

Ordinal Logistic Regression



Ordinal Logistic Regression

- ✓ What is ordinal logistic regression?
- ✓ Where do we need to apply ordinal logistic regression?
- ✓ How it is solved / worked?
- ✓ How does it predict the outcome?

What is Ordinal logistic regression?

- General logistic Regression - Two possible outcome like
 - ✓ Responder / Non responder
 - ✓ Buyer / non buyer
- At times, the dependent variable is ordinal variable.
- Like, if we conduct a survey after presentation, say on predictive machine learning, the audience can respond like

<i>Very Satisfied</i>	<i>Satisfied</i>	<i>OK</i>	<i>Disatisfied</i>	<i>Very disatisfied</i>
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

- Here there is **order** in dependent variable. They are **not nominal** variables.
- A logistic regression model trying to use independent variables to predict, what is the likely response of each individual in the survey is **ordinal logistic regression**.
- in some sense the situation is intermediate between regression and classification

Another examples of ordinal logistic regression

- What do you think about state spending on health programs
 - ✓ Poor
 - ✓ Average
 - ✓ Good
- Level of insurance coverage
 - ✓ None
 - ✓ Partial
 - ✓ Full
- Employment status
 - ✓ Unemployed
 - ✓ Employed part-time
 - ✓ Fully employed

Other names of ordinal logistic regression

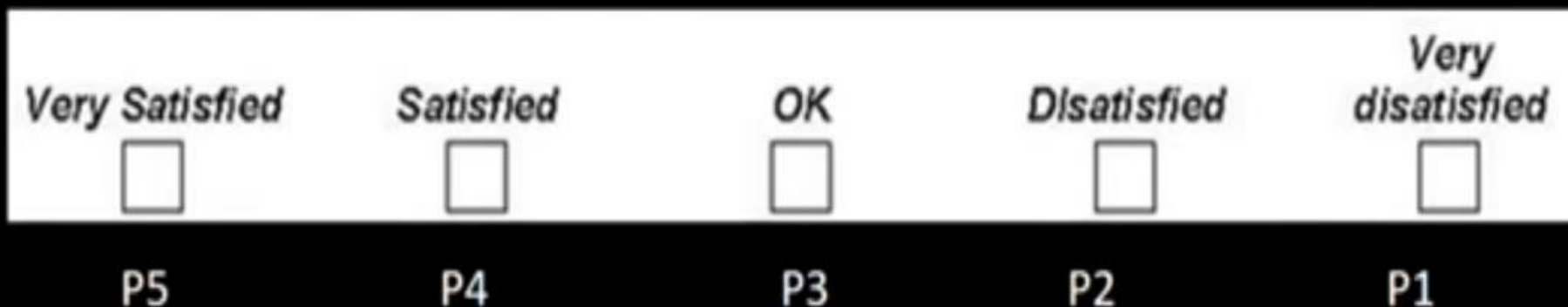
- Ordinal regression
- Ordered logit model
- Ordered probit model (very similar to ordered logit – The main difference is in the interpretation of the coefficients)
- Ordered logistic regression
- Proportional odds model
- In machine learning, ordinal regression may also be called ranking learning

How to solve the ordinal logistic regression?

- the proportional odds assumption

What is proportional odds assumption

- The model only applies to data that meet the proportional odds assumption
- Let me explain - Proportional odds assumption
- If probabilities of below events are



- Then the logarithms of odds (please note – not the probabilities) form an arithmetic series.
- Let's see it

What is proportional odds assumption

<i>Very Satisfied</i>	<i>Satisfied</i>	<i>OK</i>	<i>Dissatisfied</i>	<i>Very dissatisfied</i>
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
P5	P4	P3	P2	P1

Text meaning	Formula	Value	Value_1	Simple sequence
Very Dissatisfied	$\text{Log} \left(\frac{P1}{(P2+P3+P4+P5)} \right)$	A	B-3x	1
Dissatisfied or worse	$\text{Log} \left(\frac{(P1+P2)}{(P3+P4+P5)} \right)$	A + D	B-2x	2
Neutral or worse	$\text{Log} \left(\frac{(P1+P2+P3)}{(P4+P5)} \right)$	A + 2D	B-X	3
Satisfied, Neutral or worse	$\text{Log} \left(\frac{(P1+P2+P3+P4)}{(P5)} \right)$	A + 3D	B	4

In output

Text meaning	Formula	Value	Value_1	Simple sequence
Very Dissatisfied	$\text{Log} \left(\frac{P1}{(P2+P3+P4+P5)} \right)$	A	B-3x	1
Dissatisfied or worse	$\text{Log} \left(\frac{(P1+P2)}{(P3+P4+P5)} \right)$	A + D	B-2x	2
Neutral or worse	$\text{Log} \left(\frac{(P1+P2+P3)}{(P4+P5)} \right)$	A + 2D	B-X	3
	$\text{Log} \left(\frac{(P1+P2+P3+P4)}{(P5)} \right)$	A + 3D	B	4

Analysis of Maximum Likelihood Esti

Parameter		DF	Estimate
Intercept	03_Very_Likely	1	-4.2983
Intercept	02_Somewhat_Likely	1	-2.2029
At_Least_One_Parent_		1	1.0478
Undergraduate_Instit		1	-0.0585
GPA		1	0.6156

D

A

The left side kind of output, you can think that intercept is like D and Other part is like A

Solution

- **OLS not enough** - The coefficients in the linear combination cannot be consistently estimated using ordinary least squares.
- **MLE** - They are usually estimated using maximum likelihood.
- **Iterative process** - The maximum-likelihood estimates are computed iteratively

$$P(Y = 2) = \left(\frac{1}{1 + e^{-(a_2 + b_1 x_1 + b_2 x_2 + b_3 x_3)}} \right)$$

Maximum
Probability will
define the class

$$P(Y = 1) = \left(\frac{1}{1 + e^{-(a_1 + b_1 x_1 + b_2 x_2 + b_3 x_3)}} \right) - P(Y = 2)$$

$$P(Y = 0) \Rightarrow 1 - P(Y = 1) - P(Y = 2)$$