

## LAB SESSION 2:

### Mathematical Modeling of Physical Systems and Analysis of their Behavior using Matlab/Simulink

#### Objective:

In this lab session, we deal with the mathematical modeling of the mechanical and electrical systems and use the MATLAB commands to find the transfer function of the respective systems and plot their step and impulse responses. Furthermore, we design our electrical and mechanical models in Simulink and observe the impulse and step responses.

#### Equipment Required:

PC and MATLAB® R2017b

#### Procedure:

Consider the following mechanical system having its three main components, mass (M), spring(K) and friction(B). To analyze the behavior of this mass spring system, the first thing is to find its transfer function, which is defined as the relationship between the output over input in frequency domain ( $G(s) = \frac{X(s)}{F(s)}$ ).

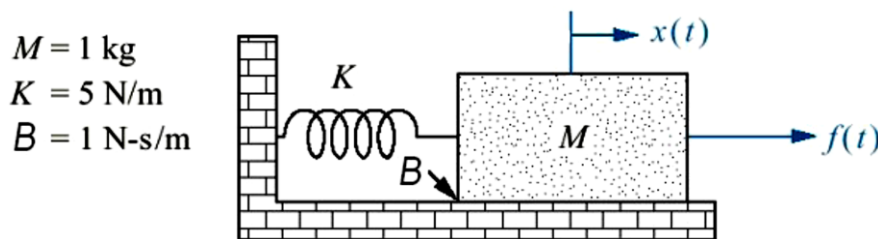


Figure 2.1: A mass-spring-damper system

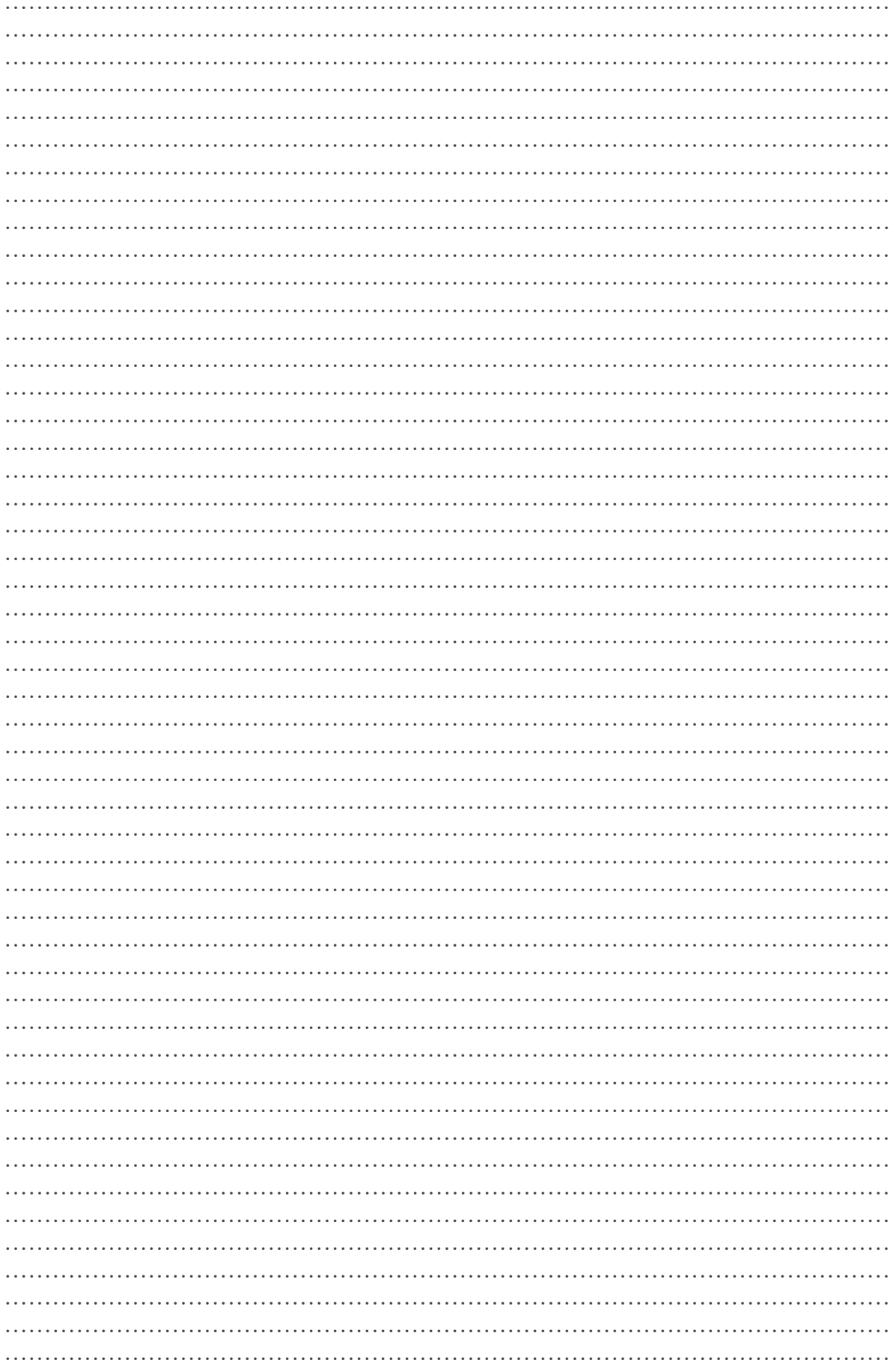
To find the transfer function of the above system follow the following steps

- I. First draw the free body diagram in time domain.
- II. Draw the second free body diagram in frequency domain.
- III. Write the equation in frequency domain by considering all the forces opposing the applied force on the one side of the equation.
- IV. By taking the common term in the form of displacement  $X(s)$  as output and  $F(s)$  as input, then write the transfer function in the form of  $G(s) = \frac{X(s)}{F(s)}$ .

#### EXERCISE 1:

Verify the given transfer function by following the above steps for the fig2.1.

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K} = \frac{1}{s^2 + Bs + K}$$



**EXERCISE 2:**

The transfer function of a system can be find by using the command of **tf (num,den)**. Now find the transfer function of the above system by giving the coefficients of numerator and denominator.

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**EXERCISE 3:**

Wat is the characteristic equation of the above system and calculate the natural frequency  $\omega_n$  of the system by using the formula

$$\omega = \omega_n \sqrt{(1 - \xi^2)}$$

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**Step Response:**

Step response of a system corresponds to its behavior when a step input is applied. In MATLAB the step response of a system can be plotted by using the step function.

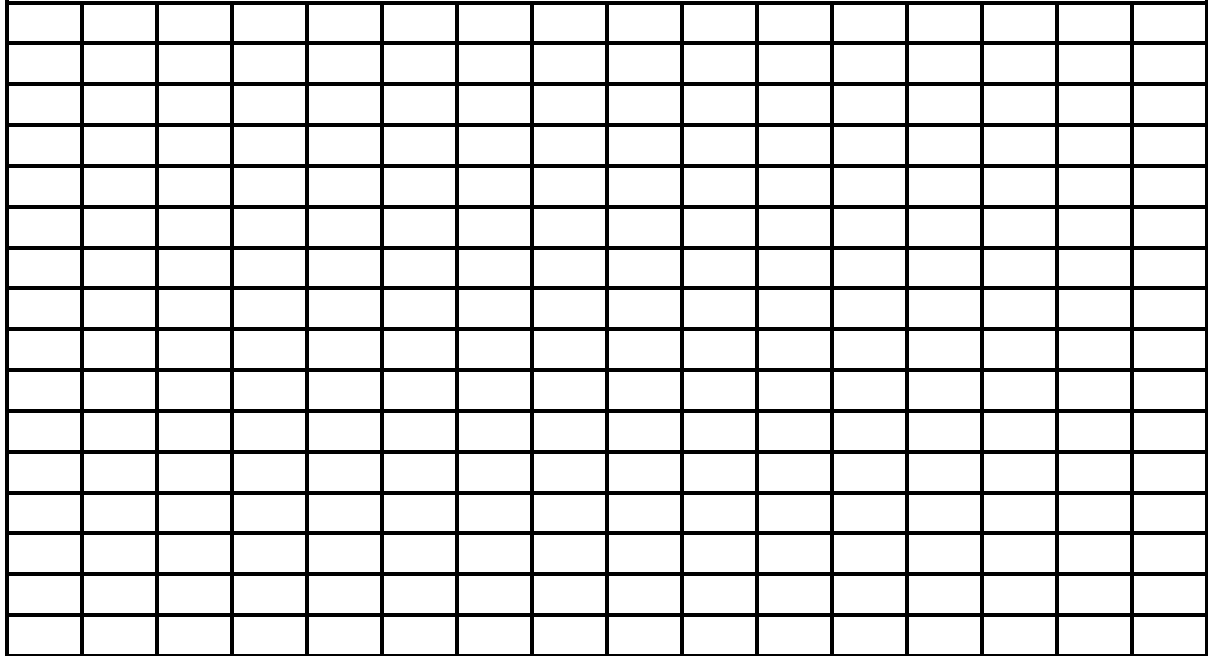
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step(sys);
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By using the above command, MATLAB terminates the output at 12 sec because the system is settled. The final time can be changed by using the following command were the final time is 30 sec.

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step(sys,30);
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**EXERCISE 4:**

Plot the step response of the system shown in the fig 2.1 by using the *step(sys)* command. Where *sys = tf(num, den)*.



**Figure 2.2 Step response of the mechanical system**

Write your comment on the step response of the above mechanical system

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**Impulse Response:**

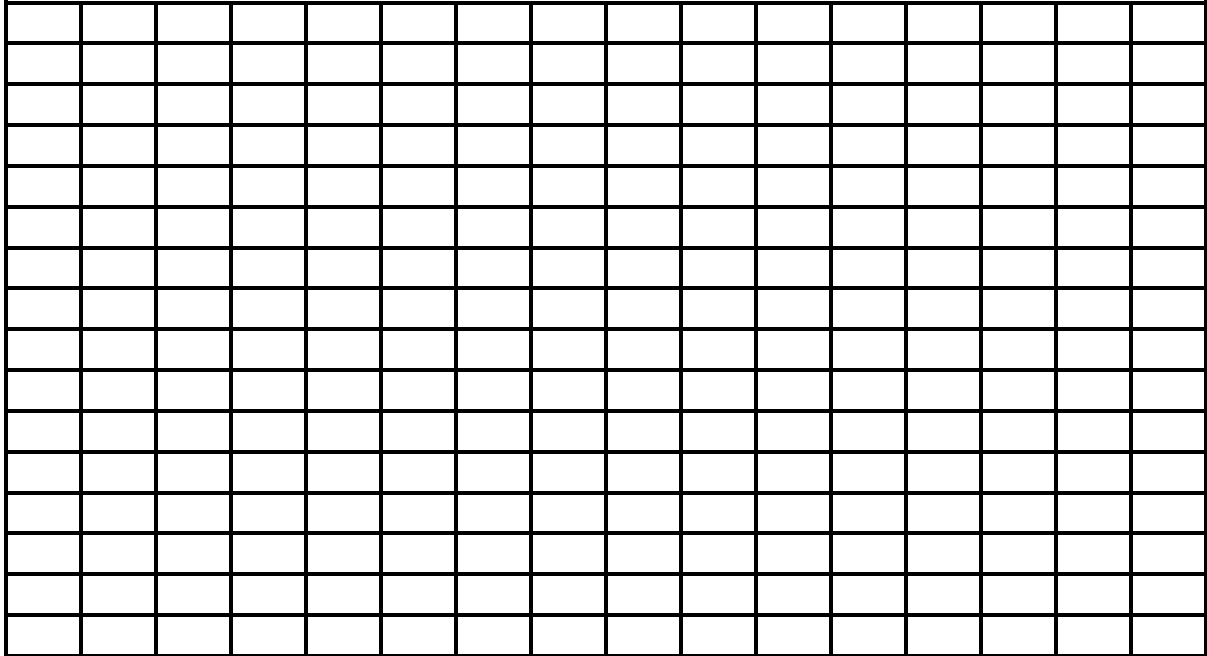
An impulse response is the reaction of any dynamic system in response to some external change. In both cases, the impulse response describes the reaction of the system as a function of time (or possibly as a function of some other independent variable that parameterizes the dynamic behaviour of the system).

The impulse response of the system is plotted in MATLAB by using the following command.

**Impulse(sys)**

**EXERCISE 5:**

Plot the impulse response of the system shown in the fig 2.1 by using the *impulse(sys)* command. Where  $sys = tf(num, den)$ .



**Figure 2.3 impulse response of the mechanical system**

Write your comment on the impulse response of the above mechanical system

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**MODELING AN ELECTRICAL SYSTEM:**

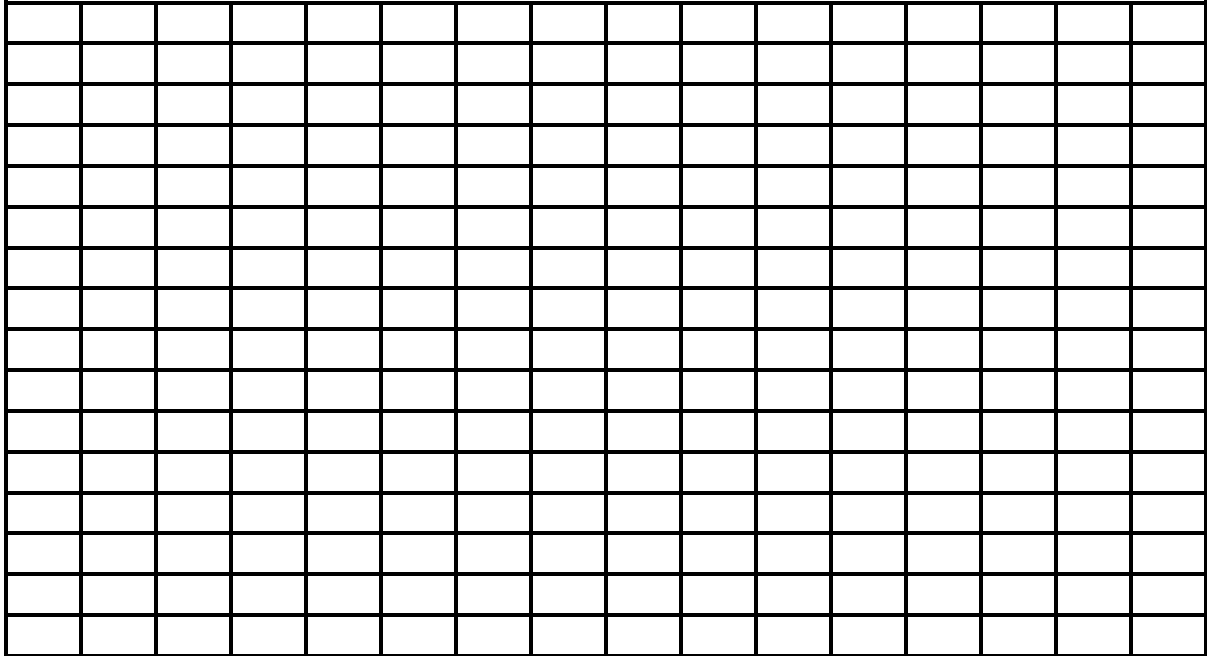
Now let's take an example of commonly used an electrical circuit and try to develop its mathematical model. Given below is a series RLC circuit.





**EXERCISE 8:**

Plot the step response of the system shown in the fig 2.2 by using the *step(sys)* command. Where *sys = tf(num, den)*.



**Figure 2.5 Step response of the Electrical System**

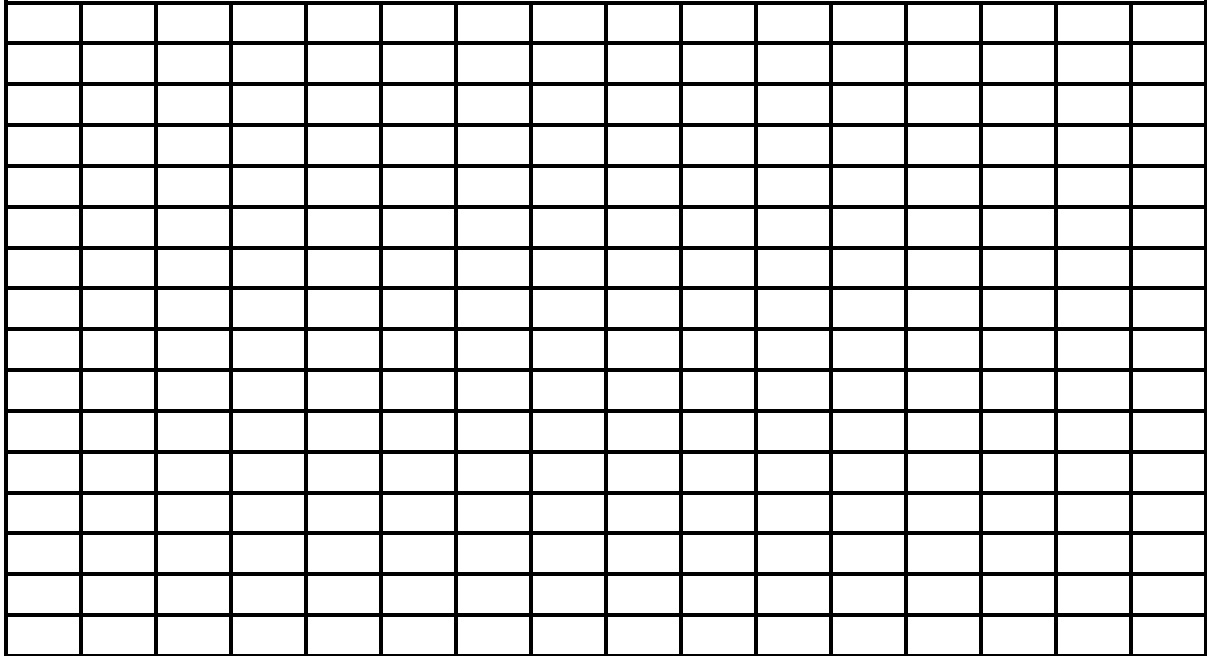
Write your comment on the step response of the above electrical system

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**EXERCISE 9:**

Plot the impulse response of the system shown in the fig 2.2 by using the *impulse(sys)* command. Where  $sys = tf(num, den)$ .



**Figure 2.6 impulse response of the electrical system**

Write your comment on the impulse response of the above electrical system.

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# SIMULINK

It is a MATLAB-based graphical programming environment for modelling, simulating and analyzing multidomain dynamical systems. Its primary interface is a graphical block diagramming tool and a customizable set of block libraries. It offers tight integration with the rest of the MATLAB environment and can either drive MATLAB or be scripted from it. Simulink is widely used in automatic control and digital signal processing for multidomain simulation and model-based design.

Again, consider the spring-mass-damper system:

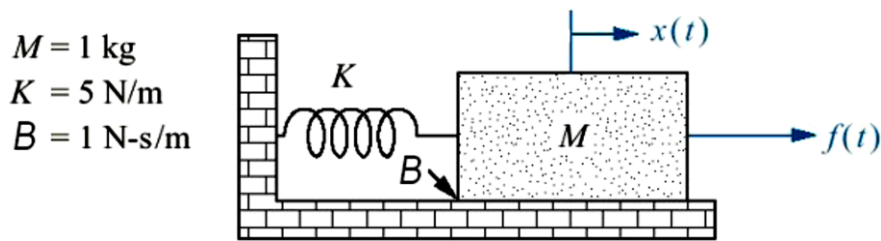


Figure 2.1: A mass-spring-damper system

The differential equation of the above model is given by:

$$M \cdot \frac{d^2x}{dt^2} + B \cdot \frac{dx}{dt} + Kx = f(t)$$

$$\frac{d^2x}{dt^2} = \frac{f - B \cdot \frac{dx}{dt} - K \cdot x}{M}$$

Simulink model of the above differential equation

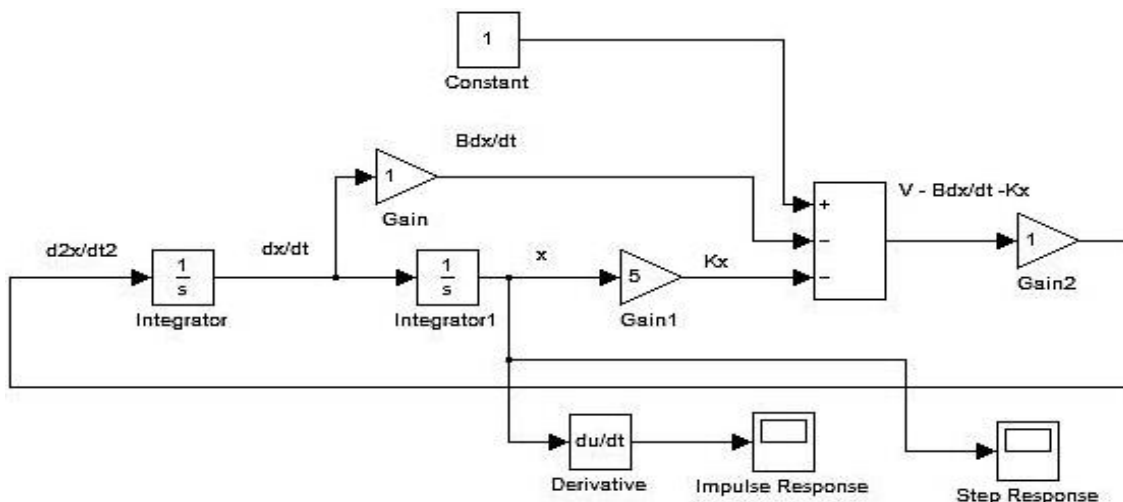
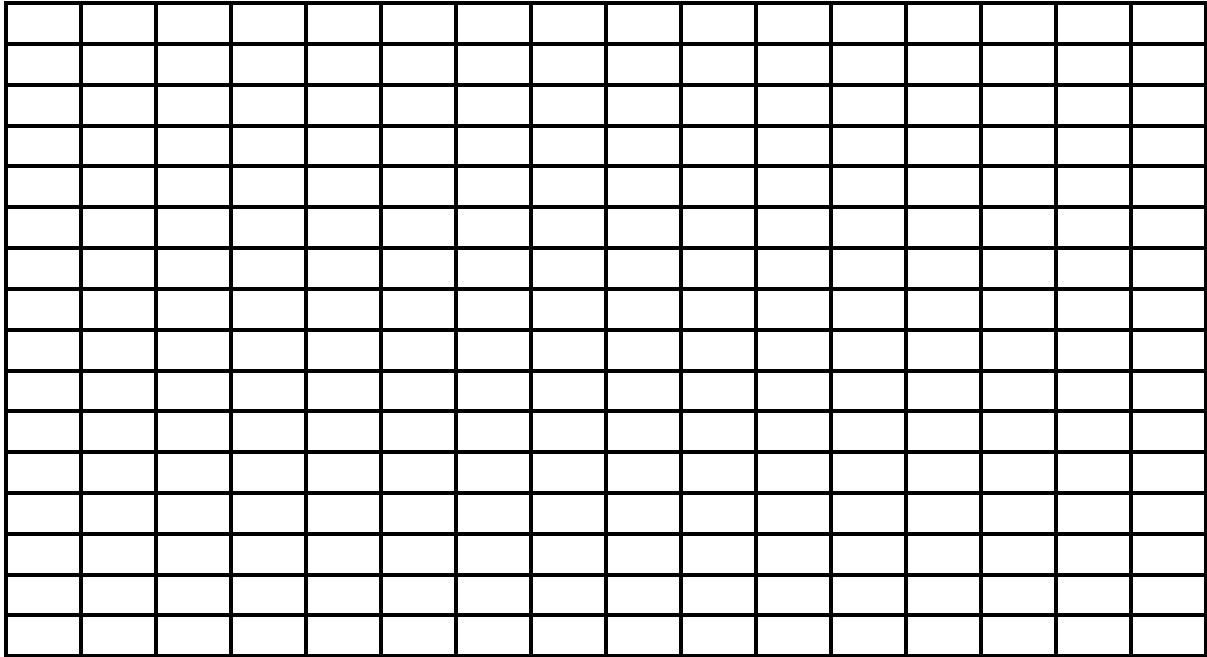


Figure 2.7: Simulink Model for differential equation of spring-mass-damper system

**EXERCISE 10:**

Implement above model and select appropriate scale and simulation parameters to get the step and impulse responses and sketch them below.



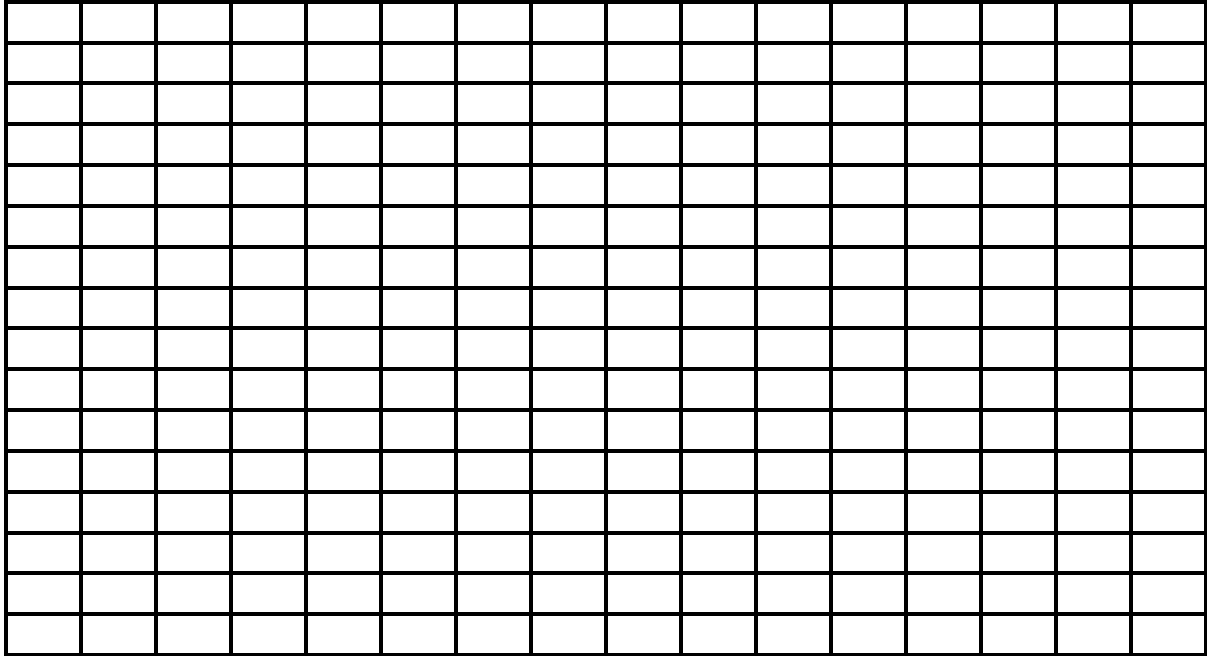
**Figure 2.7: Step response of mass-spring-damper system via Simulink**

Compare your answer with Exercise 4 and write your comment on the step response.

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**EXERCISE 11:**

Implement above model and select appropriate scale and simulation parameters to get impulse response and sketch it below.

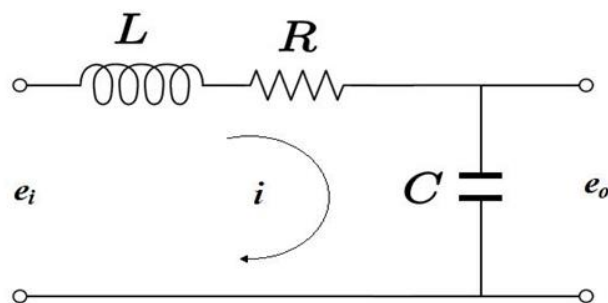


**Figure 2.8: Impulse response of mass-spring-damper system via Simulink**

Compare your answer with exercise 5 and write your comment on the impulse response .

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Consider the series RLC circuit given in the fig



**Figure 2.4 series RLC circuit**

By applying Kirchhoff's voltage law in the single loop, we have

$$L \frac{di}{dt} + Ri + \frac{1}{C} \int i dt = e_i$$

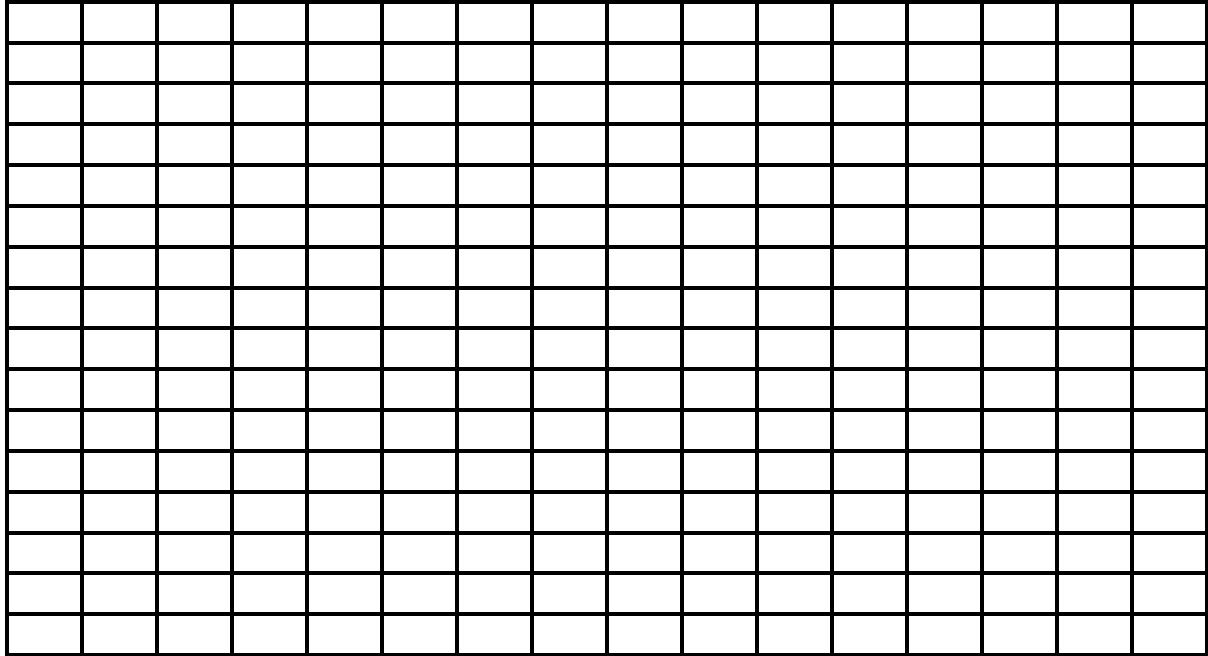
**EXERCISE 12:**

Find the second order differential form of the above series RLC circuit. Also implement and draw the Simulink model.

**Draw the block diagram here**

**EXERCISE 13:**

Implement the above Simulink model and select appropriate scale and simulation parameters to get the step response and sketch it below.



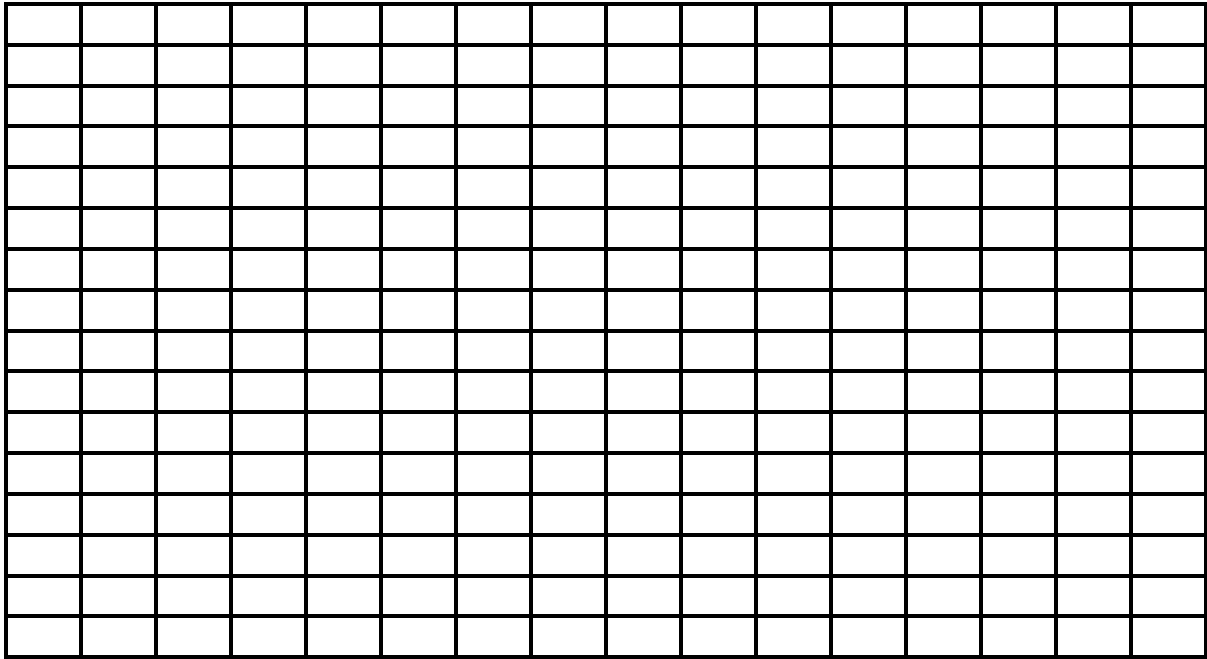
**Figure 2.7: Step response of series RLC system via Simulink**

Compare your answer with Exercise 8 and write your comment on the step response.

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**EXERCISE 14:**

Implement above model and select appropriate scale and simulation parameters to get impulse response and sketch it below.



**Figure 2.8: Impulse response of series RLC system via Simulink**

Compare your answer with exercise 9 and write your comment on the impulse response.

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