given ξ.

$$\%OS = e^{-\left(\xi\pi / \sqrt{1-\xi^2}\right)} \times 100$$

<u>Settling time, Ts</u>:

The time required for the transient's damped oscillations to reach and stay within $\pm 2\%$ of the steady-state value. Which is given as follows

$$T_s = \frac{4}{\xi\omega_n}$$

<u>EXERCISE9:</u>

Find the Ts, %OS, T_p, Tr for the following transfer function and plot the step response of the system.

$$G(s) = \frac{100}{s^2 + 15s + 100}$$

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Figure 4.11 Step response of the system

EXERCISE10:

Find the $\xi, \omega_n, T_s, T_p, T_r,$ and $\%OS$ for the given system using the MATLAB code.

$$G(s) = \frac{361}{s^2 + 16s + 361}$$

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EXERCISE11:

Explore the command of **"stepinfo",** use it to find the transient response for the system given in exercise 10 and compare your answer and comment on it.

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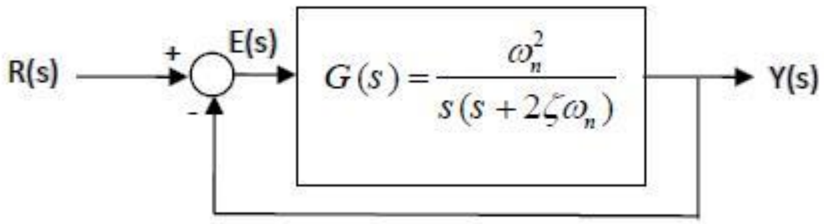
To study the effects of damping ratio on the performance measures, consider the following second-order single-loop unity feedback system.

EXERCISE12:

Find the step response of the system for values of ωn = 1 and ζ = 0.1, 0.4, 0.7,1.0 and 2.0 and also plot the combined response with different values of ξ.



Plot all the results in the same figure window and fill the following table.

$\xi(damping\ ratio)$	Rise time T_r	Peak Time T_p	Settling time T_p	% Overshoot	Steady State Error
0.1					
0.4					
0.7					
1.0					
2.0					

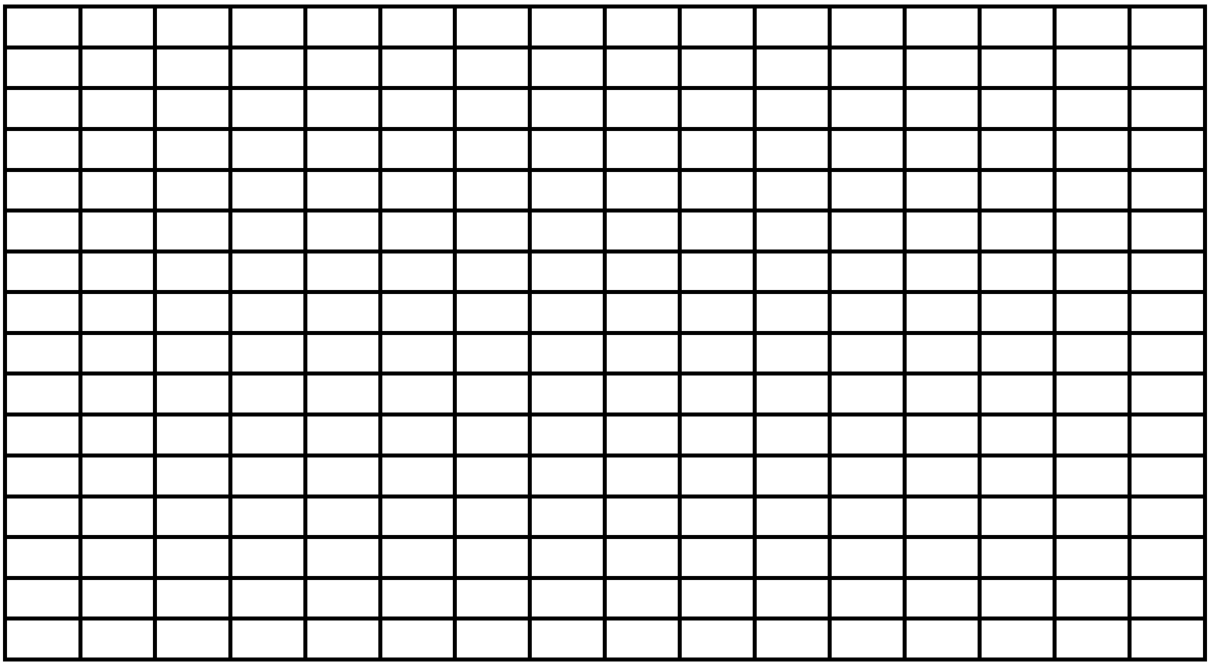


Figure 4.11 combined response for different values of ξ